ELLENBERGER'S
COURSE OF
ARITHMETIC.

THE

COURSE OF ARITHMETIC

AS TAUGHT IN THE

PESTALOZZIAN SCHOOL,

WORKSOP.

BY J. L. ELLENBERGER.

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PREFACE.

So many works on Arithmetic have been published, that the present treatise would appear superfluous, did it not exhibit a new and improved method of treating that important subject.

The majority of our predecessors in the path we are now following, appear to have had in view the convenience of the master more than the improvement of the pupil. They have imagined it an easier task to make a boy learn a rule than to develop his mental powers. We differ entirely from such a doctrine, and wish the *master* to be the *ruling* spirit of the class, and not the book which is used; he must be at the helm, and not carried along like a common passenger.

There is nothing empirical, nothing which has a tendency to "cram," in the plan we here introduce. Nature has been our guide. Profiting by the natural disposition which every child possesses of becoming interested about objects which his mind can grasp, we have, by a well graduated method, extended the field of his observation, and led him from the known to the unknown, from the easy

to the difficult. He is made to know the reason of everything he does, his memory is not loaded with rules which have not been approved by his intelligence, he is taught to discover them from established truths: and to illustrate their application, interesting examples are given.

Every question is solved on its own data, and every solution contains a reasoning process, by which it becomes a demonstration, and thus arithmetic is made a captivating system of practical logic. The pupil is in this manner trained to learn ideas, not words; his memory is not made the recipient—the instrument, *par excellence,* to make him pass through his lesson without stumbling; but his intellectual faculties are called forth and exercised.

In the production of this course we do not lay claim to much originality; for wherever we have found useful hints suitable to our purpose, they have been incorporated in the work, but the reader will find many new suggestions and applications. We have extended the scope of arithmetic by applying it to several parts of Natural Philosophy, Chemistry, Mechanics, &c., sciences which become every day more important.

For the convenience of instructors the Answers have been printed separately, as the author is fully convinced that to have introduced them into the body of the work the progress of the pupil would have been retarded.

We cannot recommend too strongly to Masters the necessity of inducing each youth to make problems for himself; at first they will be little else than copies of those in the book, but gradually they will throw off all semblance of imitation, and become quite original. Pupils rapidly acquire an astonishing degree of proficiency in this kind of work.

The tendency of the human mind at the present time is eminently towards improvement, and we see daily our knowledge extended, both in the material and intellectual world; so that the mass of information which a man must possess in order to be a worthy and useful member of society, requires that from his youth, the mind be rightly disciplined, that the food it receives be administered with prudence and caution; and this in order to develop intellectual power, to adorn the understanding, and to create an inextinguishable thirst for knowledge. This has been the intention of the author, as far as the subject would allow, and if he has succeeded in leading the student along "the royal road," he will be amply compensated.

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ERRATA.

Page 17, Exercises 4, for 100076709 *read* 1000076709.

Page 30, Art. 87, for whether a divisor *read* whether a division.

Page 49, line 10 from bottom, for $8 = \cancel{2} \times 2 \times \cancel{2}$ *read* $8 = \cancel{2} \times \cancel{2} \times \cancel{2}$.

Page 53, Exercise 4, for $1 - \frac{2}{3} \div 4\frac{1}{5} -$ &c., *read* $1 - \frac{2}{3} + 4\frac{1}{5} -$ &c.

Page 57, Exercise 10, for I paid £$\frac{7}{8}$ *read* I paid $\frac{7}{8}$.

Page 63, for Art. 175 *read* 165.

Page 66, line 13 from bottom, for $8\dot{3}$ *read* $8.\dot{3}$.

Page 76, Art. 203, for is integer *read* is an integer.

,, Art. 204, line 9 from bottom, for decimal *read* decimals.

Page 82, Exercises 5, for $-.\dot{0}\dot{8}\dot{1}$ of £3.5, *read* $+.\dot{0}\dot{8}\dot{1}$ of £3.5.

Page 111, line 16 from bottom, for 252 *read* 253.

,, line 13 from bottom, for £21 4s. 6d. *read* £2 14s. 6d.

Page 113, for Art. 253 *read* 254.

Page 120, line 5 from top, for $\frac{1}{1}\frac{3}{1}\frac{4}{2}$ *read* $\frac{1}{2}\frac{3}{1}\frac{4}{1}$.

Page 127, Exercises, 8, for £2477. 10s. 10d. *read* £2377. 10s. 10d.

Page 190, in the Malta money table, for Garni, *read* Grani.

Page 203, line 5 from bottom, for 38 Russians in 13 days, *read* 38 Prussians in 13 days

Page 266, Ex. 11, for The areas of &c., *read* The diameters of &c.

Page 279, Ex. 46, for an atom of Phosphorus, *read* 2 atoms of Phosphorus.

Page 283, Ex. 97, for 24 square feet, *read* 15 square feet.

,, Ex. 97, for 784 pores *read* 2784 pores.

COURSE OF ARITHMETIC.

PART I.

NOTATION AND NUMERATION.

1. A *line* admits of lengthening or shortening; a *surface* admits of extension or diminution; a *weight* allows its being made heavier or lighter; *time* allows of increase or decrease, so does *motion*.

2. *Quantity* or *magnitude* is anything which will admit of increase or decrease. For instance, lines, surfaces, weight, time, motion are *quantities*.

3. The science by which we become acquainted with the properties of quantity is called *mathematics*.

4. If we were told, that in a certain house, there is a room twenty feet long, sixteen broad, and twenty high, we should at once form a correct idea of it, with regard to its dimensions, because we had a previous knowledge of the length of the *foot*. *One foot* is here called the *unit* or *unity*.

Suppose a piece of cloth contain twelve yards; here *one yard* is the unit; in a basket there are eight pebbles, *one pebble* is the unit, and so on.

5. Therefore, in mathematics, the unit is a measure of any kind, arbitrarily taken, to which we refer every quantity of the same kind.

A collection of units of the same kind constitutes a *number*, thus: ten horses, sixteen yards, forty houses, &c., are numbers.

6. When numbers are considered in a general sense, without referring them to any particular thing, they are called *abstract*, as four, seven, twelve, &c., but when applied to particular objects, as two pounds, ninepence, fourteen yards, they are termed *concrete* numbers.

7. To represent numbers, to express them, to give the means of computing by them, and to apply them to practical purposes, constitute a branch of mathematics called *arithmetic*.

8. Numbers may be expressed in two different ways, by words and by signs or characters, called *figures* or *digits*.

9. The method of expressing any number by figures is called *notation*, and that of reading numbers so expressed, *numeration*.

10. Words taken from the Saxon are used for the English names of numbers; they are: one, two, three, four, five, six, seven, eight, nine, ten.

11. It is agreed to consider the number ten as a unit of the second order, which is called tens, and to proceed with the tens as we did from one unit to nine units; but, for the sake of brevity,—instead of these expressions: one tens, two tens, three tens, four tens, five tens, six tens, seven tens, eight tens, nine tens—we say: ten, twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety.

12. Between ten and twenty, there are nine numbers: ten-one, ten-two, ten-three, ten-four, ten-five, ten-six, ten-seven, ten-eight, ten-nine, which appellations have been changed into eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen.

13. Likewise, between twenty and thirty, there are also nine numbers, and we say: twenty-one, twenty-two, twenty-three, twenty-four, twenty-five, twenty-six, twenty-seven, twenty-eight, twenty-nine; thirty-one, thirty-two...thirty-nine; and so on, as far as ninety-nine. Therefore, ninety-nine is the largest number containing tens and units only.

14 Now, if nine tens and nine units, or ninety-nine, be increased by one unit, we obtain ten tens, or one hundred, which commences a third order of units, called hundreds. The hundreds ascend also from one to ten; but, for shortness, we use: one hundred, two hundred, and so on, to nine hundred. As there are ninety-nine numbers between any one hundred and the following, by adding successively the first hundred numbers to each, we have: one hundred and one, one hundred and two, &c.; two hundred and one, two hundred and two, &c.; three hundred and one, three hundred and two, &c.; and so on, to

nine hundred and ninety-nine, the largest number containing hundreds, tens, and units only.

15. Nine hundred and ninety-nine, increased by one unit, gives a collection of ten hundreds, or one thousand, forming the units of thousands, or of the fourth order. Putting successively before thousand the nine hundred and ninety-nine first numbers, we have: one thousand, two thousand, three thousand, &c.; ten thousand, eleven thousand, twelve thousand, &c.; twenty thousand, thirty thousand, &c.; one hundred thousand, two hundred thousand, &c.; and so on, to nine hundred and ninety-nine thousand. One tens of thousands gives the unit of the fifth order; one hundred thousands the unit of the sixth order.

Now, by placing between two consecutive numbers of thousands, such as twelve thousand and thirteen thousand, all the numbers less than one thousand, it is evident that every number, as far as nine hundred and ninety-nine thousand nine hundred and ninety-nine, will be expressed.

16. Increased by one, the last number becomes ten hundred thousand, or a thousand thousands, called a million, being the unity of the seventh place. It has also its units, tens, hundreds.

In the same manner as a million is the collective name of a thousand thousands, so is a billion the name given to that of a thousand millions; a trillion signifies a thousand billions; and so on, to quadrillions, quintillions, &c.

17. Hence we have the following table :—

The first order of units are briefly called			units.
Second	,,	,,	tens of units.
Third	,,	,,	hundreds of units.
Fourth	,,	,,	units of thousands.
Fifth	,,	,,	tens of thousands.
Sixth	,,	,,	hundreds of thousands.
Seventh	,,	,,	units of millions.
Eighth	,,	,,	tens of millions.
Ninth	,,	,,	hundreds of millions.
Tenth	,,	,,	units of billions.
Eleventh	,,	,,	tens of billions,
Twelfth	,,	,,	hundreds of billions.

And proceeding thus, to reckon by trillions, quadrillions, quintillions, sextillions, &c., we are enabled to write down every imaginary number.

18. Therefore, the above system of numeration is based upon this two-fold property, that: ten units of one order make a unit of a higher order, and that three orders of units form a unit of a higher period, which, on that account, is called *ternary period.*

19. This system is termed the *decimal* system, because the number ten is the basis of it.

20. Almost every nation on the earth has adopted the decimal system, probably because man used his fingers and thumbs at first, when required to reckon. Why should not, likewise, the joints of the fingers have given rise to the three orders—units, tens, hundreds—of which each period is composed? Whatever be the origin of this system, it is certainly a most admirable invention, and the author, though unknown, deserves our gratitude.

21. This method, remarkable both on account of its simplicity and elegance, is not yet universally used in this country, but is likely to become so in time. The English having found that six orders of units preceded the millions, judged it most consistent to make a unit of a superior period, and for this reason it is called *sextuple period.* The two methods agree as far as hundreds of millions, and it is seldom necessary to use larger numbers. The subjoined table illustrates this method:—

The first order of units are called			units
Second	,,	,,	tens
Third	,,	,,	hundreds
Fourth	,,	,,	thousands
Fifth	,,	,,	tens of thousands
Sixth	,,	,,	hundreds of thousands
Seventh	,,	,,	units of millions
Eighth	,,	,,	tens of millions
Ninth	,,	,,	hundreds of millions
Tenth	,,	,,	thousands of millions
Eleventh	,,	,,	tens of thousands of millions
Twelfth	,,	,,	hundreds of thousands of millions
Thirteenth	,,	,,	units of billions
Fourteenth	,,	,,	tens of billions
Fifteenth	,,	,,	hundreds of billions
Sixteenth	,,	,,	thousands of billions
Seventeenth	,,	,,	tens of thousands of billions
Eighteenth	,,	,,	hundreds of thousands of billions

The succeeding periods being called trillions, quadrillions, &c., as we have before observed.

22. The tediousness of writing down words in ordinary language is so apparent as to have early induced men to substitute signs for words. The signs, figures, or digits which are used take the names of the first nine numbers, they are:—

one,	two,	three,	four,	five,	six,	seven,	eight,	nine.
1,	2,	3,	4,	5,	6,	7,	8,	9.

23. These figures appear to have had their origin in India, and were introduced into Europe by the Arabs, in the tenth century. The learned Frenchman, Gerbert, afterwards Pope Sylvester the second, was the first who borrowed them from the Arabs.

24. In order to express every possible numerical quantity with few figures, (as had been done with few words) it was agreed that each digit placed on the left of another should represent a unit of an order immediately superior: thus, to represent the number fifty-four, which consists of four units and five tens, we write down 54. In 456, 4 expresses four units of the third order; 5, five units of the second order; and 6, six units of the first order. The number seven thousand six hundred and eighty-four is thus represented: 7684.

25. The number fifty, which consists of five units of the second order, and none of the first, is thus expressed: 50. The number seven hundred and eight containing no tens, is thus expressed: 708. The number four thousand, 4,000. The number ninety thousand four hundred and six, 90,406. Seven millions four thousand and sixty, 7,004,060.

26. A cipher, or zero, having no value, becomes necessary when any order of units is wanting. It might be placed at the beginning of a number, but would have no meaning, for 034 is the same as 34.

27. Therefore, by means of these ten signs, nine figures, or digits, and zero, any number whatever can be expressed.

28. A digit admits of two values, one *absolute* and the other *relative*. First, it has an absolute value when it expresses units of the first order only, as 1, 2, 3, 4, &c.; second, in any other case a digit has a relative value, as 5 in 54; in 506; in 5,559, &c.

29. The following table will exemplify this truth, and the pupil can point out the different values which a digit admits of, as well as express the law of variation :—

1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9

30. Read and write in words the following numbers : 84 ; 973 ; 8542 : 73961 ; 340609 ; 8760464 ; 55478035 ; 341275648.

The larger the number is, the greater the difficulty of reading it ; but this difficulty vanishes if the number be divided into periods of three figures, commencing at the right, till not more than three remain. Then read each period by itself, beginning at the left, and call the periods after their units, thus : 464,789,485,321,543, which number is thus read, 464 trillions 789 billions 485 millions 321 thousands 543 units, or briefly 543.

31. Write down in figures the following numbers : sixty-four, two hundred and forty-nine, eight thousand four hundred and twenty-four, forty-six thousand seven hundred and sixteen, six hundred and forty thousand four hundred and ninety-four, five millions four hundred and forty-four thousand seven hundred and eighty-nine.

32. In the Roman notation, seven capital letters were used to express numbers, these letters were—

I.	representing the	number	1
V.	,,	,,	5
X.	,,	,,	10
L.	,,	,,	50
C.	,,	,,	100
D.	,,	,,	500
M.	,,	,,	1000

33. When any letter is followed by one of less or equal value, the expression is equal to their simple values taken together; but when a letter precedes one of greater value, together they express

the difference of their simple values; thus the series of natural numbers, from one to a thousand, is denoted:—

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.
1	2	3	4	5	6	7	8	9	10	11

XII.	XIII.	XIV.	XV.	XVI.	XVII.	XVIII.	XIX.
12	13	14	15	16	17	18	19

XX.	XXI., &c.	XXX.	XL.	L.	LX.	LXX.	LXXX.
20	21	30	40	50	60	70	80

XC.	C.	CC.	CCC.	CCCC.	D. or IↃ	DC.	DCC.
90	100	200	300	400	500	600	700

DCCC.	DCCCC.	M. or CIↃ, &c.
800	900	1000

34. To increase a number a thousand fold, a line is drawn over the top of a letter. Thus, 5000 is written, $\overline{\text{V}}$ or IↃↃ; 70000 is written, $\overline{\text{LXX}}$.; 2001000 is written, $\overline{\text{MM}}$M.

35. The Roman characters are used at the present day, first, to number the chapters and the several parts into which books are divided; sometimes to show the year of publication of a work, and the paging of prefaces; secondly, in inscriptions for monuments and in epitaphs; thirdly, in numismatics, the science of medals and coins.

36. EXERCISES IN NUMERATION.

Write down in words the following numbers:—5, 8, 12, 17, 24, 26, 37, 49, 76, 87, 90, 99, 101, 241, 176, 087, 008, 245, 574, 809, 977, 047, 999, 1005, 1014, 1346, 1787, 2007, 3745, 9846, 10609, 10428, 82307, 110349, 137008 540423, 837457, 806004, 7640006, 7607760, 6900070, 24803765, 900706539, 6476300088, 807050700603.

37. EXERCISES IN NOTATION.

Express the following numbers in figures:—Forty-five, three hundred and sixty-five, six hundred and eight, one thousand and seventy-five, three thousand nine hundred and nine, seventy-eight thousand and seven, eight hundred and four thousand four hundred and fifty-four, five millions three hundred and forty thousand nine hundred and eleven, three hundred and four millions seven hundred and forty thousand four hundred,

eighty-four billions seven millions three hundred thousand and ninety.

38. EXERCISES IN ROMAN NOTATION.

14, 28, 74, 80, 96, 99, 107, 194, 345, 609, 7000, 9080, 5704, 5555, 60000, 33333, 800000, 659186, 999999.

39. Express in Arabic figures:—XV. XIX. XXIV. XXXVII. XCIX. LV. LXXXI. CL. CXC. CCCX. DLX. DXCIV. MDCCXCIX. $\overline{\text{VIII}}$. $\overline{\text{LX}}$DCVI. $\overline{\text{LV}}$DLV. $\overline{\text{DCCLXXVII}}$DCCLXXVII.

PART II.

ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION.

40. Every process in Arithmetic consists in the increase or diminution of numbers.

41. These processes are various, but they can be reduced to four, which are called *fundamental*, because all the others depend on them.

42. The fundamental operations are: Addition, Subtraction, Multiplication, and Division, the last two being shorter methods of the first and second.

ADDITION.

43. I had in my purse fifteen shillings; my father gives me seven shillings and my mother five shillings more. How many shall I have altogether?

To find the answer I have *to put* these three numbers of shillings together, or *to add* them up; and the process is called *addition*.

44. Therefore, *addition is the process of collecting several numbers together.*

45. The number resulting from addition is called the *sum.*

46. The sum of 6 and 4 is 10, which is expressed: $6+4=10$. The sign $+$ signifies *plus* or *and;* and the sign $=$ is called the *sign of equality*, denoting that the quantity on the left is equal to that on the right.

47. PREPARATORY EXERCISES.

1+1= 2	2+1= 3	3+1= 4	4+1= 5
1+2= 3	2+2= 4	3+2= 5	4+2= 6
1+3= 4	2+3= 5	3+3= 6	4+3= 7
1+4= 5	2+4= 6	3+4= 7	4+4= 8
1+5= 6	2+5= 7	3+5= 8	4+5= 9
1+6= 7	2+6= 8	3+6= 9	4+6=10
1+7= 8	2+7= 9	3+7=10	4+7=11
1+8= 9	2+8=10	3+8=11	4+8=12
1+9=10	2+9=11	3+9=12	4+9=13

Let it be repeated and arranged in the form of a table, like the following :—

	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	18	14	15	16	17	18
10	11	12	13	14	15	16	17	18	19
11	12	13	14	15	16	17	18	19	20
12	13	14	15	16	17	18	19	20	21

The sum of two numbers in the first vertical and the first horizontal column is found at the concourse of these two columns. The learner must afterwards be questioned in different order upon this exercise.

In the following exercise, add up the numbers outside the brackets to those that are within; let it be done both in writing and orally :—

$$10\left\{\begin{array}{l}+4=\\+6=\\+3=\\+7=\\+5=\\+9=\end{array}\right.\quad 11\left\{\begin{array}{l}+6=\\+4=\\+2=\\+7=\\+5=\\+9=\end{array}\right.\quad 12\left\{\begin{array}{l}+3=\\+7=\\+5=\\+2=\\+8=\\+6=\end{array}\right.\quad 13\left\{\begin{array}{l}+6=\\+5=\\+9=\\+7=\\+4=\\+3=\end{array}\right.\quad 14\left\{\begin{array}{l}+6=\\+4=\\+7=\\+8=\\+5=\\+9=\end{array}\right.$$

This is to be continued until the pupil answers readily questions of the same kind.

Determine the different methods of expressing 24, with the sum of two numbers. Also, 42 and 54.

Express 18 with the sum of three numbers, in all the possible ways. Likewise, 27 and 35.

What is the sum of

$$
\begin{aligned}
3+4+6&=\\
5+6+7&=\\
8+9+3&=\\
7+6+6&=\\
7+8+9&=\\
5+4+3+6&=\\
7+2+4+9&=\\
8+6+6+7&=\\
6+4+7+9&=\\
3+4+7+9+2&=\\
5+6+6+4+8&=\\
7+8+6+7+9&=\\
5+6+7+4+7+3&=\\
8+6+6+5+5+9&=
\end{aligned}
$$

&c., &c.

The same exercise ought to be also proposed under the following form, where the pupil is to add up the vertical columns in each square, and afterwards also the horizontal rows :—

4	6	7	8
3	4	2	6
5	4	5	6
3	7	6	4

5	3	4	6	7
2	3	3	5	6
6	5	7	3	9
7	6	4	5	7
5	8	6	7	8

7	3	4	5	2	4	8	9	6	7
6	1	3	6	2	7	4	3	9	8
7	3	7	4	5	7	3	4	6	7
4	2	5	6	7	3	6	4	8	7
3	6	3	9	4	7	5	2	7	3
2	3	9	4	7	6	3	5	6	4
5	4	7	8	9	3	5	4	6	7
3	5	4	6	4	3	9	6	4	3
8	6	2	4	5	3	4	9	8	5
7	9	6	5	7	8	9	4	5	6

48. To find the sum of two numbers, 36 and 45, put them under the form $30+6$ and $40+5$; the required sum is equal to $30+40+6+5$. Now, three tens and four tens are seven tens, and six units and five units are eleven units; therefore, the sum is $70+11=70+10+1=81$.

Whence we perceive that the tens are to be put together

separately, and likewise the units. The same method is followed when the numbers are hundreds, thousands, &c.

Example: What is the sum of $4631+13524+546+78$?

$$\begin{aligned} 4631 &= 4000+600+30+1 \\ 13524 &= 13000+500+20+4 \\ 546 &= 500+40+6 \\ 78 &= 70+8 \end{aligned}$$

Adding up, separately, the units, the tens, the hundreds, &c., we have 17 thousands+16 hundreds+16 tens+19 units, or $17000+1600+160+19$; the same process being repeated for these numbers, we have $1800+700+70+9$, or 18779.

This operation is much shortened by the following method:—

$$\begin{array}{r} 4631 \\ 13524 \\ 546 \\ 78 \\ \hline 18779 \end{array}$$

The numbers are placed under each other, with the units of the same order in the same vertical column; then adding each number thus: $1+4=5$, $5+6=11$, $11+8=19$ units or one tens +nine units, we set down the nine units, and the one tens we carry mentally to the next or tens column, which sum is 17 tens, or one hundreds and seven tens; the seven is set down in the place of the tens, and the one hundreds are carried to the third column, and so on, which explains why we begin with the right hand column.

49. From the definition of number, it is evident that, to be added up, numbers must be of the same kind, unless they have a common name. Example: 5 lions+4 bears+6 foxes=15 animals.

50. To ascertain whether an addition is correct, the operation is repeated, beginning at the bottom of each column; and if the results agree, it is *probable* the work is right. Or, add the figures of each column, omitting the upper row, then to this sum add the upper row; if the result is equal to the previous answer, the work is *probably* correct.

51. EXERCISES.

1. On Monday, 7645 persons passed over a bridge; on Tuesday, 8965; on Wednesday, 4905; on Thursday, 3679; on

Friday, 6844; on Saturday, 7638; on Sunday, 2944. How many persons crossed it during the whole time?

It is evident that we have here to find the sum of the seven given numbers:—

$$\begin{array}{rl}
7645 & \\
8965 & \\
4905 & \\
3679 & \\
6844 & \\
7638 & \\
2944 & \\
\hline
42620 & \text{number of persons.} \\
\hline
\end{array}$$

$$\text{Proof}\left\{\begin{array}{rl}
34975 & \text{number on the last six days.} \\
7645 & \text{number on the first day.} \\
\hline
42620 & \text{which is equal to the sum required.}
\end{array}\right.$$

2. $86420135+9873546+998739+6978+407369=$?

3. $439+2704+1743206+290704+7680+15647=$?

4. Find the sum of thirty thousand seven hundred and seven + forty-six thousand nine hundred and seventy-five + four hundred and sixty-four thousand eight hundred and seventy-eight + three thousand and seven + nine millions three hundred and forty-nine thousand three hundred and fifty-six.

5. In a school, the pupils are at study in four rooms; in the first, there are 128 pupils; in the second, 84; in the third, 93; and in the fourth, 65. The number of pupils in the school is required.

6. How many times does a clock (which only strikes the hour) strike, whilst the hour hand goes round the dial?

7. In what year did a man die who was born in 1766, and died at the age of 85?

8. A person is born in 1793; in what year will he be 74 years of age?

9. Virgil was born near Mantua, 70 years B.C. How many years ago is it?

10. Noah's flood occurred 2356 years B.C. How long is it since that event?

11. An army which consists of 24647 infantry and 3249 cavalry, is increased by 6473 infantry and 2464 cavalry. What is the whole number of men?

12. From January 19th, 1852, to June 17th, inclusive, how many days?

13. The smaller of two numbers is 349, the larger is 124 more. What is the sum of both numbers?

14. Required, the sum of two numbers, such, that when 48 is taken away from one, and 244 from the other, the remainder is 173.

15. A father is 49 years older than his son, who is 12 years old; the daughter is five years older than the son, and the mother is 23 years older than the daughter. What are the ages of the father and mother?

16. The land on the earth's surface is divided into five great parts; Europe contains 3720000 square miles; Asia, $17\frac{1}{2}$ millions; Africa, 12 millions; America, 15050000; Australia, four millions. The number of square miles of the land is required.

17. The first of five numbers is 247, and the four others increase respectively by 48, 49, 50, and 51. Required, those numbers, and their sum.

18. Six partners, A, B, C, D, E, and F, divide their profits; A gets £2458, B gets as much as A and D together, C as much as B and F, D gets £1500, E as much as C and D, and F receives £800. Find each partner's share and the whole profit.

SUBTRACTION.

52. In an orchard, there are 36 trees; if 13 be cut down, how many will be left.

Here we are led, by the meaning of the question, to take away, or subtract 13 trees from 36 trees, and the operation is called a *subtraction;* the resulting number is called *remainder*, *difference*, or *excess*.

53. Therefore, *to subtract is to find how much one number exceeds another.* The greater number given is called the *minuend*, and the less, *subtrahend*.

54. The difference of 9 and 4 is 5, which is briefly expressed, $9-4=5$. The sign — meaning *minus* or *less*.

55. PREPARATORY EXERCISES.

$19-1=18$	$13-1=12$	$7-1=6$
$18-1=17$	$12-1=11$	$6-1=5$
$17-1=16$	$11-1=10$	$5-1=4$
$16-1=15$	$10-1=9$	$4-1=3$
$15-1=14$	$9-1=8$	$3-1=2$
$14-1=13$	$8-1=7$	$2-1=1$
		$1-1=0$

$19-2=17$	$19-3=16$	$19-4=15$
$18-2=16$	$18-3=15$	$18-4=14$
$17-2=15$	$17-3=14$	$17-4=13$
&c., &c.	&c., &c.	&c., &c.

$24-4=20$	$20-4=16$	$16-4=12$
$48-6=42$	$42-6=36$	$36-6=30$
$56-8=48$	$48-8=40$	$40-8=32$
$86-9=77$	$77-9=68$	$68-9=59$ &c.

$$17\left\{\begin{array}{l}-2= \\ -4= \\ -6= \\ -7= \\ -5= \\ -3=\end{array}\right. \qquad 23\left\{\begin{array}{l}-7= \\ -6= \\ -3= \\ -8= \\ -4= \\ -5=\end{array}\right. \qquad 37\left\{\begin{array}{l}-8= \\ -9= \\ -6= \\ -3= \\ -4= \\ -5=\end{array}\right. \text{ \&c.}$$

56. From 7569 take 4354.

Write the smaller number below the larger, placing units under units, tens under tens, &c.

$$\begin{array}{rcl} 7569 & \text{or} & 7000+500+60+9 \\ \underline{4354} & \text{or} & \underline{4000+300+50+4} \\ 3215 & \text{or} & 3000+200+10+5 \end{array}$$

Now,	4 units	from 9 units	leaves 5 units
	5 tens	from 6 tens	leaves 1 tens
	3 hundreds	from 5 hundreds	leaves 2 hundreds
	4 thousands	from 7 thousands	leaves 3 thousands

Therefore, the difference of $7569-4354=3215$.

57. The difference of 93546 and 37874 is required.

The numbers being placed as before—

93546	=9 tens of thous.	+3 thous.		+5 hunds.		+4 tens		+6 units	
37874	=3 ,, ,,	+7	,,	+8	,,	+7	,,	+4	,,
55672	5 ,, ,,	+5	,,	+6	,,	+7	,,	+2	

Beginning at the right, take 4 units from 6 units, there remain 2 units; 7 tens from 4 tens cannot be done, but if we add mentally 1 hundreds, which is = 10 tens to the 4 tens, we shall now have 7 tens from 14 tens, there remain 7 tens, which are written down in the tens' place; proceeding to the next figures, it must be observed that the 8 hundreds must be increased by 1 hundreds, (since 1 hundreds has been added to the top row) then say 9 hundreds from 5 hundreds, which cannot be done, in the same manner add 1 thousands, or 10 hundreds, to the 5 hundreds, and we have now 9 hundreds from 15 hundreds, leave 6 hundreds, which are set down in the hundreds' place; for the same reason as before, we have 8 thousands from 13 thousands, leave 5 thousands, which are set down; and, lastly, 4 tens of thousands from 9 tens of thousands, leave 5 tens of thousands.

The operation, with the artifices used, might be put under this form :—

Tens of Thous.	Thous.	Hunds.	Tens.	Units.
9	13	15	14	6
4	8	9	7	4
5	5	6	7	2

58. The inference drawn from this operation is, that *the difference of two numbers is not altered by adding the same number to the minuend and the subtrahend.* It is easily shown that this is also true when the same number *is subtracted* from both.

For instance, 18—13 = 5, adding 7 to each, we have 25—20 = 5; subtracting 7 from each, we have 11—6 = 5.

59. Let it be required to take 234639 from 700607.

Minuend	700607
Subtrahend	234639
Remainder	465968

9 units from 17 units leave 8 units; 4 tens from 10 tens leave 6 tens; 7 hundreds from 16 hundreds leave 9 hundreds;

5 thousands from ten thousands leave 5 thousands; 4 tens of thousands from 10 tens of thousands leave 6 tens of thousands; and 3 hundreds of thousands from 7 hundreds of thousands leave 4 hundreds of thousands. Therefore, the greater number exceeds the lesser by 465968.

60. Enough has been said to show that subtraction might be defined to be *an operation in which it is required to find a number called difference, which, being added to a proposed number, called subtrahend, the sum is equal to another proposed number, called minuend.*

61. Therefore, to ascertain if a subtraction be correct, add the difference to the subtrahend; if the sum be equal to the minuend, it is presumed the work is right.

62. Another proof is: subtract the difference from the minuend, and if the remainder be equal to the subtrahend, the work, in all probability, is correct; for the minuend may be considered as the sum of the subtrahend and difference.

63. EXERCISES.

1. The authorised version of the Old Testament contains 592439 words, and the New 181253. How many more words are there in the Old than in the New?

Operation.	Proof by Addition.	Proof by Subtraction.
592439 minuend	181253 subtrahend	592439 minuend
181253 subtrahend	411186 difference	411186 difference
411186 difference	592439 minuend	181253 subtrahend

2. 97—43; 549—327; 784—376; 2947—578; 14748—13942.

3. 340639—247546; 461201201—359758467.

4. 2122090046—1217484566; 100076709—934567788.

5. 7900000000—4756739523.

6. Napoleon was born in 1769, and died in 1821. How long did he live?

7. The University of Cambridge was founded A.D. 915. How many years have elapsed from that date till now?

8. The expenses of an establishment are £74656, and the receipts £93758; the profits are to be ascertained.

9. Printing was invented in 1449. How many years from that time to 1853?

10. In summer, when the earth is in its aphelion, or greatest distance from the sun, that distance is about 95600000 miles; and in winter, in its perihelion, or least distance, it is about 93500000 miles. How much nearer the sun is the earth in winter than in summer?

11. An empty box weighs 76lbs., and when filled with goods, 464lbs. The weight of the goods is required.

12. Halley's comet, which appeared in 1835, was 76 years invisible. When was it seen previously?

13. If A lived 36 years longer he would be 100 years old. Find A's age.

14. The greater of two numbers is 278, and the difference 169. What is the smaller number?

15. Find the greater of two numbers, the smaller of which is 111, and the sum 327.

16. A property was bought for £9867, and sold for £8978. What is the loss?

17. If 18 years were taken from the age of the father, and added to his son's age, they would each be 30 years old. Find the age of each.

18. Two persons started from two towns, 347 miles apart, when they met one had travelled 198 miles. How far had the other travelled?

19. If I had £500 more, I could repay £1200 which I owe, and have £19 over. What sum have I?

20. The father is born in 1796, the mother in 1801, the son in 1823, and the daughter in 1827. Find the age of each person, the differences, and the sum of their ages in 1853.

MULTIPLICATION.

64. A person spends seven shillings daily. How much will he spend in six days?

Since in one day he spends 7 shillings, in two days his expenses will be $7+7$ shillings, in three days $7+7+7$ shillings, &c.; therefore, it is evident that we have to repeat as many times as there are days, or as many times as there are units in 6. Then the expenses are 6 times 7 or 42 shillings, which is thus expressed, $6\times 7=42$. The same result would have been obtained had we added 7 shillings 6 times to itself, as follows: $7+7+7+7+7+7=42$. *To repeat a number several times is to multiply it.*

65. Therefore, *to multiply is to repeat a number called multiplicand as many times as there are units in another, called multiplier.*

The result of a multiplication is called *product.*

In the previous question, which is the multiplicand, which the multiplier, and which the product?

66. The multiplier and multiplicand are also named *terms* or *factors.*

67. PREPARATORY EXERCISES.

$1\times2=2$	$1\times3=3$	1×4	1×5	1×8	1×11
$2\times2=4$	2×3	2×4	2×5	2×8	2×11
$3\times2=6$	3×3	3×4	3×5	3×8	3×11
4×2	4×3	4×4	&c.	&c.	&c.
5×2	5×3	5×4	1×6	1×9	12×11
6×2	6×3	6×4	2×6	2×9	
7×2	7×3	7×4	3×6	3×9	1×12
8×2	8×3	8×4	&c.	&c.	2×12
9×2	9×3	9×4	1×7	1×10	3×12
10×2	10×3	10×4	2×7	2×10	&c.
11×2	11×3	11×4	3×7	3×10	12×12
12×2	12×3	12×4	&c.	&c.	

2×1	3×1	4×1	5×1	6×1	7×1
2×2	3×2	4×2	5×2	6×2	7×2
2×3	3×3	4×3	5×3	6×3	7×3
&c.	&c.	&c.	&c.	&c.	&c.
2×12	3×12	4×12	5×12	6×12	7×12

8×1	9×1	10×1	11×1	12×1
8×2	9×2	10×2	11×2	12×2
8×3	9×3	10×3	11×3	12×3
&c.	&c.	&c.	&c.	&c.
8×12	9×12	10×12	11×12	12×12

2 × 4	3 × 6	4 × 3	5 × 6	6 × 8	7 × 3
2 × 7	3 × 1	4 × 7	&c.	&c.	&c.
2 × 6	3 × 12	4 × 6			
2 × 3	3 × 6	4 × 10			
2 × 9	3 × 7	4 × 5			
2 × 11	3 × 2	4 × 9			
2 × 5	3 × 9	4 × 12			
2 × 12					

4 × 3	5 × 4	6 × 4	4 × 7	7 × 5	10 × 4
5 × 7	6 × 11	7 × 9	6 × 7	6 × 12	11 × 9
6 × 9	8 × 2	8 × 5	9 × 8	9 × 12	12 × 6
7 × 3	7 × 7	8 × 6	2 × 8	9 × 9	&c.

The learner will construct the following table, called Multiplication Table, or Pythagoras' Table, from its inventor. This he may do several times, in order to get very perfect in its contents. Let him write down the first horizontal and vertical lines, then repeating every figure in the horizontal line by the figures in the vertical one, the results will be the several horizontal lines.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

68. What is the product of 4 and 5? Answer, 20. Therefore, 4 and 5 are factors of 20.

Of what number are 6 and 8 the factors?
" " 8 " 7 "
" " 4 " 9 "

What are the factors of 8, 12, 15, &c.?

69. Such numbers as 5, 7, 19, &c., which have no factors, but unity and themselves, are named *prime numbers;* the others are called *multiples.*

70. The following table is very useful, and may be made by the student :—

Prime Numbers.	Multiples.	Factors.
1	$4=2\times 2$.	
2	$6=2\times 3$.	
3	$8=2\times 4$.	
5	$9=3\times 3$.	
7	$10=2\times 5$.	
11	$12=2\times 6$,	3×4
13	$14=2\times 7$.	
17	$15=3\times 5$.	
19	$16=2\times 8$,	4×4.
23	$18=2\times 9$,	3×6.
29	$20=2\times 10$,	4×5.
31	$21=3\times 7$.	
37	$22=2\times 11$.	
41	$24=2\times 12$,	3×8, 4×6.
43	$25=5\times 5$.	
47	$26=2\times 13$.	
53	$27=3\times 9$.	
59	$28=2\times 14$,	4×7.
61	$30=2\times 15$,	3×10, 5×6.
67	$32=2\times 16$,	4×8.
71	$33=3\times 11$.	
73	$34=2\times 17$.	
79	$35=5\times 7$.	
83	$36=2\times 18$,	3×12, 4×9, 6×6.
89	$38=2\times 19$.	
97	$39=3\times 13$.	
101	$40=2\times 20$,	4×10, 5×8.
103	$42=2\times 21$,	3×14, 6×7.
107	$44=2\times 22$,	4×11.
109	$45=3\times 15$,	5×9.
113	$46=2\times 23$.	
127	$48=2\times 24$,	3×16, 4×12, 6×8.
131	$49=7\times 7$.	
137	$50=2\times 25$,	5×10.
139	$51=3\times 17$.	

Multiples. **Factors.**

$52=2\times 26$, 4×13.
$54=2\times 27$, 3×18, 6×9.
$55=5\times 11$.
$56=2\times 28$, 4×14, 7×8.
$57=3\times 19$.
$58=2\times 29$.
$60=2\times 30$, 3×20, 4×15, 5×12, 6×10.
$62=2\times 31$.
$63=3\times 21$, 7×9.
$64=2\times 32$, 4×16, 8×8.
$65=5\times 13$.
$66=2\times 33$, 3×22, 6×11.
$68=2\times 34$, 4×17.
$69=3\times 23$.
$70=2\times 35$, 5×14, 7×10.
$72=2\times 36$, 3×24, 4×18, 6×12, 8×9.
$74=2\times 37$.
$75=3\times 25$, 5×15.
$76=2\times 38$, 4×19.
$77=7\times 11$.
$78=2\times 39$, 3×26, 6×13.
$80=2\times 40$, 4×20. 5×16, 8×10.
$81=3\times 27$, 9×9.
$82=2\times 41$.
$84=2\times 42$, 3×28, 4×21, 6×14, 7×12.
$85=5\times 17$.
$86=2\times 43$.
$87=3\times 29$.
$88=2\times 44$, 4×22, 8×11.
$90=2\times 45$, 3×30, 5×18, 6×15, 9×10.
$91=7\times 13$.
$92=2\times 46$, 4×23.
$93=3\times 31$.
$94=2\times 47$.
$95=5\times 19$.
$96=2\times 48$, 3×32, 4×24, 6×16, 8×12.
$98=2\times 49$, 7×14.
$99=3\times 33$, 9×11.
$100=2\times 50$, 4×25, 5×20, 10×10.
$102=2\times 51$, 3×34, 6×17.
$104=2\times 52$, 4×26, 8×13.

Multiples.	Factors.
105	$=5\times 21$, 3×35, 7×15.
106	$=2\times 53$.
108	$=2\times 54$, 3×36, 4×27, 6×18, 9×12.
110	$=2\times 55$, 5×22, 10×11.
111	$=3\times 37$.
112	$=2\times 56$, 4×28, 7×16, 8×14.
114	$=2\times 57$, 3×38, 6×19.
115	$=5\times 23$.
116	$=2\times 58$, 4×29.
117	$=3\times 39$, 9×13,
118	$=2\times 59$,
119	$=7\times 17$.
120	$=2\times 60$, 3×40, 4×30, 5×24, 6×20, 8×15, 10×12.
121	$=11\times 11$.
122	$=2\times 61$.
123	$=3\times 41$.
124	$=2\times 62$, 4×31.
125	$=5\times 25$.
126	$=2\times 63$, 3×42, 6×21, 7×18, 9×14.
128	$=2\times 64$, 4×32, 8×16.
129	$=3\times 43$.
130	$=2\times 65$, 5×26, 10×13.
132	$=2\times 66$, 3×44, 4×33, 6×22, 11×12.
133	$=7\times 19$.
134	$=2\times 67$.
135	$=5\times 27$, 3×45, 9×15.
136	$=2\times 68$, 4×34, 8×17.
138	$=2\times 69$, 3×46, 6×23.
140	$=2\times 70$, 4×35, 5×28, 7×20, 10×14.
141	$=3\times 47$.
142	$=2\times 71$.
143	$=11\times 13$.
144	$=2\times 72$, 3×48, 4×36, 6×24, 8×18, 9×16, 12×12.

71. A man gains £2347 yearly. How many pounds does he gain in six years?

The answer to this question might be found in adding six numbers equal to 2347, as it is done below:—

$$2347+2347+2347+2347+2347+2347=14082.$$

But we notice that we have taken 6 times the 7 units of the multiplicand, 6 times the four tens, &c. Therefore, we are led

to place the multiplier under the multiplicand, and to proceed as follows :—

$$\begin{array}{rl} 2347 & \text{multiplicand} \\ 6 & \text{multiplier} \\ \hline 14082 & \text{product} \end{array}$$

6 times 7 units are 42 units, or 4 tens and 2 units, set down 2 in the place of the units, and reserve 4 tens for the next place; 6 times 4 tens are 24 tens, and the 4 tens reserved are 28 tens, or 2 hundreds and 8 tens, set down the 8 tens and carry the 2 hundreds to the next place; 6 times 3 hundreds are 18 hundreds, and 2 are 20 hundreds, or 2 thousands and 0 hundreds, set down 0 in the hundreds' place, and reserve 2 thousands; 6×2 thousands$=12$ thousands, and 2 thousands are 14 thousands, set down 14. The product is 14082.

Therefore, this operation consists in multiplying each figure of the multiplicand by the multiplier, beginning with the units, and setting down the results under the figure multiplied by, reserving, as in addition.

72. Let it now be required to multiply 4598 by 365. The meaning of which is to repeat 4598, 5 times$+60$ times$+300$ times, and to add the results. Place the multiplier under the multiplicand, so that the units of the same order be under each other :—

$$\begin{array}{rl} 4598 & \\ 365 & \\ \hline 22990 & = 5 \times 4598 \\ 27588 & = 60 \times 4598 \\ 13794 & = 300 \times 4598 \\ \hline 1678270 & = 365 \times 4598 \end{array}$$

The product of 4598 and 5 is found as in (§ 71); to multiply 4598 by 6 tens, we proceed as if we multiplied by 6 units, taking care to place the first figure obtained in the tens' place, because 6 tens$\times 8$ units are 48 tens, or 4 hundreds$+8$ tens; 6 tens$\times 9$ tens$=54$ hundreds, and 4 hundreds reserved$=58$ hundreds, or 5 thousands$+8$ hundreds, &c.; lastly, to multiply 4598 by 3 hundreds, multiply as if it were by 3 units, writing the first figure in the column of the hundreds, because 3 hundreds$\times 8$ units are 24 hundreds, or 2 thousands$+4$ hundreds, &c. Add the partial products, and the sum is the product required.

From which it is inferred that we must multiply one of the factors by each figure of the other, placing the unit of each line in the place under the figure of the multiplier from which it came, and add the several lines together.

73. It has been explained in numeration, that the respective values of figures increase ten-fold as we proceed from the units' place towards the left hand. Let us take any number, 24, for instance, put a zero at the right hand, it becomes 240, that is to say, the units have become tens and the tens hundreds, or the number has acquired ten times its former value. If two zeros had been placed at the end of 24, the number would have one hundred times its previous value, and so on.

Therefore, to multiply a number by 10, 100, 1000, &c., place at the right hand of that number 1, 2, 3, &c., zeros.

74. Let us multiply 30 by 20. If we were to omit the zeros, and say $3 \times 2 = 6$, this result would be evidently 10×10, or 100 times too small, then affix both zeros at the end of the product and we have 600 for the answer. Therefore, when one or both factors have ciphers on the right hand multiply them together, omitting the ciphers, and then place on the right hand of the product as many ciphers as there are on the right hand of the factors.

75. Since $5 \times 7 = 7 \times 5$, or $14 \times 25 = 25 \times 14$, &c., we shall be able to ascertain that a multiplication is right by transposing the factors ; and if the product thus obtained agree with the product found previously, the operations have been correctly performed in both cases.

76. EXERCISES.

1. How many lines are there in a book containing 247 pages, and each page 18 lines?

Since each page contains 18 lines, 247 pages will contain 247×18 lines.

Multiplication.

$$\text{Factors}\begin{cases} 18 & \text{multiplicand} \\ 247 & \text{multiplier} \end{cases}$$

$$\begin{array}{r} 126 \\ 72 \\ 36 \\ \hline 4446 \end{array}$$

The book contains 4446 lines = product

Proof.

$$\begin{array}{r} 247 \\ 18 \\ \hline 1976 \\ 247 \\ \hline 4446 \end{array}$$

2. Sound moves about 1142 feet per second. How many feet will it move in 144 seconds?

By the question, we know that in one second sound travels 1142 feet; therefore, in 144 seconds it travels 144×1142 feet.

Multiplication.		Proof.
Factors $\begin{cases} 1142 \\ 144 \end{cases}$	multiplicand multiplier	144 1142
4568 4568 1142		288 576 144 144
164448	feet = product	164448

3. In a plantation there are 274 rows of trees, each containing 485 trees. How many trees are there in the plantation?

4. How many times does the hammer of a clock strike in 365 days, or one year, at 156 strokes per day?

5. One mile contains 1760 yards. How many yards are there from the earth to the moon, the distance being 240000 miles?

6. Perform the following multiplications:—24×762; 63×196; 5704×487; 7800×365; 3600×49200.

7. How many hours are there in 365 days, of 24 hours each?

8. The equator, like every other circumference, is supposed to be divided into 360 parts, or degrees, each containing 69 miles. How many miles is it round the earth?

9. Light travels with a velocity of 192000 miles per second, and the sun's light reaches us in 8 minutes 13 seconds, or in 493 seconds. What is our distance from the sun?

10. Two persons start at the same time, one from London and the other from Worksop; the first travels 18 miles per day, and the other 19 miles; they meet after travelling four days. How far are the two places from each other?

11. A person owns 79 horses, worth £26 each, on an average; and 347 head of cattle, valued at £13 each. What is the value of all?

12. A draper bought 347 yards of cloth, at 27 shillings per yard, and 276 yards at 18 shillings, for which he paid 12337 shillings. How much has he yet to pay?

13. Reckoning every year of 365 days 6 hours, how many hours is it since the birth of our Lord, which took place 1853 years ago?

14. A boy who is 14 years 8 months and 14 days old, wishes to know how many seconds he has lived; three years of his life were leap years; of the 8 months, 5 were of 31 days, 2 of 30 days, and the last of 28 days.

15. A wheel of a locomotive engine makes 25 revolutions in one second. How many revolutions will it make in 7 hours 17 minutes and 24 seconds? Also, how many yards has it run over, the wheel being six yards round?

16. A merchant employs 12 clerks, three at the rate of £240 annually, five at the rate of £180, and four at £80; 14 servants, eight of them at £22 each, on the average, and the six others at £15 each; for house rent, &c., he spends £1400; the house-keeping expenses amount to £2200. In one year he did business to the amount of £224600, upon which he laid out £206700. What were his profits at the year's end?

DIVISION.

77. If eight apples are to be parted, or divided equally between two persons, how many will each receive?

In this question it is proposed to make as many equal shares, or portions of the apples, as there are persons, or to divide 8 by 2, or to see how many times 2 is contained in 8, and the answer is 4.

Other questions similar to this might be proposed by the teacher, and solved mentally.

78. Therefore, *division consists in dividing numbers into equal parts*, or *in finding how many times one number is contained in another*.

79. The number which is divided is called the *dividend*, the number we divide by, the *divisor*, and the number resulting from the operation is named the *quotient*.

In the previous question, which number is the dividend, which the divisor, and which the quotient?

8, divided by 2, equal 4, is expressed thus: $8 \div 2$, or $\frac{8}{2} = 4$.

80. PREPARATORY EXERCISES.

$2\div2=$	$3\div3=$	$4\div4=$	$5\div5=$	$6\div6=$
$4\div2=$	$6\div3=$	$8\div4=$	$10\div5=$	$12\div6=$
$6\div2=$	$9\div3=$	$12\div4=$	$15\div5=$	$18\div6=$
$8\div2=$	$12\div3=$	$16\div4=$	$20\div5=$	$24\div6=$
$10\div2=$	$15\div3=$	$20\div4=$	$25\div5=$	$30\div6=$
$12\div2=$	$18\div3=$	$24\div4=$	$30\div5=$	$36\div6=$
$14\div2=$	$21\div3=$	$28\div4=$	$35\div5=$	$42\div6=$
$16\div2=$	$24\div3=$	$32\div4=$	$40\div5=$	$48\div6=$
$18\div2=$	$27\div3=$	$36\div4=$	$45\div5=$	$54\div6=$
$20\div2=$	$30\div3=$	$40\div4=$	$50\div5=$	$60\div6=$
$22\div2=$	$33\div3=$	$44\div4=$	$55\div5=$	$66\div6=$
$24\div2=$	$36\div3=$	$48\div4=$	$60\div5=$	$72\div6=$

$7\div7=$	$49\div7=$	$8\div8=$	$56\div8=$	$9\div9=$	$63\div9=$
$14\div7=$	$56\div7=$	$16\div8=$	$64\div8=$	$18\div9=$	$72\div9=$
$21\div7=$	$63\div7=$	$24\div8=$	$72\div8=$	$27\div9=$	$81\div9=$
$28\div7=$	$70\div7=$	$32\div8=$	$80\div8=$	$36\div9=$	$90\div9=$
$35\div6=$	$77\div7=$	$40\div8=$	$88\div8=$	$45\div9=$	$99\div9=$
$42\div7=$	$84\div7=$	$48\div8=$	$96\div8=$	$54\div9=$	$108\div9=$

81.

$\frac{4}{2}=$	$\frac{9}{3}=$	$\frac{10}{5}=$	$\frac{28}{4}=$	$\frac{63}{9}=$	$\frac{72}{8}=$	$\frac{81}{9}=$
$\frac{6}{3}=$	$\frac{18}{2}=$	$\frac{24}{3}=$	$\frac{40}{5}=$	$\frac{63}{7}=$	$\frac{72}{9}=$	$\frac{84}{7}=$
$\frac{6}{2}=$	$\frac{18}{3}=$	$\frac{24}{4}=$	$\frac{40}{10}=$	$\frac{64}{8}=$	$\frac{72}{12}=$	$\frac{96}{12}=$
$\frac{12}{4}=$	$\frac{18}{6}=$	$\frac{24}{2}=$	$\frac{48}{6}=$	$\frac{44}{4}=$	$\frac{72}{6}=$	$\frac{77}{7}=$
$\frac{12}{3}=$	$\frac{18}{9}=$	$\frac{24}{6}=$	$\frac{48}{4}=$	$\frac{36}{4}=$	$\frac{56}{8}=$	$\frac{88}{11}=$
$\frac{12}{6}=$	$\frac{27}{3}=$	$\frac{24}{8}=$	$\frac{48}{8}=$	$\frac{49}{7}=$	$\frac{45}{5}=$	$\frac{80}{8}=$
$\frac{12}{2}=$	$\frac{27}{9}=$	$\frac{32}{4}=$	$\frac{56}{7}=$	$\frac{54}{6}=$	$\frac{42}{7}=$	$\frac{99}{9}=$

82. Let it be required to divide 64 by 4.

There are various methods of solution, for instance, $\frac{64}{4}=\frac{60}{4}+\frac{4}{4}=\frac{40}{4}+\frac{20}{4}+\frac{4}{4}=10+5+1=16$, or $\frac{64}{4}=\frac{48}{4}+\frac{16}{4}=12+4=16$,

&c. But the best method of solution is this : $\frac{64}{4}=10+6=16.$ 4 is contained in 6 tens, 1 tens, and 2 tens over, which are added to the 4 units, 2 tens+4 units=24 units. 4 is contained in 24 units 6 units times.

Divide 16422 by 6.

$$\frac{16422}{6}=2737.$$

Solution: 6 is contained in 16 thousands 2 thousand times, and 4 thousands over, 4 thousands+4 hundreds=44 hundreds; 6 is contained in 44 hundreds 7 hundred times, and 2 hundreds over, 2 hundreds+2 tens=22 tens; 6 is contained in 22 tens 3 tens, and 4 over, 4 tens+2 units=42 units; 6 is contained in 42 units 7 times; therefore, the quotient is 2737.

83. EXAMPLES FOR PRACTICE.

$\frac{536}{4}$; $\frac{3846}{6}$; $\frac{7624}{8}$; $\frac{37134}{9}$; $\frac{493767}{9}$; $\frac{56772}{12}$.

84. The process is rather longer when the divisor is larger than 12. Example: Divide 6164 by 46.

```
46)6164(134
   46
   ---
   156
   138
   ---
    184
    184
    ---
    ...
```

The divisor is placed to the left of the dividend, separated by a small curved line; on the right, draw another line. Take the least number of figures on the left of the dividend, that will contain the divisor, find how many times they contain it, and place the result on the right as the first figure of the quotient; and the product arising from the multiplication of the divisor by this figure being subtracted from the dividend, annex to the remainder the next figure of the dividend, and divide the result as before; and proceed thus till all the figures of the dividend have been brought down or used.

If, after having annexed a figure to the remainder, the divisor is not contained in it, we place zero in the quotient, annex the

next figure of the dividend to the remainder, and proceed as before.

85. EXAMPLES.

$$\frac{25608}{37};\quad \frac{1554768}{216};\quad \frac{1674918}{189};\quad \frac{251667}{239};\quad \frac{6765158}{7894}.$$

86. It very often occurs that the divisor is not contained exactly in the dividend, then the last remainder may be joined to the quotient, under this form: +remainder...... (or see § 92).

87. Since the quotient is the number of times the divisor is contained in the dividend, it follows that the quotient multiplied by the divisor must necessarily be equal to the dividend, minus the remainder, if there be any. This affords a method of ascertaining whether a divisor has been correctly performed.

88. EXERCISES.

1. The annual income of a person is £38360. How much is that per day?

The income must be divided into 365 equal parts, and one of them is the daily income; therefore, £38360 must be divided by 365.

Division.	Proof.
365)38360(105+remainder 35.	365
365	105
1860	1825
1825	3650
35	35
	38360

2. 648 chests of tea cost £7128. What is the price of one chest?

3. What number is 25 times less than 3675?

4. £8400 are to be divided equally amongst six persons. How much would each individual's share be?

5. The dividend is 156970, and the divisor 2854. Find the quotient.

6. The dividend is 8760, and the quotient 24. Find the divisor.

7. What number, multiplied by 54, gives 9990 as product?

8. In how many days will a traveller go from London to Geneva if he travel at the rate of 7 miles per hour? The distance between the two cities is 1176 miles.

9. The earth moves in her orbit 583416000 miles in one year. At what rate does she go in one minute?

10. A gentleman's annual income is £2920; he wishes to lay by £1 every day. What must be his daily expenditure?

PART III.

FRACTIONS.

89. In the previous parts we have only considered whole numbers, or integers, viz., numbers containing once or several times unity; but, were we to suppose unity divided, or broken into several parts, we should obtain other quantities, called *fractions;* we are, therefore, led to the consideration of that most important part of arithmetic named *Vulgar Fractions.*

90. If we suppose an apple to be divided into two equal parts, one of the parts is called *one-half;* if it be divided into three equal parts, each part is *one-third;* if into four equal parts, each part is *one-quarter;* into five equal parts, *one-fifth*, &c.

If a rod, a line, &c., was also cut into 2, 3, 4, 5, &c., equal parts we should obtain 2-halves, 3-thirds, 4-fourths, 5-fifths, &c.

Therefore, *one-half* signifies 1 portion of a whole, or unit, which has been divided into 2 equal portions.

It follows that 2 halves are equal to the whole.

Also, 1-third is one part of a whole, which has been divided into 3 equal parts ; hence, 3-thirds are equal to one whole. Apply similar reasoning to the other divisions of unity.

91. When two or more equal parts are taken together, we have quantities, as the following :—2-thirds, 3-fourths, 5-eighths, &c., meaning that unity has been divided into 3 parts, and 2 of them taken, or that two units have been divided into 3 equal parts, and 1 of these taken. In 3-fourths, the whole has been divided into 4 equal parts, and 3 of them taken, or 3 wholes have been divided into 4 equal parts, and 1 of these parts taken, &c.

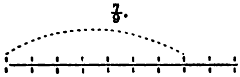
92. Hence it follows that every numerator is supposed to be divided by its denominator.

Thus, we perceive that fractions are expressed with two numbers : one the *denominator*, which shows into how many equal

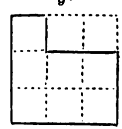
parts unity is divided; and the other the *numerator*, denoting how many such equal parts are taken. The denominator and numerator are called *terms* of the fraction; they are thus written: $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{9}$, &c., and read three-fourths, five-eighths, seven-ninths, &c.

94. The pupil must be exercised upon such questions as the following:—What is the meaning of $\frac{7}{9}$? It implies that unity is divided into 9 equal parts, and 7 of them are taken; 9 is the denominator, and 7 the numerator. He should also represent the value of those fractions by lines, squares, cubes, &c. The line or the square represent unity, which is divided into 9 equal parts, and 7 are marked out.

$\frac{7}{9}$.



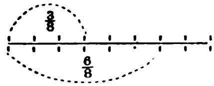
$\frac{7}{9}$.



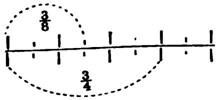
95. When speaking of fractions in connection with each other, it is evident they must be parts of the same, or equal units; it would, for instance, be too ridiculous to attempt to connect or compare $\frac{1}{2}$ lb. of butter to $\frac{1}{2}$ yard of linen.

96. The greater the number of parts into which unity is divided, the smaller the parts are; thus, $\frac{1}{2}$ is greater than $\frac{1}{3}$; $\frac{1}{5}$, greater than $\frac{1}{8}$; $\frac{3}{4}$, greater than $\frac{3}{7}$; $\frac{7}{9}$, greater than $\frac{7}{12}$, &c. Also, the greater the number of parts taken, the greater the fraction is; $\frac{7}{10}$ is greater than $\frac{3}{10}$; $\frac{11}{12}$, greater than $\frac{5}{12}$; $\frac{14}{17}$, greater than $\frac{5}{17}$, &c. Therefore, the larger the denominator of a fraction is, the smaller is the fraction, if the numerators remain equal; and the larger the numerator is, the larger is the fraction, if the denominators remain the same.

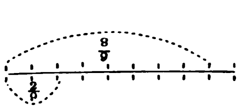
97. If the numerator of a fraction be less than the denominator, the value of the fraction is less than unity, since it does not contain all the parts into which the unit has been divided, it is called a *Proper Fraction:* $\frac{4}{5}$, $\frac{3}{7}$. If the numerator be equal to the denominator, it is unity in the form of a fraction: $\frac{3}{3}$, $\frac{8}{8}$. If the numerator be greater than the denominator, the value of the fraction is more than one unit, and it is called an *Improper Fraction:* $\frac{7}{4}$, $\frac{14}{11}$

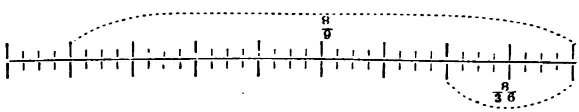


98. If the numerator of the fraction $\frac{3}{8}$ be multiplied by 2, without altering the denominator, the product is $\frac{6}{8}$, which fraction is twice as great as $\frac{3}{8}$, for $\frac{6}{8}$ contains twice as many equal parts as $\frac{3}{8}$ does. If the denominator be divided by 2, keeping the numerator as it is, the quotient is $\frac{3}{4}$, and as the unit is now divided into half as many equal parts, each of these is twice as great; therefore, $\frac{3}{4}$ = twice $\frac{3}{8}$. It follows that, to multiply a fraction by any whole number, we may either multiply its numerator or divide its denominator by that number.



99. Let us take the fraction $\frac{8}{9}$: if its numerator be divided by 4, the quotient is $\frac{2}{9}$, which is 4 times as small, for it contains 1-fourth of this number of equal parts. If the denominator 9 be multiplied by 4 the result is $\frac{8}{36}$, a fraction 4 times as small as $\frac{8}{9}$, for it contains the same number of parts,





each of which is 4 times less. Therefore, to divide a fraction by any whole number, we may either divide its numerator or multiply its denominator by that number.

100. EXERCISES UPON ARTICLE 98.

1. Multiplication of a fraction by a whole number, when operating upon its numerator.

Multiply by 2 : $\frac{2}{5}$, $\frac{4}{9}$, $\frac{5}{8}$, $\frac{7}{11}$, $\frac{8}{17}$.
Multiply by 3 : $\frac{2}{7}$, $\frac{4}{5}$, $\frac{6}{13}$, $\frac{2}{15}$, $\frac{11}{25}$.

2. Multiplication of a fraction by any whole number operating on its denominator.

Multiply by 4 : $\frac{5}{8}$, $\frac{7}{16}$, $\frac{11}{20}$, $\frac{5}{36}$, $\frac{5}{44}$.
Multiply by 6 : $\frac{5}{12}$, $\frac{7}{30}$, $\frac{11}{24}$, $\frac{17}{42}$, $\frac{5}{54}$.

3. Multiplication of a fraction by a whole number, operating on its numerator, or on its denominator, *if possible*.

Multiply by 3 : $\frac{8}{9}$, $\frac{6}{11}$, $\frac{7}{12}$, $\frac{11}{42}$, $\frac{7}{8}$, $\frac{13}{16}$, $\frac{14}{17}$, $\frac{5}{27}$.
Multiply by 8 : $\frac{7}{12}$, $\frac{19}{24}$, $\frac{13}{16}$, $\frac{15}{23}$, $\frac{11}{48}$, $\frac{25}{32}$, $\frac{8}{25}$, $\frac{17}{80}$.

101. EXERCISES UPON ARTICLE 99.

1. Division of a fraction by a whole number, operating on its numerator.

Divide by 3 : $\frac{6}{7}$, $\frac{15}{16}$, $\frac{12}{13}$, $\frac{18}{19}$, $\frac{21}{25}$.
Divide by 7 : $\frac{14}{17}$, $\frac{28}{37}$, $\frac{21}{31}$, $\frac{60}{61}$, $\frac{35}{36}$.

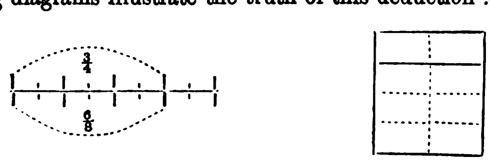
2. Division of a fraction by any whole number, operating on its denominator.

Divide by 5 : $\frac{11}{15}$, $\frac{3}{10}$, $\frac{12}{25}$, $\frac{17}{30}$, $\frac{14}{55}$.
Divide by 8 : $\frac{9}{24}$, $\frac{13}{32}$, $\frac{15}{56}$, $\frac{1}{24}$ $\frac{7}{64}$.

3. Division of a fraction by any whole number, operating on the numerator, *if possible*, or on the denominator.

Divide by 4 : $\frac{12}{17}$, $\frac{8}{9}$, $\frac{14}{15}$, $\frac{7}{8}$, $\frac{11}{24}$, $\frac{20}{27}$, $\frac{28}{31}$. $\frac{16}{19}$.
Divide by 6 : $\frac{17}{24}$, $\frac{42}{54}$, $\frac{36}{37}$, $\frac{18}{25}$, $\frac{6}{9}$, $\frac{24}{48}$, $\frac{42}{47}$, $\frac{11}{12}$.

102. Since a fraction is multiplied by an integer, when we multiply its numerator, and is divided by an integer when its denominator is multiplied, what will result if we multiply both the numerator and denominator by the same quantity? For instance, $\frac{3\times 2}{4\times 2}=\frac{6}{8}$. Now, evidently, on one hand, we have made the fraction twice as great as $\frac{3}{4}$, and on the other twice as small; therefore, the value of the fraction is not altered. The accompanying diagrams illustrate the truth of this deduction :—



It is easily shown that the same result is obtained if the numerator and denominator be both divided by the same number; for instance, $\frac{6\div 2}{8\div 2}=\frac{3}{4}$. Therefore, if the terms of a fraction be multiplied or divided by the same number, the value of the fraction will be unaltered.

103. EXERCISES.

1. Multiplication of the terms of a fraction by the same number.

Multiply by 4 : $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{7}{13}$, $\frac{13}{16}$.

Multiply by 7 : $\frac{3}{4}$, $\frac{5}{8}$ $\frac{7}{9}$, $\frac{11}{12}$, $\frac{14}{17}$.

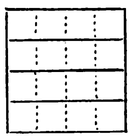
2. Division of the terms of a fraction by the same number.

Divide by 3 : $\frac{6}{9}$. $\frac{12}{15}$, $\frac{21}{24}$, $\frac{27}{36}$, $\frac{39}{48}$.

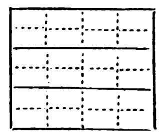
Divide by 8 : $\frac{16}{24}$, $\frac{32}{40}$, $\frac{56}{64}$, $\frac{112}{120}$, $\frac{88}{104}$.

104. By article 102, we see that a fraction can be expressed by different terms, for instance :—

$\frac{12}{16}=\frac{3}{4}$.



$\frac{16}{24}=\frac{2}{3}$.



But as we form a clearer idea of a fraction when expressed by small numbers than large ones, and as the numbers being small shorten the operations, it will be useful to simplify fractions, or to divide the terms by any *common factor*, or *common measure ;* the fraction will be expressed in its *lowest terms* when the factor is the *greatest common measure.*

Thus, in $\frac{32}{40}$, it is found that 8 is contained 4 times in the numerator and 5 times in the denominator; then, $\frac{32}{40}$ is equivalent to $\frac{4}{5}$, or $\frac{32}{40}=\frac{4}{5}$, and 8 is called the greatest common measure of 32 and 40. It must be observed that the same result can be obtained by dividing the terms by 2, three times in succession, $\frac{32}{40}=\frac{16}{20}=\frac{8}{10}=\frac{4}{5}$; 2 is only a *common measure* of 32 and 40.

105. A number which contains another number exactly, is said to be a multiple of it (see Art. 69). 32 and 40 are multiples of 8.

106. Beginners find it easier to reduce fractions by dividing the terms successively by common measures, as follows : $\frac{24}{60}=\frac{12}{30}=\frac{6}{15}=\frac{2}{5}$. The terms are divided by 2 as often as possible, and then by 3.

107. The following observations will be found useful :—

1. If the terms of the fraction are even, or if the figures in the units' place be divisible by 2, the numbers are divisible by 2.

2. If 4 is a measure of the figures of the last two places, the terms are divisible by 4. Ex. $\frac{124}{156}$. Since 24 and 56 are divisible by 4 the fraction is reducible by 4 : $\frac{124\div4}{156\div4}=\frac{31}{39}$.

3. If 8 is a measure of the figures in the three last places, then the terms are divisible by 8. Ex. $\frac{2168}{2760}$. Since 8 is a measure both of 168 and 760, therefore the fraction is reducible by 8. $\frac{2168\div8}{2760\div8}=\frac{271}{345}$.

4. If the terms of the fraction end in 0, cut the same number of ciphers from each. Ex. $\frac{2300}{3100}=\frac{23}{31}$.

5. If the terms end in 0 or 5, they are divisible by 5. Ex. $\frac{30\div5}{35\div5}=\frac{6}{7}$; $\frac{25\div5}{45\div5}=\frac{5}{9}$.

6. If the sum of the figures of each term be divisible by 3 or 9, the fraction is reducible by 3 or 9. Ex. $\frac{471}{573}$. Now, $4+7+1=12$; and $5+7+3=15$. Both 12 and 15 are divisible by 3, therefore the terms are reducible by 3, and $\frac{471\div3}{573\div3}=\frac{157}{191}$.

7. If the sum of the figures of the even places be equal to the sum of the figures of the uneven places, or if one sum exceeds the other by 11, or by any multiple of eleven, in each term, then the fraction is reducible by 11. Ex. $\frac{8459}{19162}$.

Sum of figures of even places of numerator $=4+9=13$ } dif. 0.
" odd " $=8+5=13$ }

Sum of figures of even places of denominator $=9+6=15$ } dif. 11.
" uneven " $=1+1+2=4$ }

Therefore, $\frac{8459\div11}{19162\div11}=\frac{769}{1742}$.

8. Every number consisting of more than 2 figures is divisible by 7, if after having doubled the units' figure and subtracted that product from the figure on the left hand of the units' place the remainder is 7 or a multiple of 7. If the remainder contains more than 2 figures, the units' figure of the remainder must be doubled, and the product subtracted from the figures on the left of the units' place, &c. The only exceptions to this law are 98 and 119. Here the figures on the left of the units must be subtracted from the units

doubled. Ex. $\frac{252}{1015}$. Multiply the units' figure of the numerator by 2: $2\times2=4$, subtract 4 from 25 leaves 21, double the units' figure of the denominator, $2\times5=10$, subtract 10 from 101 leaves 91; then, because 21 and 91 are multiples of 7, the fraction admits of being reduced by 7: $\frac{252\div7}{1015\div7}=\frac{36}{145}$.

108. It would save time, and very often useless labour, if we had a method to ascertain directly *the greatest common divisor, factor*, or *measure* of two numbers. In order to make this inquiry, let us establish a few principles.

109. Since the common measure of two numbers is a number which divides them both exactly, we shall find that the measures of 36 and 60 are,

For 36: 1, 2, 3, 4, 6, 9, 12, 18, 36.
For 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60.

The measures common to both are: 1, 2, 3, 4, 6, 12. We see, then, that 12 is the greatest common measure of 36 and 60.

110. If we examine two numbers, as 36 and 60, and find their sum and difference, $36+60=96$, $60-36=24$, we notice that the common measure of the numbers 1, 2, 3, 4, 6, 12 measure also their sum and difference. Therefore, *if one quantity measures two others, it measures likewise their sum and difference.*

111. But as the multiple of any number is the sum of several numbers equal to the one expressed, it follows that *if any number measures another number, it measures any multiple of the latter.* Ex. 4 measures 8; and, therefore, measures 2×8, 3×8, 4×8, &c.

112. We may observe that the remainder of the division of two numbers is nothing but the difference between the dividend and the product of the divisor by the quotient, viz., a multiple of the latter. Hence, *every number which measures both the dividend and the divisor, measures, likewise, the remainder.* Ex. 13 measures both 234 and 65. Divide 234 by 65: the quotient is 3, and the remainder 39, which 13 measures.

113. These principles being understood, let it be required to find the greatest common measure of 1749 and 477.

Divide 1749 by 477, and the remainder is 318. We infer that 477 is not the greatest measure; but (by § 112), any number which measures 1749 and 477 measures also 318, or the greatest common measure must be a factor of the three numbers, 1749, 477, and 318.

We are thus guided in finding the greatest common measure of 477 and 318. Divide 477 by 318, and the remainder is 159; in the same manner, it is shown that the greatest common measure of 477, and 318 is a factor of the three numbers, 477, 318, and 159, Let us now ascertain the greatest common measure of 318 and 159. Divide 318 by 159, and the remainder is 0. Therefore, 159 is the greatest common measure of 318 and 159; but the greatest common measure of these two numbers is a measure of 477, 318, and 159; and, therefore, also of the four numbers 1749, 477, 318, and 159, or, in short, of 1749 and 477. Thus 159 is the greatest common measure required.

The operation is effected in the following manner:—

```
477)1749(3
    1431
    ----
     318)477(1
         318
         ---
         159)318(2
             318
             ---
             ...
```

114. Therefore, to find the greatest common measure of two numbers, divide the greater number by the less; if there is a remainder, divide the less by it; continue dividing the last divisor by the last remainder till there is nothing remaining, and the last divisor is the common measure required.

115. If one of the given numbers were a *prime* number, there would be no common measure, unless the less number is the measure of the greater; of course, in this case, the less number is the prime number.

116. Strictly speaking, all numbers have a common measure, because they are all measured by 1; but when two numbers have no common measure greater than 1, or when they are *prime to each other*, the last divisor will be 1. If, in the operation, any divisor be a prime number, and leave a remainder, it is useless to proceed further, as unity is the only common measure.

117. If it were required to find the greatest common measure of three or more given numbers, we should ascertain the greatest common measure of any two of them; then that of this greatest common measure and a third number, and so on, to the end.

EXERCISES.

1. Reduce to their lowest terms: $\frac{8}{12}$, $\frac{6}{15}$, $\frac{9}{27}$, $\frac{25}{75}$, $\frac{72}{360}$.
2. Simplify, as much as possible, the following fractions: $\frac{30}{36}$, $\frac{345}{625}$, $\frac{840}{1440}$, $\frac{5565}{8533}$, $\frac{592}{999}$.
3. Express, in their simplest forms, the fractions $\frac{594}{1386}$, $\frac{799}{2961}$, $\frac{1575}{4095}$, $\frac{8379}{9575}$, $\frac{2618}{11781}$.
4. Reduce $\frac{8595}{18711}$, $\frac{644}{1728}$, $\frac{43365}{44688}$, $\frac{8280}{8352}$, $\frac{94380}{141570}$ to their most simple expressions.

(The four following Articles may be omitted until pupils have acquired a better knowledge of fractions.)

118. We have established this principle: that fractions are not altered in value when both numerator and denominator are divided by the same number. But it often happens that fractions, even when reduced to their lowest terms, are expressed in numbers inconveniently high, and it is sometimes required to find approximate values of them in smaller numbers.

[Note: $>$ signifies *greater than*, as $20>6$; $<$ signifies *less than*, as $6<20$; $\therefore$ signifies *therefore*; $\because$ signifies *since* or *because*.]

Suppose we want to find an approximate value of $\frac{353}{1213}$, which is expressed in its lowest terms. Let us divide the terms of the fraction by the numerator, then the fraction becomes $\dfrac{1}{\frac{1213}{353}}=\dfrac{1}{3+\frac{154}{353}}$. Were we to omit the fraction $\frac{154}{353}$, the denominator being decreased, the result $\dfrac{1}{3}>\dfrac{1}{3+\frac{154}{353}}$ and $\therefore$ $\dfrac{1}{3}>\dfrac{353}{1213}$.

Now, if we divide the fraction of the denominator by 154, we find $\dfrac{154}{353}=\dfrac{1}{2+\frac{45}{154}}$, $\therefore$ $\dfrac{353}{1213}=\dfrac{1}{3+\dfrac{1}{2+\frac{45}{154}}}$; let us omit $\dfrac{45}{154}$, it

follows that $\frac{1}{2} > \frac{154}{353}$, $\therefore \frac{1}{3+\frac{1}{2}} < \frac{353}{1213}$, but $\frac{1}{3+\frac{1}{2}} = \frac{2}{7}$, hence $\frac{2}{7} < \frac{353}{1213}$, from which we infer that the value of $\frac{353}{1213}$ is between $\frac{1}{3}$ and $\frac{2}{7}$.

Proceeding with the fraction of the denominator as before, we find $\frac{45}{154} = \frac{1}{3+\frac{19}{45}}$, whence $\frac{353}{1213} = \cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3+\frac{19}{45}}}}$, if we omit the fraction $\frac{19}{45}$, we have $\frac{353}{1213} = \cfrac{1}{3+\cfrac{1}{2+\frac{1}{3}}}$, but $2+\frac{1}{3} = \frac{7}{3}$, also $\frac{1}{2+\frac{1}{3}}$ $= \frac{1}{\frac{7}{3}} = \frac{3}{7}$; then $3+\frac{1}{2+\frac{1}{3}} = 3+\frac{3}{7} = \frac{24}{7}$; and $\cfrac{1}{3+\cfrac{1}{2+\frac{1}{3}}} = \frac{1}{\frac{24}{7}} = \frac{7}{24}$

$\therefore \frac{353}{1213} = \frac{7}{24}$.

Since $3 < 3+\frac{19}{45}$, it follows that $\frac{1}{3} > \frac{1}{3+\frac{19}{45}}$, also $2+\frac{1}{3}$ or $\frac{7}{3} > 2+\frac{1}{3+\frac{19}{45}}$ and $\frac{1}{\frac{7}{3}}$ or $\frac{3}{7} < \cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3+\frac{19}{45}}}}$ and hence $\frac{7}{24} > \frac{353}{1213}$.

It follows that the value of the given fraction is between $\frac{7}{24}$ and $\frac{2}{7}$.

To continue the approximation, let us divide both terms of $\frac{19}{45}$ by 19, the result is $\frac{1}{2+\frac{7}{19}}$, neglecting the fraction $\frac{7}{19}$ we

obtain $\frac{353}{1213}=\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3+\frac{1}{2}}}}$, but $3+\frac{1}{2}=\frac{7}{2}$, then $\cfrac{1}{3+\frac{1}{2}}=\cfrac{1}{\frac{7}{2}}$ or $\frac{2}{7}$,

also $2+\cfrac{1}{3+\frac{1}{2}}=2+\frac{2}{7}$ or $\frac{16}{7}$, and $\cfrac{1}{2+\cfrac{1}{3+\frac{1}{2}}}=\cfrac{1}{\frac{16}{7}}$ or $\frac{7}{16}$; likewise

$3+\cfrac{1}{2+\cfrac{1}{3+\frac{1}{2}}}=3+\frac{7}{16}$ or $\frac{55}{16}$, hence $\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3+\frac{1}{2}}}}=\cfrac{1}{\frac{55}{16}}$ or $\frac{16}{55}$.

Since $2<2+\frac{7}{19}$ it follows that $\frac{1}{2}>\cfrac{1}{2+\frac{7}{19}}$; also $3+\frac{1}{2}>3+\cfrac{1}{2+\frac{7}{19}}$

therefore $\cfrac{1}{3+\frac{1}{2}}<\cfrac{1}{3+\cfrac{1}{2+\frac{7}{19}}}$; likewise $2+\cfrac{1}{3+\frac{1}{2}}<2+\cfrac{1}{3+\cfrac{1}{2+\frac{7}{19}}}$

hence $\cfrac{1}{2+\cfrac{1}{3+\frac{1}{2}}}>\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{2+\frac{7}{19}}}}$; similarly $3+\cfrac{1}{2+\cfrac{1}{3+\frac{1}{2}}}$

$>3+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{2+\frac{7}{19}}}}$, it follows that $\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3+\frac{1}{2}}}}<\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{2+\frac{7}{19}}}}}$

or $\frac{16}{55}<\frac{353}{1213}$; therefore the value of $\frac{353}{1213}$ is between $\frac{7}{24}$ and $\frac{16}{55}$.

By continuing the process in a similar manner, we find other values which approximate more and more towards $\frac{353}{1213}$, until at last we obtain the given fraction. If the remaining fraction after each division be omitted, except when the numerator is unity, we have the following converging fractions:—

$$\frac{1}{3};\quad \cfrac{1}{3+\cfrac{1}{2}}=\frac{2}{7};\quad \cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3}}}=\frac{7}{24};\quad \cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{2}}}}=\frac{16}{55};\quad \cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{2}}}}}=\frac{25}{86};$$

$$\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{1}}}}}}=\frac{55}{184};\qquad \cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2}}}}}}}=\frac{149}{512};$$

$$\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{2}}}}}}}}=\frac{353}{1213}.$$

119. Ex. Find the fractions converging to $\frac{135}{347}$.

The following process is convenient to determine the several fractions approximating to $\frac{135}{347}$:—

$$\frac{135}{347}=\cfrac{1}{2+\cfrac{77}{135}};\qquad \frac{77}{135}=\cfrac{1}{1+\cfrac{58}{77}};\qquad \frac{58}{77}=\cfrac{1}{2+\cfrac{19}{58}};\qquad \frac{19}{58}=\cfrac{1}{3+\cfrac{1}{19}}.$$

Therefore, the converging fractions are :

$$\frac{135}{347}=\frac{1}{2};\quad \frac{135}{347}=\cfrac{1}{2+\cfrac{1}{1}}=\frac{1}{3};\quad \frac{135}{347}=\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1}}}=\frac{2}{5};$$

$$\frac{135}{347}=\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{3}}}}=\frac{7}{18};\quad \frac{135}{347}=\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{19}}}}}=\frac{135}{347}.$$

We observe that the approximation is alternately greater and less than the real value, and that the denominators of the continued fractions are the quotients, which are found by the method of ascertaining the greatest common measure of two numbers. This last observation leads us to a short process of finding the denominator of the continued fractions, the numerators of which are unity.

120. Ex. Express $\frac{7529}{11513}$ in a continued fraction, and find the converging fractions.

```
7529)11513(1
     7529
     ----
     3984)7529(1
          3984
          ----
          3545)3984(1
               3545
               ----
                439)3545(8
                    3512
                    ----
                      33)439(13
                         429
                         ---
                          10)33(3
                             30
                             --
                              3)10(3
                                 9
                                 -
                                 1)3(3
                                   3
                                   -
                                   .
```

Therefore the continued fraction is :

$$\frac{7529}{11513}=\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{8+\cfrac{1}{13+\cfrac{1}{3+\cfrac{1}{3+\cfrac{1}{3}}}}}}}},$$

and the approximating values are : 1, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{17}{26}$, $\frac{223}{341}$, $\frac{686}{1049}$, $\frac{2281}{3488}$, $\frac{7529}{11513}$.

121. EXERCISES.

1. Find the approximating values of $\frac{38597}{75937}$; also of $\frac{887}{1103}$.
2. What are the converging fractions to $\frac{275}{338}$; also to $\frac{1947}{3359}$?
3. On an average of 100 years, it is found that the lunar month consists of 27.321661 days. From this datum, it would follow that the moon describes 1000000 revolutions round the earth in 27321661 days. Express, approximately, the fractions which give its daily revolution.
4. It is found that the circumference of a circle, whose diameter is 1, is 3.1415926535, nearly. Find the converging fractions expressing nearly the ratio of the diameter to the circumference.
5. The planet Mercury revolves in 87.969255 days, and Venus in 224.700817. Express these times of revolution approximately, by small numbers.

122. Suppose it was required to reduce $\frac{44}{9}$ into a whole number, or a *mixed quantity* ; that is, into a whole number and a fraction. Since 9-ninths make 1 whole, 18-ninths are equal to 2 wholes, 27-ninths to 3 wholes; therefore, as many times as 9 is contained in 44, so many wholes will there be, now 9 is contained in 44, 4 times + the remainder $\frac{8}{9}$, then $\frac{44}{9}=4\frac{8}{9}$.

Therefore, to reduce an improper fraction to a whole or mixed number, divide the numerator by the denominator; the quotient expresses the whole number, and if there be a remainder, place it over the denominator for the fractional part.

123. Exercises : Reduce the following fractions to a whole or mixed number :—$\frac{7}{4}$, $\frac{24}{9}$, $\frac{24}{8}$, $\frac{42}{33}$, $\frac{100}{3}$, $\frac{171}{40}$, $\frac{3710}{69}$, $\frac{3335}{12}$, $\frac{2000}{27}$.

124. Let us consider the converse of the preceding case; for instance, suppose it were required to reduce the mixed quantity $5\frac{2}{7}$ to an improper fraction. Since unity is supposed to be divided into 7 equal parts, every unit = 7-sevenths; therefore, 5 units = 5×7-sevenths, or 35-sevenths, and 2-sevenths make altogether 37-sevenths. Therefore, $5\frac{2}{7} = \frac{37}{7}$.

Hence, to reduce a mixed quantity to an improper fraction, multiply the integer by the denominator of the fraction, and to the product add the numerator of the fraction; the sum is the new numerator, and the denominator of the fraction will be its denominator.

125. Exercises: Express the following mixed quantities as improper fractions:—$3\frac{2}{5}$, $7\frac{4}{7}$, $16\frac{1}{2}$, $14\frac{7}{8}$, $26\frac{8}{9}$, $35\frac{1}{12}$, $48\frac{1}{15}$, $124\frac{21}{25}$.

126. RECAPITULATION.—EXERCISES.

1. Express that an object is divided into 25 equal parts, and 17 of them taken away.
2. Exhibit the quantity represented by an object broken into 134 equal parts, and 21 of them taken away.
3. Write down the following fractions in words:—$\frac{1}{5}$, $\frac{2}{7}$, $\frac{5}{9}$, $\frac{17}{24}$, $\frac{35}{41}$.
4. What is the meaning of $\frac{5}{7}$, $\frac{4}{9}$, $\frac{2}{3}$, $\frac{13}{15}$, $\frac{11}{14}$?
5. Write down in figures the following expressions:—five-sixths, one-thirteenth, seven-tenths, thirteen-fifteenths, three-fifths, four-ninths, three-nineteenths, fifteen-twenty-eighths.
6. Which is the greater $\frac{5}{7}$ or $\frac{3}{7}$? Why?
   " " $\frac{7}{11}$ or $\frac{10}{11}$? Why?
   " " $\frac{3}{4}$ or $\frac{3}{5}$? Why?
   " " $\frac{8}{11}$ or $\frac{8}{13}$? Why?
   " " $\frac{7}{10}$ or $\frac{7}{15}$? Why?
7. Which is the greater of $\frac{5}{8}$, $\frac{5}{6}$, $\frac{5}{11}$, $\frac{5}{12}$, $\frac{5}{16}$?
   " " $\frac{9}{13}$, $\frac{7}{13}$, $\frac{6}{13}$, $\frac{10}{13}$, $\frac{4}{13}$?
8. Express in figures that a workman has done but three-quarters of his work.
9. Two workmen are in a factory, one works five-sevenths of the time, and the other five-sixths. Express in figures the time each worked, and also mention which worked the longer time.
10. Two men work in the same manufactory, the first performs one-fifth less than his daily work, and the second one-fifth more. How much does each perform?

11. What name is given to quantities of this form: $\frac{15}{7}$, $\frac{11}{3}$, $\frac{19}{4}$, $\frac{21}{5}$, $\frac{76}{9}$, $\frac{100}{3}$, $\frac{134}{13}$? and express them in mixed quantities.

12. Which quantity is twice as great as $\frac{2}{5}$?
 ,, ,, three times ,, $\frac{3}{10}$?
 ,, ,, five times ,, $\frac{4}{27}$?

13. The treble of $\frac{2}{11}$ is required; the quadruple of $\frac{6}{11}$; the double of $\frac{1}{5}$?

14. A mason builds, in one day, $\frac{5}{48}$ of a wall. How much will he do in 7 days?

15. A man goes, in one day, $\frac{7}{72}$ of his journey. How much will he accomplish in 8 days?

16. Which quantity is three times less than $\frac{2}{5}$?
 ,, ,, one-third of $\frac{3}{4}$?
 ,, ,, one-sixth of $\frac{8}{11}$?
 ,, ,, one-ninth of $\frac{12}{13}$?

17. How much is one-quarter of one quarter?
 ,, one fifth of one-fifth?
 ,, one-seventh of one-seventh?

18. Make $\frac{11}{12}$ six times less.
 ,, $\frac{7}{10}$ nine ,,

19. In 5 days, $\frac{3}{4}$ of a work is performed; how much is done in one day? In nine days, $\frac{5}{6}$; how much in one day? In seven days, $\frac{4}{5}$; how much in one day?

20. A and B divide a sum of money equally between them; A divides his share equally between his three children, and B divides his equally between his four children. What part of the whole money does each of the children of A and B receive?

21. A person received $\frac{3}{8}$ of an inheritance, which was trebled in trade; his son quadrupled that fortune; but the grandson, failing in business, lost one-seventh of his inheritance. What part of the whole inheritance was left?

22. Required, the greatest common measure of 28 and 98; of 345 and 2415; of 22893 and 79245.

23. Reduce to their lowest terms, if possible, $\frac{18}{63}$, $\frac{317}{873}$, $\frac{504}{1296}$, $\frac{432}{720}$, $\frac{1408}{1664}$.

24. A shopkeeper was asked for the $\frac{3}{6}\frac{1}{3}\frac{7}{4}$ of a yard of cloth, but not understanding the question, what must be done?

24. A boy who was offered the $\frac{4}{9}\frac{7}{4}\frac{4}{8}$ part of an orange, asked that the quantity be put in a simpler form. How is this to be done?

ADDITION OF FRACTIONS.

127. If to $\frac{3}{7}$ of a yard $\frac{2}{7}$ be added, the sum is $\frac{5}{7}$ of a yard. The sum of $\frac{5}{9}$ and $\frac{2}{9}$ is $\frac{7}{9}$.

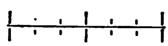
Find the sum of $\frac{5}{8}+\frac{3}{8}+\frac{7}{8}$. $\frac{5}{8}+\frac{3}{8}=\frac{8}{8}$; $\frac{8}{8}+\frac{7}{8}=\frac{15}{8}=1\frac{7}{8}$

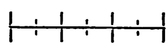
Find the sum of $\frac{9}{13}+\frac{7}{13}+\frac{8}{13}+\frac{11}{13}$. $\frac{9}{13}+\frac{7}{13}=\frac{16}{13}$; $\frac{16}{13}+\frac{8}{13}=\frac{24}{13}$; $\frac{24}{13}+\frac{11}{13}=\frac{35}{13}=2\frac{9}{13}$.

Therefore, when fractions have the same denominator, add all the numerators together, and place their sum over the denominator, the result will be the sum of the fractions, which must be reduced to a whole or mixed quantity, when possible.

128. How much are $\frac{1}{2}+\frac{1}{3}$?

These fractions, not having the same denominator, cannot be put together without transforming them to fractions of the same denominator, for instance:—

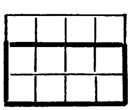
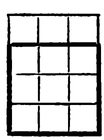
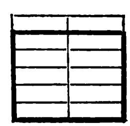
$\frac{1}{2}=\frac{3}{6}$ 

$\frac{1}{3}=\frac{2}{6}$ 

Hence, $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$. We observe that $\frac{1}{2}$ is transformed into sixths by multiplying the terms by 3: $\frac{3\times1}{3\times2}=\frac{3}{6}$; and $\frac{1}{3}$ is transformed by multiplying the terms by 2: $\frac{2\times1}{2\times3}=\frac{2}{6}$.

129. The sum of $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ is required.

Here the fractions can all be transformed into twelfths:—

$\frac{2}{3}=\frac{8}{12}$ ; $\frac{3}{4}=\frac{9}{12}$ ; $\frac{5}{6}=\frac{10}{12}$ 

Therefore, $\frac{2}{3}+\frac{3}{4}+\frac{5}{6}=\frac{8}{12}+\frac{9}{12}+\frac{10}{12}=\frac{27}{12}=2\frac{3}{12}=2\frac{1}{4}$.

To transform $\frac{2}{3}$ into twelfths the terms must be multiplied by 4.
" $\frac{3}{4}$ " " " 3.
" $\frac{5}{6}$ " " " 2.

And we have $\frac{4\times2}{4\times3}=\frac{8}{12}$; $\frac{3\times3}{3\times4}=\frac{9}{12}$; $\frac{2\times5}{2\times6}=\frac{10}{12}$.

Therefore, when the fractions proposed have not the same denominator, reduce them to a common denominator, and then proceed as before.

130. We have seen that fractions cannot be added together without being of the same kind, or of the same denominator. Let us now determine a method which will enable us to reduce fractions to a *common denominator*. The common denominator of two or more fractions is such a number as will contain the several proposed denominators, without remainders; thus, in our last example, 12 is a common denominator of 3, 4, 6; so is 24, 36, &c., any multiple of 12; but 12 is *the least*, and it will be preferable to find that one, because we shall thereby shorten the operations.

131. What is the least common denominator of $\frac{5}{6}$, $\frac{3}{8}$, $\frac{7}{18}$, $\frac{11}{24}$, $\frac{29}{45}$?

First, let it be understood that to resolve a number into its *prime factors* is to divide it by 2, and the quotient by 2, and this second quotient again by 2. When the quotient can no longer be divided by 2, we must try by 3, then by 5, by 7, &c., until the quotient is a prime number. The several divisors and the last quotient are the prime factors of the numbers.

Let the above denominators be resolved into their prime factors.

$$\begin{aligned}
6&=\cancel{2}\times\cancel{3}\\
8&=\cancel{2}\times2\times\cancel{2}\\
18&=\cancel{2}\times\cancel{3}\times\cancel{3}\\
24&=2\times2\times2\times\cancel{3}\\
45&=3\times3\times5.
\end{aligned}$$

Now, we observe that the two of the first line, the three twos of the second, and the two of the third, are contained in the three twos of the fourth line, for which reason they are struck out; the three of the first line, the two threes of the third, and the three of the fourth are contained in the two threes of the fifth line, then they are also struck out. The product of the remaining

factors will evidently be the least common denominator, or $2\times2\times2\times3\times3\times5=360$.

132. In practice, the operation is carried on in this manner:—

$$\begin{array}{l}
2)\underline{6\ ,,\ 8\ ,,\ 18\ ,,\ 24\ ,,\ 45}\\
2)\underline{3\ ,,\ 4\ ,,\ \ 9\ ,,\ 12\ ,,\ 45}\\
2)\underline{3\ ,,\ 2\ ,,\ \ 9\ ,,\ \ 6\ ,,\ 45}\\
3)\underline{3\ ,,\ 1\ ,,\ \ 9\ ,,\ \ 3\ ,,\ 45}\\
3)\underline{1\ ,,\ 1\ ,,\ \ 3\ ,,\ \ 1\ ,,\ 15}\\
\quad 1\ ,,\ 1\ ,,\ \ 1\ ,,\ \ 1\ ,,\ \ 5
\end{array}$$

The several divisors, and the last quotient 5, are the factors found above. But, by inspection, we easily see that 6 and 8 are contained in 24; and, therefore, may be neglected, and the process is thus abridged:—

$$\begin{array}{l}
2)\underline{18\ ,,\ 24\ ,,\ 45}\\
3)\underline{\ 9\ ,,\ 12\ ,,\ 45}\\
3)\underline{\ 3\ ,,\ \ 4\ ,,\ 15}\\
\quad\ 1\ ,,\ \ 4\ ,,\ \ 5;
\end{array}$$

and the least common denominator is $2\times3\times3\times4\times5=360$, as before.

133. To transform the proposed fractions into others, having 360 for their denominator, it is merely necessary to multiply the terms of every fraction by the quotient resulting from dividing 360 by its denominator.

Now, 6 is contained in 360, 60 times; therefore, $\frac{5}{6}=\frac{60\times\ 5}{60\times\ 6}=\frac{300}{360}$.

8	,,	360, 45	,,	$\frac{5}{8}=\frac{45\times\ 5}{45\times\ 8}=\frac{225}{360}$.
18	,,	360, 20	,,	$\frac{7}{18}=\frac{20\times\ 7}{20\times18}=\frac{140}{360}$.
24	,,	360, 15	,,	$\frac{11}{24}=\frac{15\times11}{15\times24}=\frac{165}{360}$.
45	,,	360, 8	,,	$\frac{29}{45}=\frac{8\times29}{8\times45}=\frac{232}{360}$.

Add together $5\frac{1}{2}$, $17\frac{2}{3}$, $7\frac{3}{4}$, $8\frac{5}{6}$, $12\frac{3}{5}$, $16\frac{11}{20}$, and $13\frac{5}{8}$.

$$\begin{array}{rl} 5\frac{1}{2} & \frac{60}{120} \\ 17\frac{2}{3} & \frac{80}{120} \\ 7\frac{3}{4} & \frac{90}{120} \\ 8\frac{5}{6} & \frac{100}{120} \\ 12\frac{3}{5} & \frac{72}{120} \\ 16\frac{11}{20} & \frac{66}{120} \\ 13\frac{5}{8} & \frac{75}{120} \\ \hline 82\frac{21}{40} & \frac{543}{120}=4\frac{21}{40} \end{array}$$

Here we notice that 2 and 4 are contained in 8, 3 in 6, and 5 in 20 ; thus, we have only to consider 6, 20, and 8 to determine the least common denominator :—

$$\begin{array}{r|l} 2 & 6\ ,,\ 20\ ,,\ 8 \\ \hline 2 & 3\ ,,\ 10\ ,,\ 4 \\ \hline & 3\ ,,\ \ 5\ ,,\ 2. \end{array}$$

Therefore, $2 \times 2 \times 3 \times 5 \times 2 = 120$. Transforming all the fractions to this denomination, and adding together the numerators, we find $\frac{543}{120}$, or $4\frac{21}{40}$. Put down $\frac{21}{40}$ in the column of the fractions, and the 4 is added with the wholes, and we find the sum to be $82\frac{21}{40}$.

134. EXERCISES.

1. What is the sum of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{3}{4}$? $\frac{4}{9}$, $\frac{5}{8}$, $\frac{2}{3}$, and $\frac{3}{7}$?

2. Add together $3\frac{1}{4}$, $4\frac{7}{8}$, $5\frac{6}{7}$, $\frac{1}{12}$, and $1\frac{7}{9}$.

3. Find the sum of $3\frac{1}{3}+7\frac{11}{16}+124\frac{13}{24}+14\frac{17}{18}+195\frac{17}{36}$.

4. A draper has 4 pieces of black cloth, the first contains $32\frac{1}{2}$ yards ; the second, $25\frac{5}{6}$ yards ; the third, $44\frac{5}{8}$ yards ; and the fourth, $36\frac{7}{12}$ yards. How many yards are there altogether.

5. A workman performs, in 5 days, a work which it takes another 8 days to do. What part of the work would both together perform in 1 day ?

6. In an army, the cavalry equal $\frac{1}{6}$, and the artillery $\frac{3}{10}$ of the infantry. What part of the infantry are the cavalry and artillery together ?

7. A weaving machine makes, the first day, $\frac{3}{20}$ of a piece of cloth ; the second day, $\frac{7}{50}$; the third day, $\frac{2}{15}$; and on the fourth day, $\frac{11}{60}$. How much has been done during these four days ?

8. Two persons, intending to meet, start at the same time, from two different towns; the first could perform the distance in 8 days, and the second in 9 days. What part of the distance will they have achieved in 2 days?

9. Water is admitted into a reservoir through four apertures; the first would fill it in 24 hours, the second in 36 hours, the third in 30 hours, and the fourth in 40 hours. What part of the reservoir is filled up in 1 hour? Also in 3 hours?

10. A requires $5\frac{1}{2}$ yards of cloth for a coat, $2\frac{2}{3}$ yards for a pair of trowsers, $1\frac{3}{4}$ yards for a waistcoat, and $\frac{7}{12}$ yard for a pair of gaiters. How much does he want altogether?

SUBTRACTION OF FRACTIONS.

135. From a piece of cloth, $\frac{4}{5}$ yard long, a tailor makes use of $\frac{3}{5}$. How much remains?

We are led by the question to subtract $\frac{3}{5}$ from $\frac{4}{5}$, viz., to take away the numerator of the lesser from the numerator of the greater, then $\frac{4}{5}-\frac{3}{5}=\frac{1}{5}$.

Also, $\frac{11}{12}-\frac{5}{12}=\frac{6}{12}$ or $\frac{1}{2}$; $\frac{14}{17}-\frac{9}{17}=\frac{5}{17}$; $\frac{19}{21}-\frac{10}{21}=\frac{9}{21}$ or $\frac{3}{7}$.

From	$13\frac{5}{6}$	From	$194\frac{11}{15}$
Take	$7\frac{1}{6}$	Take	$176\frac{8}{15}$
Remainder	$6\frac{4}{6}$ or $6\frac{2}{3}$	Remainder	$18\frac{3}{15}$ or $18\frac{1}{5}$

136. A performs $\frac{13}{16}$ of a work whilst B does $\frac{7}{9}$ of it. How much does one perform more than the other?

Evidently, the lesser work must be taken away from the greater; but it happens sometimes that there is some difficulty in ascertaining at once the greater of two fractions, which have different denominators; the difficulty is easily removed by reducing them to the same denominator, and then the lesser is subtracted from the greater as before. Now, we find that $\frac{13}{16}=\frac{117}{144}$, and $\frac{7}{9}=\frac{112}{144}$; therefore, $\frac{13}{16}-\frac{7}{9}=\frac{117}{144}-\frac{112}{144}=\frac{5}{144}=$ the difference between the work performed by A and by B.

In like manner: $\frac{7}{12}-\frac{3}{8}=\frac{14}{24}-\frac{9}{24}=\frac{5}{24}$; $\frac{4}{5}-\frac{5}{9}=\frac{36}{45}-\frac{25}{45}=\frac{11}{45}$.

137. Let it be required to subtract $5\frac{1}{3}$ from $11\frac{1}{2}$.

1st method: $11\frac{1}{2}-5\frac{1}{3}=\frac{23}{2}-\frac{16}{3}=\frac{69}{6}-\frac{32}{6}=\frac{37}{6}=6\frac{1}{6}$.

2nd method: $11\frac{1}{2}-5\frac{1}{3}=11\frac{3}{6}-5\frac{2}{6}=6\frac{1}{6}$.

Find the difference of $17\frac{3}{8}$ and $14\frac{6}{7}$.

1st method: $17\frac{3}{8}—14\frac{6}{7}=\frac{139}{8}—\frac{104}{7}=\frac{973}{56}—\frac{832}{56}=\frac{141}{56}=2\frac{29}{56}$.
2nd method: $17\frac{3}{8}—14\frac{6}{7}=17\frac{21}{56}—14\frac{48}{56}=2\frac{29}{56}$;

or this method may be put under the following form, as in addition :—

$$\begin{array}{ll} 17\frac{3}{8} & \frac{21}{56} \\ 14\frac{6}{7} & \frac{48}{56} \\ \hline 2\frac{29}{56}. & \end{array}$$

Since $\frac{48}{56}$ cannot be subtracted from $\frac{21}{56}$, we add 1 whole to $\frac{21}{56}$ and $\frac{56}{56}+\frac{21}{56}=\frac{77}{56}$, from which $\frac{48}{56}$ is taken away, the remainder is $\frac{29}{56}$; now, add 1 to 14, (because 1 was added to the top quantity,) and we say 15 from 17 leaves 2.

138. EXERCISES.

1. Required, the difference of $\frac{7}{8}$ and $\frac{2}{3}$; of $\frac{48}{121}$ and $\frac{34}{195}$; of $\frac{124}{205}$ and $\frac{89}{360}$; of $\frac{243}{288}$ and $\frac{144}{576}$.
2. Subtract $5\frac{1}{8}$ from $8\frac{3}{4}$; $17\frac{3}{5}$ from $22\frac{3}{4}$; $12\frac{58}{60}$ from $16\frac{50}{51}$.
3. Find the value of $\frac{3}{4}+\frac{2}{3}—\frac{5}{6}$; $1\frac{1}{2}—\frac{2}{3}+\frac{6}{7}—\frac{2}{5}$; $2\frac{4}{5}+7\frac{5}{8}—3\frac{2}{7}—4\frac{1}{12}$.
4. Find the value of $1—\frac{2}{3}\div4\frac{1}{5}—3\frac{1}{7}+14\frac{5}{6}—9\frac{3}{4}—\frac{5}{8}+\frac{1}{2}$.
5. $\frac{4}{7}$ of an oil barrel is emptied. What part remains?
6. $\frac{7}{12}$ of a work is achieved. How much remains to be done?
7. The first day A performed $\frac{1}{3}$ of his work; the second, $\frac{1}{4}$. How much has he to finish?
8. From a piece of linen, $30\frac{7}{8}$ yards long, $5\frac{1}{4}$ yards $+7\frac{2}{3}+10\frac{5}{6}$ were sold. How much is there left?
9. A jar weighs $5\frac{3}{4}$ lbs., when filled with water it weighs $36\frac{7}{8}$ lbs. The weight of the water is required.
10. $\frac{1}{5}$ of a pole stands in the ground and $\frac{1}{3}$ in the water. How much is there above the water?
11. A basin would be filled in 7 hours by a tap, and emptied in 10 hours by a pipe. If both tap and pipe be opened at the same time, when the basin is empty, how much water would it contain at the end of 1 hour?
12. The sum of two numbers is $7\frac{3}{4}$; one is $2\frac{4}{5}$. The other is required.

13. If I had $1\frac{5}{8}$ acres less land, I should have $1\frac{3}{4}$ acres more than you, who have $15\frac{5}{8}$ acres. How many acres belong to me?

MULTIPLICATION OF FRACTIONS.

139. Under this head, we have the following cases:—

1st, to multiply a fraction by a whole number.
2nd, to multiply a whole number by a fraction.
3rd, to multiply a mixed quantity by a whole number.
4th, to multiply a whole number by a mixed quantity.
5th, to multiply a fraction by a fraction.
6th, to multiply a mixed quantity by another.
7th, to multiply a mixed quantity by a fraction.
8th, to multiply a fraction by a mixed quantity,

140. 1st, to multiply a fraction by a whole number. Let it be required to multiply $\frac{2}{9}$ by 4.

From the definition of multiplication (§ 65), we have to repeat $\frac{2}{9}$ as many times as there are units in 4. Now, we know that $4 \times 2 = 8$; and since $\frac{2}{9}$ are 9 times less than two wholes, $4 \times \frac{2}{9} = 9$ times less than 8, or $\frac{8}{9}$.

Therefore, to multiply a fraction by a whole number, multiply the numerator by the whole number, and divide the product by the denominator.

141. 2nd, to multiply an integer by a fraction. Multiply 8 by $\frac{6}{7}$.

If it were necessary to multiply 8 by 6, the product would be 48; but 8 must be multiplied by $\frac{6}{7}$, or by a quantity $7\times$ less than 6 wholes. Hence, the product must be $7\times$ smaller, or $\frac{48}{7} = 6\frac{6}{7}$.

Therefore, to multiply an integer by a fraction, the integer must be multiplied by the numerator, and the product divided by the denominator.

142. These two cases often admit of contractions which must not be neglected. For instance, multiply $\frac{11}{12}$ by 8.

Here we have $8 \times \frac{11}{12} = \frac{8 \times 11}{12} = \frac{2 \times 4 \times 11}{3 \times 4}$; but (by § 102) to

multiply the numerator and the denominator by the same number does not alter the value of the fraction; therefore, $\frac{2\times4\times11}{3\times4}=\frac{2\times11}{3}=\frac{22}{3}=7\frac{1}{3}$. The contraction consists, then, in *cancelling* a measure common to the terms.

Also, $\frac{5}{6}\times18=\frac{5\times18}{6}=\frac{5\times3}{1}=15$.

143. 3rd, to multiply a mixed quantity by a whole number. For instance, $7\frac{4}{5}$ by 12. Every mixed quantity may be transformed into an improper fraction; therefore, the question is, $12\times7\frac{4}{5}=\frac{12\times39}{5}=93\frac{3}{5}$. Thus, we have to reduce the mixed quantity to an improper fraction, and proceed as in case 1; or, since $12\times7\frac{4}{5}=12\times7+12\times\frac{4}{5}=84+\frac{48}{5}=84+9\frac{3}{5}=93\frac{3}{5}$; that is to say, to the product of the integer of the mixed quantity by the multiplier, add the product of the fraction by the multiplier.

Example: $15\times6\frac{7}{8}$.

1st method: $15\times6\frac{7}{8}=\frac{15\times55}{8}=\frac{825}{8}=103\frac{1}{8}$.

2nd method: $15\times6\frac{7}{8}=15\times6+15\times\frac{7}{8}=90+13\frac{1}{8}=103\frac{1}{8}$.

144. 4th, to multiply a whole number by a mixed quantity. Multiply 10 by $14\frac{3}{4}$. Reducing the mixed quantity into an improper fraction, we have: $14\frac{3}{4}=\frac{59}{4}$; then, $14\frac{3}{4}\times10=\frac{59\times10}{4}$ $=\frac{59\times5}{2}=\frac{295}{2}=147\frac{1}{2}$. Another method is this: $14\frac{3}{4}\times10=$ $14\times10+\frac{3}{4}\times10=140+\frac{15}{2}=147\frac{1}{2}$.

Therefore, reduce the mixed quantity to an improper fraction, and proceed as in case 2; or, add the product of the integer and multiplicand to the product of the fraction and multiplicand. Thus:—

1st method: $9\frac{5}{6}\times8=\frac{59\times8}{6}=\frac{236}{3}=78\frac{2}{3}$.

2nd method: $9\frac{5}{6}\times8=9\times8+\frac{5}{6}\times8=72+6\frac{2}{3}=78\frac{2}{3}$.

145. 5th, to multiply a fraction by a fraction. Multiply $\frac{3}{4}$ by $\frac{2}{3}$.

If it were required to multiply $\frac{3}{4}$ by 2, the result would be $\frac{2\times3}{4}$; but, by the question, $\frac{3}{4}$ must be multiplied by a quantity

5 times less than 2 wholes, and (by § 99) it is known that a fraction is made 5 × less by multiplying its denominator by 5; therefore, $\frac{2}{5} \times \frac{3}{4} = 5$ times less than $\frac{2 \times 3}{4}$, or $\frac{2 \times 3}{4 \times 5} = \frac{3}{10}$.

Therefore, multiply the numerators together for the numerator of the product, and the denominators together for its denominator.

Thus, the product of $\frac{7}{8}$ by $\frac{5}{12} = \frac{35}{96}$.
" " $\frac{8}{9}$ " $\frac{9}{16} = \frac{1}{2}$.

146. 6th, to multiply a mixed quantity by another. Multiply $3\frac{1}{4}$ by $2\frac{2}{3}$.

$$3\frac{1}{4} \times 2\frac{2}{3} = \frac{13}{4} \times \frac{8}{3} = \frac{26}{3} = 8\frac{2}{3}.$$

Thus, we see that, after having reduced the mixed quantities to improper fractions, we proceed as in case 5.

The multiplication might have been effected by multiplying 3 by 2, $\frac{1}{4}$ by 2, $\frac{2}{3}$ by 3, and $\frac{1}{4}$ by $\frac{2}{3}$, then adding those four products; but this operation would be much longer.

Example: Multiply $5\frac{3}{8}$ by $7\frac{12}{13}$.

1st method: $5\frac{3}{8} \times 7\frac{12}{13} = \frac{43}{8} \times \frac{103}{13} = 42\frac{61}{104}$.

2nd method: $5 \times 7 + \frac{3}{8} \times 7 + 5 \times \frac{12}{13} + \frac{3}{8} \times \frac{12}{13} = 35 + 2\frac{5}{8} + 4\frac{8}{13} + \frac{36}{104} = 42\frac{61}{104}$.

147. The 7th and 8th cases are easily explained, by § 146.

148. We know that $\frac{1}{2} \times 6 = 3$; also, $\frac{1}{2}$ of $6 = 3$.

Therefore, $\frac{1}{2} \times 6 = \frac{1}{2}$ of 6.

Likewise, $\frac{2}{3} \times 8 = \frac{16}{3} = 5\frac{1}{3}$; also, $\frac{2}{3}$ of $8 = 2 \times \frac{1}{3}$ of $8 = 2 \times \frac{8}{3} = \frac{16}{3} = 5\frac{1}{3}$. Therefore, $\frac{2}{3} \times 8 = \frac{2}{3}$ of 8.

Since $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$, and $\frac{1}{2}$ of $\frac{3}{4} = \frac{3}{8}$; therefore, $\frac{1}{2} \times \frac{3}{4} = \frac{1}{2}$ of $\frac{3}{4}$. Likewise, since $\frac{5}{6} \times \frac{7}{8} = \frac{35}{48}$; also, $\frac{5}{6}$ of $\frac{7}{8} = 5 \times \frac{1}{6}$ of $\frac{7}{8} = 5 \times \frac{7}{48} = \frac{35}{48}$; therefore, $\frac{5}{6} \times \frac{7}{8} = \frac{5}{6}$ of $\frac{7}{8}$.

From these different examples, it follows that the word *of*, connecting two fractions, is equivalent to the sign ×.

149. When the word *of* connects two or more fractions, it is usual to call such an expression a *compound fraction*. Thus, $\frac{3}{4}$ of $\frac{1}{3}$ of $\frac{6}{7}$ is a compound fraction.

150. It is sometimes necessary to find the result of a quantity expressed thus: $\frac{3}{5}$ of $\frac{7}{8} \times \frac{2}{5}$ of $7\frac{3}{4} \times \frac{5}{6}$. By what has been said before, it may be written in this manner: $\frac{3}{5} \times \frac{7}{8} \times \frac{2}{5} \times \frac{31}{4} \times \frac{5}{6} = \frac{3 \times 7 \times 2 \times 31 \times 5}{5 \times 8 \times 5 \times 4 \times 6}$. Cancelling the factors and the measures common to the terms, we have: $\frac{7 \times 31}{5 \times 8 \times 2 \times 2} = \frac{217}{160} = 1\frac{57}{160}$.

151. The expression $\frac{\frac{2}{3}}{\frac{4}{5}}$ is simplified by multiplying the numerator and the denominator by a common multiple (the least is preferable) of 3 and 5, or 15; thus, $15 \times \frac{2}{3} = 10$, and $15 \times \frac{4}{5} = 12$; therefore, $\frac{\frac{2}{3}}{\frac{4}{5}} = \frac{10}{12} = \frac{5}{6}$.

152. It is observed that, in the multiplication of fractions, the product of two quantities is often less than either quantity. Now, this ambiguity, when compared with the multiplication of whole numbers, is easily explained. We know that $1 \times$ a quantity $=$ that quantity; for instance: $1 \times \frac{3}{4} = \frac{3}{4}$; and since fractions are less than 1, $\frac{3}{4}$ multiplied by a quantity less than $1 =$ less than $\frac{3}{4}$. Example: $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$.

153. RECAPITULATORY EXERCISES.

1. Multiply $\frac{6}{7}$ by 36; 5 by $\frac{7}{8}$; $\frac{5}{8}$ by $\frac{9}{11}$; 7 by $6\frac{5}{7}$.
2. Required, the product of $\frac{2}{5} \times \frac{6}{7} \times \frac{7}{9} \times 1\frac{1}{5}$.
3. Multiply together $\frac{4}{8} \times \frac{4\frac{1}{5}}{7\frac{6}{7}} \times 6\frac{1}{4}$ of $\frac{1}{2}$.
4. Find the value of $\frac{6\frac{2}{3}}{4\frac{4}{5}}$ of $7\frac{1}{4}$ of $\frac{1\frac{1}{7}}{3\frac{4}{5}} \times \frac{10\frac{2}{5}}{14\frac{7}{8}}$.
5. Find the product of $24\frac{3}{8} \times 12\frac{4}{9} \times 27\frac{9}{13} \times \frac{7}{12}$.
6. What is $\frac{3}{4}$ of £80?
7. How much is $\frac{5}{6}$ of $3\frac{4}{7}$?
8. Required, the $\frac{17}{25}$ of 750.
9. Find the sum of $\frac{3}{7}$ of £40 $+ \frac{5}{8}$ of the same sum.
10. I paid £$\frac{7}{8}$ of £36, and my debt was $\frac{3}{5}$ of £$74\frac{1}{6}$. How much do I owe still?
11. A does, on the first day, $\frac{1}{3}$ of his work; on the second, $\frac{2}{3}$ of the remainder. How much has he yet to do?
12. A pump gives daily $64\frac{4}{5}$ gallons. How much will it give in $6\frac{3}{4}$ days?
13. On the first day, A performs $\frac{1}{6}$ of his work; on the second, $\frac{1}{4}$ of the remainder; on the third, $\frac{1}{2}$ of what is left; and on the fourth day, he finishes it. How much did he do on the last day?

14. I spent $\frac{3}{4}$ of $\frac{2}{3}+\frac{1}{2}$ of $\frac{5}{6}$ of what I had, and I have £10 left. How much had I?

15. With £1, $4\frac{4}{5}$ yards are bought. How much can be bought for £$\frac{2}{7}$.

16. A traveller had to go 144 miles in 3 days; on the first day he went $\frac{2}{5}$ of the way; on the second, $\frac{1}{3}$. How many miles did he travel each day?

17. 6 lbs. of copper are melted with 4 lbs. of tin. How much of each metal is there in $\frac{3}{4}$ lb.?

18. A room is $34\frac{2}{3}$ feet in length, and $17\frac{4}{5}$ in breadth. Find its area, or the number of square feet in its floor.

DIVISION OF FRACTIONS.

154. We are enabled to make as many cases in division as were made in multiplication.

1st, to divide an integer by a fraction.
2nd, to divide a fraction by an integer.
3rd, to divide an integer by a mixed quantity.
4th, to divide a mixed quantity by an integer.
5th, to divide a fraction by a fraction.
6th, to divide a mixed quantity by a fraction.
7th, to divide a fraction by a mixed quantity.
8th, to divide a mixed quantity by another.

155. 1st, to divide an integer by a fraction, as $4\div\frac{5}{6}$.

Now, $4\div\frac{5}{6}=\frac{4}{\frac{5}{6}}=\frac{4\times 6}{5}=4\frac{4}{5}$. If 4 were divided by 5, the result would be $\frac{4}{5}$; but 4 is to be divided by a quantity 6 times less than 5, viz., $\frac{5}{6}$; therefore, the quotient must be $6\times\frac{4}{5}$, or $4\frac{4}{5}$.

Therefore, to divide an integer by a fraction, multiply the integer by the denominator of the fraction, and divide by the numerator.

Ex $9\div\frac{7}{10}=\frac{9\times 10}{7}=12\frac{6}{7}$; $14\div\frac{11}{12}=\frac{14\times 12}{11}=15\frac{3}{11}$.

156. To divide a fraction by an integer, as $\frac{7}{8} \div 5$.

Here $\frac{7}{8} \div 5 = \frac{\frac{7}{8}}{5} = \frac{7}{5 \times 8} = \frac{7}{40}$. When 7 is to be divided by 5, the result is $\frac{7}{5}$; but when $\frac{7}{8}$, or a quantity 8 times less than 7 is to be divided by 5, then the quotient must be 8 times less than $\frac{7}{5}$, or $\frac{7}{8 \times 5} = \frac{7}{40}$.

Therefore, to divido a fraction by an integer, multiply the integer by the denominator, and divide the numerator by the product.

Thus: $\frac{4}{5} \div 8 = \frac{4}{40} = \frac{1}{10}$; $\frac{11}{12} \div 10 = \frac{11}{120}$.

157. 3rd, to divide an integer by a mixed quantity, as $6 \div 3\frac{2}{5}$.

We have $6 \div 3\frac{2}{5} = 6 \div \frac{17}{5} = \frac{6}{\frac{17}{5}} = \frac{30}{17} = 1\frac{13}{17}$.

Having reduced the mixed quantity into an improper fraction, the explanation is like case 1st.

The quotient of $12 \div 16\frac{3}{4} = 12 \div \frac{67}{4} = \frac{12}{\frac{67}{4}} = \frac{48}{67}$.

The quotient of $127 \div 24\frac{4}{5} = 127 \div \frac{124}{5} = \frac{127}{\frac{124}{5}} = \frac{635}{124} = 5\frac{15}{124}$.

158. 4th, to divide a mixed quantity by an integer, as $4\frac{2}{3} \div 7$.

Here $4\frac{2}{3} \div 7 = \frac{14}{3} \div 7 = \frac{14}{3 \times 7} = \frac{2}{3}$.

When the mixed quantity is transformed into an improper fraction, we have $\frac{14}{3} \div 7$, which brings it under case 2nd.

Ex. Divide $5\frac{5}{7}$ by 9. We have $5\frac{5}{7} \div 9 = \frac{40}{7} \div 9 = \frac{40}{63}$.
„ $36\frac{5}{8}$ by 12? „ $36\frac{5}{8} \div 12 = \frac{293}{8} \div 12 = \frac{293}{96} = 3\frac{5}{96}$.

159. 5th, to divide a fraction by a fraction, as $\frac{7}{9} \div \frac{5}{8}$.

If 7 were divided by 5, the quotient would be $\frac{7}{5}$; then $\frac{7}{9}$, a quantity 9 times less than 7, when divided by 5, gives a quotient 9 times less than $\frac{7}{5}$, or $\frac{7}{45}$; but if $\frac{7}{9}$ be divided by $\frac{5}{8}$, a quantity 8 times less than 5, the result must be 8 times larger than $\frac{7}{45}$, or $\frac{56}{45} = 1\frac{11}{45}$.

Thus, $7 \div 5 = \frac{7}{5}$; therefore, $\frac{7}{9} \div 5 = \frac{7}{45}$; and, therefore, $\frac{7}{9} \div \frac{5}{8} = \frac{56}{45} = 1\frac{11}{45}$.

Therefore, to divide a fraction by another, multiply the numerator of the dividend by the denominator of the divisor, and

divide the product, by the product of the numerator of the divisor by the denominator of the dividend.

$$\text{Ex. } \tfrac{7}{12} \div \tfrac{8}{11} = \frac{11 \times 7}{8 \times 12} = \tfrac{77}{96}$$

$$\tfrac{21}{16} \div \tfrac{8}{15} = \frac{21 \times 15}{8 \times 16} = \tfrac{315}{128} = 2\tfrac{59}{128}.$$

160. 6th, to divide a mixed quantity by a fraction, as $3\frac{3}{5} \div \frac{5}{6}$.

Now, $3\frac{3}{5} \div \frac{5}{6} = \frac{18}{5} \div \frac{5}{6} = \dfrac{18 \times 6}{5 \times 5} = \frac{108}{25} = 4\frac{8}{25}$.

After the mixed quantity has been reduced to an improper fraction, we proceed as in case 5th.

161. 7th, to divide a fraction by a mixed quantity, as $\frac{4}{5} \div 7\frac{2}{3}$.

Evidently, $\frac{4}{5} \div 7\frac{2}{3} = \frac{4}{5} \div \frac{23}{3} = \dfrac{3 \times 4}{5 \times 23} = \frac{12}{115}$.

This case is like case 5th, after the mixed quantity has been transformed into an improper fraction.

162. 8th, to divide a mixed quantity by another mixed quantity as $2\frac{3}{4} \div 5\frac{5}{6}$.

We know that $2\frac{3}{4} \div 5\frac{5}{6} = \frac{11}{4} \div \frac{35}{6} = \frac{66}{140} = \frac{33}{70}$.

Here we are led again into case 5th, after having reduced both mixed quantities to improper fractions.

163. RECAPITULATORY EXERCISES.

1. Find the quotient of $17 \div \frac{7}{8}$; $\frac{11}{12} \div 12$; $36 \div 7\frac{3}{4}$; $15\frac{1}{2} \div 9$; $\frac{8}{11} \div \frac{4}{5}$; $4\frac{7}{8} \div \frac{3}{5}$; $\frac{11}{12} \div 8\frac{1}{6}$; $15\frac{1}{7} \div 4\frac{5}{7}$.

2. Required, the quotient of $(\frac{1}{2} + \frac{3}{5}) \div \frac{3}{4}$; $(2\frac{3}{4} - \frac{7}{8}) \div 1\frac{2}{3}$; $(\frac{5}{6}$ of $3\frac{4}{5}) \div 5\frac{4}{9}$; $43\frac{1}{4} \div (2\frac{1}{2}$ of $\frac{4}{7})$.

3. Divide $\dfrac{14\frac{1}{7}}{15\frac{1}{3}}$ by $\dfrac{25\frac{1}{4}}{36\frac{5}{6}}$; $\left(\dfrac{5\frac{5}{7}}{31\frac{5}{6}} \text{ of } \frac{9}{14}\right)$ by $\dfrac{3\frac{2}{3}}{3\frac{3}{4}}$ of 15.

4. What is the result of $(\frac{1}{3} \times \frac{11}{12} \times 16) \div (\frac{5}{9} \times 11\frac{1}{4} \times 15)$?

5. ,, $\left(4\frac{4}{5} + 1\frac{2}{3} \text{ of } \dfrac{9}{3\frac{3}{4}} - \dfrac{5\frac{1}{6}}{7\frac{1}{2}}\right) \div (\frac{5}{8} \text{ of } 36\frac{7}{9})$?

6. Find the value of $4\frac{3}{4} + \frac{5}{8} - \frac{5}{6} \div 3\frac{1}{4} \div 1\frac{1}{7}$.

7. ,, $(2\frac{1}{3} + 3\frac{1}{4}) \times (1\frac{2}{5} - \frac{7}{9}) \div (3\frac{2}{7} + 5\frac{7}{8})$.

8. Simplify the expression $(4\frac{1}{7} + \frac{1}{3} - \frac{2}{5}) \times 16\frac{2}{7} \div 4\frac{5}{8}$.

9. Reduce $\dfrac{\frac{1}{2}\frac{5}{6}-\frac{1}{3}\frac{7}{8}}{1\frac{4}{7}+6\frac{9}{10}}$; $(3\frac{1}{4}-\frac{4}{5}\times\frac{11}{16})\div(3\frac{5}{6}+3\frac{1}{5}\times\frac{3}{8})$ to their simplest expressions.

10. Determine the value of $\dfrac{\left\{(18\frac{5}{8}+17+4\frac{1}{3}8\frac{3}{4})-(34\frac{1}{6}+26\frac{1}{2})\right\}7\frac{5}{8}}{1\frac{3}{4}\frac{2}{5}}$.

11. Prove that the expression $\dfrac{(5\frac{2}{5}+7\frac{1}{3}+\frac{2}{5}-\frac{3}{4})57\frac{4}{7}}{5\frac{2}{5}}=100$.

12. Find the value of

$$\frac{\left\{(4\frac{3}{4}+2\frac{3}{4})-(4\frac{1}{3}+\frac{7}{8})\right\}\times\frac{3}{10}\times\frac{4}{11}}{\dfrac{7\frac{1}{2}\times\frac{5}{8}}{3\frac{1}{3}}}-(\tfrac{1}{32}+\tfrac{1}{64})+(25\tfrac{2}{3}\tfrac{1}{2}-3\tfrac{1}{2})$$

13. 36 is the product of two numbers, one of which is $10\frac{5}{8}$. Required, the other.

14. If $\frac{4}{7}$ of a railway share cost £$30\frac{1}{2}$, how much will one share cost?

15. In $5\frac{3}{4}$ hours, a wheel makes 11500 revolutions. How many revolutions are made per hour?

16. The $\frac{3}{4}$ of $\frac{4}{5}$ of a sum of money is $36\frac{1}{2}$. What is the sum?

17. A man gave away $\frac{5}{8}$ of his money and had £$2\frac{1}{2}$ left? How much had he?

18. The area of a rectangular field is $892\frac{4}{5}$ yards; its breadth is $32\frac{3}{4}$ yards. Required, its length.

19. A can perform a piece of work in $5\frac{1}{2}$ days, B in $7\frac{1}{4}$ days, and C in $9\frac{1}{2}$ days. In how many days will the three do it together?

20. It is required to place five persons, A, B, C, D, and E, according to their statures; A is $6\frac{1}{4}$ feet high, B is $\frac{3}{4}$ of A's, C is $\frac{4}{5}$ of B's, D is $\frac{4}{5}$ of C's, and E is $\frac{5}{6}$ of D's height.

21. Three contractors propose to dig a canal; the first could do it in $\frac{3}{4}$ of a year, the second in $\frac{3}{5}$ of a year, and the third in $\frac{4}{5}$ of a year. In what time could the three contractors do it?

22, How many yards of linen, $\frac{3}{4}$ yard broad, would be required to line 48 yards of cloth, $\frac{5}{8}$ broad.

23. Two men, 225 miles apart, start at the same time, to meet each other; A goes 5 miles per hour, and B goes 6 miles per hour. In how many hours will they meet?

24. The difference between $\frac{1}{3}$ and $\frac{1}{4}$ of a number is 20. What is the number?

25. A ship's crew has 15 days' provisions, but circumstances oblige them to be at sea 20 days. To what must each man's daily allowance be reduced?

26. A post is coloured so that $\frac{1}{3}$ of it is white, $\frac{1}{5}$ black, $\frac{2}{9}$ blue, and the 12 remaining feet are red. The length of the post is required?

27. A cistern has two feeding pipes; the first fills it in 2 hours, the other in 3 hours; the water is let out through a tap, that empties it in $1\frac{1}{2}$ hours. Suppose the cistern empty, and the two pipes and the tap act at the same time, how long will it take to fill the cistern.

PART IV.

DECIMAL FRACTIONS.

164. In the common system of numeration, it has been shown that a figure has a value ten times smaller than if it occupied the place on its left side; the same law being extended to the right side of the units (after which let a point be placed), we shall find that the first figure next to the units expresses tenths, the second hundredths, the third thousandths, &c.

Thus, 4.4 signify 4 wholes and $\frac{4}{10}$; 34.07 is the same as $34\frac{7}{100}$; .507 is $\frac{5}{10}+\frac{7}{1000}$ or $\frac{507}{1000}$.

175. Then, there are two ways of expressing a fraction, (when the denominator is 10, 100, 1000, &c.,) either as a common fraction, or by writing the numerator after the units' place, taking care to set a point between.

166. Quantities of this kind, which are a part of unity, and whose denominator is 1, followed by one or more ciphers, which denominator, however, is not set down, are called *Decimal Fractions.*

167. Therefore, in *decimals*, the figures after the point express the numerator; and the denominator, which is not set down, is always 1, followed by as many ciphers as there are figures after the point.

168. It will be well to exercise the pupils upon the following scheme, and make them find the value of the figures, taken singly or combined together, in each line, the middle column being that of the units.

Ascending.									Units.	Descending.								
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9

Thus, we perceive that 3 in the units' place has a value of 3 units; moved one place to the left, its value is 10×3 or 30; and moved one place to the right its value is one-tenth of 3, or 3 tenths, and so on.

169. If it were required to write one-tenth, without integer, the expression would be .1; one hundredth, .01; one thousandth, .001, &c. Six-tenths would be .6; five hundredths, .05; eight thousandths, .008, &c.

170. Read and write down the following quantities: 3.6, 50.24, 36.07, 19.004, 31.042, 124.103, 95.0003, .6007, 1.011, 7.00042, 17.200042, 5.00024, 95.0000241, 7.07081.

171. Now, let us notice what alteration takes place in the value of a decimal, when one or more ciphers are added to it, thus: .4, .40, .400, &c. If these fractions be expressed as common fractions, we have; $\frac{4}{10}$, $\frac{40}{100}$, $\frac{400}{1000}$, which are evidently of the same value, Therefore, ciphers annexed to the right of a decimal have no effect upon its value. Then, $.5 = .50 = .500$, &c.; $.26 = .260$; $.005 = .0050 = .00500$, &c.

172. These properties of decimals show us at once their advantages over vulgar fractions, for the tedious process of reduction to the same denominator is here done away with, since decimals which have the same number of figures, have the same denominator. The following examples will illustrate this:— 5.9, 4.24, 7.0072, are the same as 5.9000, 4.2400, 7.0072,

173. If ciphers be prefixed to the left of a decimal, after the point, what effect has it on its value? Let .7, .07, .007, &c., be the fractions.

We know that $.7=\frac{7}{10}$; $.07=\frac{7}{100}$, $.007=\frac{7}{1000}$, &c.; each fraction being one-tenth part of that which immediately precedes it. Therefore, every cipher affixed to the left hand of a decimal, after the point, decreases its value ten-fold.

174. This property leads us to a brief method of multiplying and dividing decimals by 10, 100, 1000, &c. Take, for instance, 24.345, and let the point be moved two places to the right, we have 2434.5. Now, $24.345=\frac{24345}{1000}$, and $2434.5=\frac{24345}{10}$; hence, we see that the second fraction has 100 times the value of the first, because its denominator is 100 times smaller.

175. On the other hand, let the point be moved one place to the left, then 24.345 becomes 2.4345; and, since $24.345=\frac{24345}{1000}$, and $2.4345=\frac{24345}{10000}$, we perceive that the second fraction has a denominator 10 times larger than the first; therefore, the last fraction is 10 times smaller. Hence, a decimal is multiplied by 10, 100, 1000, &c., if the point be shifted one, two, three, &c., places towards the right; and, conversely, a decimal is divided by 10, 100, 1000, &c., if the point be shifted one, two, three, &c., places towards the left. $34.63\times10=346.3$; $1.14\times100=114$; $63.2304\times1000=63230.4$. $241.63\div10=24.163$; $354.2\div100=3.542$; $763.9\div1000=.7639$.

176. To ascertain the greater of two decimal fractions, it is not the number of figures which must be considered, but their magnitude and position. Thus, $.4<.51$; $.7>.5432$; $.005>.00087$; $.09<.1$; $.548>.5437$.

177. Suppose it were necessary to express a vulgar fraction by a decimal. In every fraction the numerator may be considered as the dividend, and the denominator as the divisor; $\frac{3}{5}$ for instance, signifies 3 divided by 5. If we affix decimal ciphers to the numerator, we have: $\frac{3}{5}=\frac{3.0}{5}=\frac{3.0}{5}=.6$; for 5 not being contained in 3 units, is contained in 3.0 or 30-tenths, 6-tenths or .6.

To reduce $\frac{5}{8}$ to a decimal fraction:—

```
8)50(.625
  48
  --
   20
   16
   --
    40
    40
    --
```

In this example, 8 is not contained in 5, but it is contained in 50 tenths, 6 tenths and 2 over, which is altered into 20 hundredths by the addition of a cipher; 8 is contained in 20 hundredths, 2 hundredths, with the remainder 4, which becomes 40 thousandths by adding one cipher, and 8 in 40 thousandths gives 5 thousandths times exactly; therefore, $\frac{5}{8}=\frac{5.000}{8}=.625$. Hence, to reduce a vulgar fraction to a decimal, divide the numerator with as many ciphers added to the right of it, as are necessary, by the denominator, and the quotient will consist of as many decimal places as there are ciphers added.

178. Transform the vulgar fractions $\frac{3}{4}$; $\frac{11}{25}$; $\frac{13}{40}$; $\frac{399}{128}$; $\frac{1399}{2560}$; $\frac{119}{3125}$ to decimals.

179. It sometimes happens that the division will not terminate, but the same figures will recur over again perpetually.

Ex. $\frac{75}{9}=75\div 9=8.333$, &c.; $\frac{3}{7}=.4285714$, &c.; $\frac{5}{18}=.2777$, &c.

Decimals of this kind, in which the figures are continually repeated in some certain order, are called *recurring*, *repeating*, or *circulating decimals*; the part repeated is termed the *period* or *repetend*.

The circulating decimal is *pure* when the period begins immediately after the point, and *mixed* if it consists of a *non-recurring* and a *recurring part*.

The period is distinguished by points placed over the first and last figures of the repetend, and the above results become: $\frac{75}{9}=8.\dot{3}$; $\frac{3}{7}=.\dot{4}2857\dot{1}$; $\frac{5}{18}=.2\dot{7}$.

180. The operation of determining a long period of a decimal whose denominator is large, is shortened by the following process:—

Let $\frac{8}{19}$ be the fraction to be converted into a decimal.

By division we find that $\frac{1}{19}=.05263\frac{3}{19}$;
therefore, $\frac{3}{19}=3\times .05263\frac{3}{19}=.15789\frac{9}{19}$;
and hence, $\frac{1}{19}=.0526315789\frac{9}{19}$;
again, $\frac{9}{19}=9\times .0526315789\frac{9}{19}=.4736842105\frac{5}{19}$;
and, therefore, $\frac{1}{19}=.05263157894736842105\frac{5}{19}$;
hence, $\frac{8}{19}=.\dot{4}2105263157894736\dot{8}42$.

Then the period comprises 18 figures. It will have been observed, that throughout the process, the period consists of the

same figures, whatever the numerator of the fraction is, though they do not begin with the same figure.

181. EXERCISES.

Express as decimals: $\frac{3}{4}$; $\frac{5}{8}$; $\frac{4}{5}$; $\frac{7}{10}$; $\frac{27}{50}$; $\frac{254}{25}$; $\frac{45}{64}$; $2\frac{5}{12}$; $7\frac{9}{16}$; $\frac{1}{3}$ of $\frac{3}{4}$; $\frac{5}{6}$ of $\frac{3}{4}$; $\frac{8}{15}$ of $\frac{11}{16}$; $\frac{5}{6}$; $\frac{2}{3}$; $\frac{4}{11}$; $\frac{19}{99}$; $\frac{6}{37}$; $\frac{21}{63}$; $\frac{7}{9}$; $\frac{1}{36}$; $\frac{11}{7}$; $17\frac{5}{33}$.

182. Let it be required to reduce a terminating decimal to a vulgar fraction.

Since the decimal is the numerator of a common fraction, whose denominator is 1, to which as many ciphers are added as there are digits after the point. Therefore:

$.5=\frac{5}{10}=\frac{1}{2}$; $.75=\frac{75}{100}=\frac{3}{4}$; $.048=\frac{48}{1000}=\frac{6}{125}$; $54.6=\frac{546}{10}$ $=\frac{273}{5}$.

183. EXERCISES.

Convert the following decimals into vulgar fractions: .125; .30; .65; .72; .945; .97; .705; .075; 7.245; 9.027; 16.00075; 18.1642.

184. It has been explained how to reduce any vulgar fraction to a decimal; the converse is, therefore, true. Any decimals, whether pure or mixed recurring, can be represented by a vulgar fraction; we shall now determine a method to find the exact value of a circulating decimal. First, of a pure recurring decimal:

By dividing, we find that $\frac{1}{9}=.1111$, &c. $=.\dot{1}$; hence, $.\dot{1}$ is the decimal equivalent to $\frac{1}{9}$.

Also, $\frac{2}{9}=.2222$ &c. $=.\dot{2}$; hence, $.\dot{2}$ is the decimal equivalent to $\frac{2}{9}$.

$\frac{3}{9}=.3333$ &c. $=.\dot{3}$,, $.\dot{3}$,, ,, $\frac{3}{9}$.

$\frac{4}{9}=$ &c.

Hence, every pure recurring decimal, having one figure in the period, is equivalent to a common fraction, whose numerator is that figure and the denominator 9.

Again, $\frac{1}{99}=.010101$ &c. $=.\dot{0}\dot{1}$; consequently, $\frac{2}{99}=.0202$ &c. $=\dot{0}\dot{2}$; $\frac{56}{99}=\dot{5}\dot{6}$; $\frac{74}{99}=.\dot{7}\dot{4}$.

Hence every pure recurring decimal, having two figures in the period, is equivalent to a common fraction, whose numerator is composed of those figures, and the denominator 99.

Likewise, $\frac{1}{999}=.001001$ &c. $=.\dot{0}0\dot{1}$, and $\frac{754}{999}=.\dot{7}5\dot{4}$ &c.

Similarly, $\frac{1}{9999}=.00010001$ &c. $=.\dot{0}00\dot{1}$, and $\frac{2356}{9999}=.\dot{2}35\dot{6}$ &c.

Hence it follows that any pure circulating decimal is equivalent to a vulgar fraction whose numerator is the period, and denominator as many 9's as there are figures in the period.

Thus: $.\dot{9}6\dot{3}=\frac{963}{999}=\frac{107}{111}$; $.\dot{1}56\dot{9}=\frac{1569}{9999}=\frac{523}{3333}$: $.\dot{0}0072\dot{9}=\frac{729}{999999}=\frac{27}{37037}$

185. **Secondly, when the recurring decimal is mixed.**

Such a decimal may always be decomposed into the sum of two vulgar fractions.

Ex. $.8\dot{5}=\frac{8}{10}+\frac{5}{90}=\frac{8\times 9+5}{90}=\frac{8(10-1)+5}{90}=\frac{80+5-8}{90}=\frac{85-8}{90}=\frac{77}{90}$. Therefore, $.8\dot{5}=\frac{77}{90}$.

In the same manner, $.83\dot{6}\dot{7}=\frac{83}{100}+\frac{67}{9900}=\frac{83\times 99+67}{9900}=\frac{83(100-1)+67}{9900}=\frac{8300+67-83}{9900}=\frac{8284}{9900}=\frac{2071}{2475}$.

$$.31\dot{9}30\dot{6}=\frac{31}{100}+\frac{9306}{999900}=\frac{31\times 9999+9306}{999900}=$$

$$\frac{31(10000-1)+9306}{999900}=\frac{319306-31}{999900}=\frac{319275}{999900}=\frac{129}{404}.$$

From the consideration of the preceding examples, we infer the following law :—

To find the value of a mixed recurring decimal, take for the numerator the difference between the number expressed by the mixed decimal, and that expressed by the non-recurring part, and for the denominator as many nines as there are recurring figures followed by as many ciphers as there are non-recurring figures.

186. Another method, for the reduction of a recurring decimal into an equivalent vulgar fraction, which is well adapted to practice, is the following :—

Let $.\dot{7}$ be the decimal; any symbol x may be used to represent the value required.

Then, $x=.7777$ &c.;

also, $10\,x=7.7777$.

And subtracting the former from the latter, we have :—

$$10x-x=7.7777\text{ \&c.}-.7777\text{ \&c.};$$

or, $9x=7$;

therefore, $x=\frac{7}{9}$.

Let, also, $.\dot{7}\dot{2}$ be the decimal ;

$$\text{then, } x=.\dot{7}\dot{2}\,;$$
$$\text{also, } 100x=72.\dot{7}\dot{2}\,;$$
$$\text{therefore, } 100x-x, \text{ or } 99x=72\,;$$
$$\text{and } x=\tfrac{72}{99}=\tfrac{8}{11}.$$

Again, let $.\dot{3}17\dot{6}$ be the decimal;

$$\text{then, } x=.\dot{3}17\dot{6}\,;$$
$$\text{also, } 10000x=3176.3176,$$
$$\text{and } 9999x=3176\,;$$
$$\text{therefore, } x=\tfrac{3176}{9999}.$$

From these several examples, we arrive at the same conclusion as in § 184.

187. If the decimal was $.3\dot{8}$, let x represent, as before, the fraction required ;

$$\text{then, } x=.3\dot{8},$$
$$\text{and } 10x=3.\dot{8}\,;$$
$$\text{also, } 100x=38.\dot{8}.$$

Subtracting the second line from the third, we find :

$$90x=35,$$
$$\text{and } x=\tfrac{35}{90}=\tfrac{7}{18}.$$

Also, let $.09\dot{0}0\dot{9}$ be a decimal ;

$$\text{then, } x=.09\dot{0}0\dot{9},$$
$$\text{and } 100x=9.\dot{0}0\dot{9}\,;$$
$$100000x=9009.009.$$

By subtracting line two from line three, we have :

$$99900x=9000,$$
$$\text{and } x=\tfrac{9000}{99900}=\tfrac{10}{111}.$$

From which example the same inference is drawn as in § 185.

188. EXERCISES.

1. What are the least vulgar fractions equivalent to $.\dot{6}$; $.5\dot{3}$; $5\dot{9}2\dot{5}$?

2. Required, the least vulgar fractions equivalent to $4.7\dot{5}4\dot{3}$; $.0\dot{0}044\dot{4}$; $.022\dot{7}$.

3. Convert $.\dot{0}2\dot{7}$; $.\dot{6}76190\dot{4}$; $9.37\dot{0}2\dot{4}$ into vulgar fractions.

ADDITION OF DECIMALS.

189. The addition of decimals is performed like simple addition, taking care to set the figures in their respective places.

Suppose we are to determine the sum of 4852.791, 4.00745, 2.7, .049.

$$\begin{array}{r} 4852.79100 \\ 4.00745 \\ 2.70000 \\ .04900 \\ \hline 4859.54745 \end{array}$$

Having set the numbers under each other as mentioned, and reduced to the same denominator or not, we proceed to add up, beginning at the right, putting the point in the sum under those of the quantities proposed.

190. If there were recurring decimals in the quantities to be added up, we should repeat the longest period twice, and sometimes two or three places more, and bring all the others to the same denomination, or reduce into vulgar fractions the recurring decimals, and the sum of all these is reduced to a circulating decimal.

Thus, find the sum of $71.\dot{4}+6.7\dot{2}+18.\dot{7}0\dot{3}+.47\dot{2}+.6\dot{1}8\dot{1}+.54\dot{6}2\dot{9}$.

$$\begin{array}{r} 71.444444444 \\ 6.722222222 \\ 18.703703703 \\ .472222222 \\ .618118118 \\ .546296296 \\ \hline 98.507007005 \end{array}$$

or $98.50\dot{7}0\dot{0}$

Required, the sum of $.\dot{3}$, $.\dot{9}4\dot{5}$, and $.\dot{7}6923\dot{0}$.

$$\begin{aligned} .\dot{3} &= \tfrac{3}{9} &&= \tfrac{333333}{999999} \\ .\dot{9}4\dot{5} &= \tfrac{945}{999} &&= \tfrac{945945}{999999} \\ .\dot{7}6923\dot{0} &= \tfrac{769230}{999999} &&= \tfrac{769230}{999999} \\ & && \overline{\tfrac{2048508}{999999}} = 2.0\dot{4}851 00\dot{4}. \end{aligned}$$

SUBTRACTION OF DECIMALS.

191. We must observe the same order as in addition, in setting down the figures, and proceed as in simple subtraction.

Find the difference of 100.011 and 2.07568 :

$$\begin{array}{r} 100.01100 \\ 2.07568 \\ \hline 97.93532 \end{array}$$

192. When the operation is to be performed with recurring decimals, we must carry out the periods sufficiently, viz., twice the longest period, and two or three places more.

$$\begin{array}{lr} \text{Ex. From} & 5.8300 \\ \text{Take} & 4.1\dot{7}77 \\ \hline & 1.6523 = 1.6\dot{5}2 \end{array} \qquad \begin{array}{lr} \text{From} & 3.3\dot{4}\dot{2}42 \\ \text{Take} & 1.7\dot{5}555 \\ \hline & 1.58687 = 1.5\dot{8}\dot{6} \end{array}$$

Or we might convert the decimals into vulgar fractions, and having found the difference and reduced it to a decimal, it will be the answer required.

Ex. Find the value of $82.\dot{8}54\dot{6} - 8.\dot{7}\dot{2}$.

Here $82.8546 = 82.\frac{8546}{9999}$, and $8.\dot{7}\dot{2} = 8\frac{72}{99} = 8\frac{7272}{9999}$; therefore, $82\frac{8546}{9999} - 8\frac{7272}{9999} = 74\frac{1274}{9999} = 74.\dot{1}27\dot{4}$.

193. EXERCISES IN ADDITION AND SUBTRACTION OF DECIMALS.

Find the value of :

1. $8400 + 5.2007 + 18.63752 + .08 + .000001 + 1.1 + 17.34$.
2. $.\dot{4} + .\dot{0}\dot{7} + 273.14\dot{1}5\dot{6} + .\dot{3}5\dot{7} + 6.534\dot{2}\dot{5} + 111.1$.
3. $9.\dot{9} + 76320.0\dot{2}3\dot{4} + 96.1\dot{6} + 2372.33\dot{4} + 1954.00\dot{7}\dot{5} + 6.3997$.
4. Five ten thousandths + seven thousandths + eight tenths + twenty-five thousandths + four hundreths + two thousandths.
5. Three thousand and five hundredths + forty-five tenths + three hundred and fifty-five thousand and twenty-nine hundredths + two hundred thousand and twelve thousandths + forty-nine thousand and forty-seven tenths.
6. Find the value of $3.4 - .045$; $496.02 - 1.0046$; $1.2 - .5673$.
7. From $.\dot{9}450\dot{1}$ take $.\dot{5}2\dot{7}$; from $.\dot{7}2536\dot{5}$ take $.\dot{3}6747\dot{3}$.

8. From $11.03\dot{6}$ take $6.05\dot{7}\dot{5}$.

9. From 27 tenths take 7984 ten thousandths.

10. Take $4036.\dot{6}9\dot{0}$ from $5658.\dot{6}0\dot{5}$.

MULTIPLICATION OF DECIMALS.

194. We shall have three cases to consider.

1st, when both fractions are terminating.
2nd, when one factor is finite and the other recurring.
3rd, when both factors are recurring.

195. First, to multiply 43.7 by 3.91, we must observe that those quantities are equivalent to $\frac{437}{10}$ and $\frac{391}{100}$; and we know they will be multiplied if the product of the numerators be divided by the product of the denominators;

$$\text{then, } 43.7 \times 3.91 = \tfrac{437}{10} \times \tfrac{391}{100} = \tfrac{170867}{1000} = 170.867.$$

Therefore, the product of two decimals is found like the product of two integers, and point off from the right of the product as many places of decimals as there are decimal places in both factors. If the number of places in the product is not sufficient, as many ciphers must be placed at the beginning as will make up the deficiency. Thus:

2.4542	6.14	.04
.0053	4.06	.007
73626	3684	.00028
122710	24560	
.01300726	24.9284	

196. Secondly, find the product of 7.35 and $.\dot{5}\dot{4}$.

Here we have $7.35 \times .\dot{5}\dot{4} = 7.35 \times \frac{54}{99} = 7.35 \times \frac{6}{11} = \frac{44.10}{11} = 4.\dot{0}0\dot{9}$; or $7.35 \times .\dot{5}\dot{4} = 7.35 \times .545454$, &c. $= 4.00908690$.

Thus, we may either reduce the recurring decimals to a vulgar fraction, and multiply as in (§ 141), then convert the product into a circulating decimal; or multiply as in the first case, taking care to take a sufficient number of periods to ascertain the period of the product.

197. Thirdly, find the value of $3.\dot{3}\dot{6} \times .0\dot{3}48\dot{5}$.

We have: $3.\dot{3}\dot{6} \times .0\dot{3}48\dot{5} = 3\frac{36}{99} \times \frac{3485}{99990} = \frac{37}{11} \times \frac{697}{19998} = \frac{25789}{219978} = \cdot 117234450718.$

The method in which we reduce both factors to vulgar fractions is shorter than the other, because it is necessary to repeat the periods many times, in order to obtain anything like an approximate value.

198. In the generality of problems, it is sufficient if the result be correct within a certain number of decimals, much less than those obtained by the multiplication of the factors. It would then be of a great advantage could we establish a method, by means of which the figures required, alone could be obtained.

199. Let it be proposed to multiply 34.253467 by 5.4637, retaining only three decimal places.

Then we have only to notice, in the several multiplications, the thousandths, the hundredths, the tenths, the units, &c.; but it will be well to mind the ten thousandths, or the fourth place of decimals, on account of their influence on the thousandths, or third place.

Let us now reverse one of the factors, the multiplier, for instance, which becomes 7364.5 ; then place it under the multiplicand, so that 5, the units' place, be under the ten thousandths of the multiplicand, and the tenths' place shall necessarily be under the thousandths' place of the multiplicand, the hundredths under the hundredths', and so on.

By this method, every figure of the multiplier is placed under a figure of the multiplicand, the product of which figures produces ten thousandths.

$$\begin{array}{r}
34.253467 \\
7364.5 \\
\hline
1712673 \\
137014 \\
20552 \\
1027 \\
240 \\
\hline
187.150\cancel{6}
\end{array}$$

This being granted, we proceed to multiply as usual, beginning each figure of the multiplier with the one above it. We then

say, $5 \times 4 = 20$, and 3 carried, for 5×6, make 23; 3 is set down and 2 carried; the rest of the multiplication by 5 proceeds in the usual way. Then, in multiplying the multiplicand by 4, we add 2 to the product, because 18 is nearer 20 than 10, and, therefore, it is nearer the truth to carry 2 than 1; and this result is placed under the former one, so that the last figures are under one another, since they both express ten thousandths.

Proceed in the same manner with regard to the other figures of the multiplier, not neglecting to arrange the partial products, so that their last figures may stand in the same column. The sum is found as usual; four places are marked off from the right for decimals, and the last is omitted, since only three decimal places are required.

Ex. Multiply 763.05403678956 by 254.4630578, so that the decimal in the product may contain five figures, or be correct to .00001.

$$\begin{array}{r}
763.05403678956 \\
\underline{8750364.452} \\
152610807358 \\
38152701839 \\
3052216147 \\
305221614 \\
45783242 \\
2289162 \\
38152 \\
5341 \\
\underline{610} \\
194169.06346\cancel{5}
\end{array}$$

In this example, the units' place of the multiplier reversed, is under the millionth of the multiplicand, and the other figures are easily set down, then proceed as in the last example.

Ex. Multiply .681472 by .01286, so that the decimal in the product may contain five figures.

$$\begin{array}{r}
.681472 \\
\underline{68210.0} \\
6814 \\
1363 \\
545 \\
\underline{41} \\
.00876\cancel{3}
\end{array}$$

Here a cipher is placed below the sixth place of the multiplicand, because the multiplier contains no integer. Ciphers are set to the right of the product, to make up the decimal required.

Ex. Multiply 2.656419 by $1.7\dot{2}\dot{3}$, correct to 6 places.

$$
\begin{array}{r}
2.6564190 \\
3232327.1 \\
\hline
26564190 \\
18594933 \\
531284 \\
79692 \\
5313 \\
797 \\
53 \\
8 \\
\hline
4.577627\cancel{0}
\end{array}
$$

The multiplier has been carried out so as to ensure correctness to six decimal places.

200. EXERCISES ON THE MULTIPLICATION OF DECIMALS.

1. Find the product of 24.36 and 8; 97.25 and 100; 75.05 and 1000.
2. 25.75×3.5; 45.05×15.05; $60.007 \times .027$; $.03054 \times .023$.
3. Multiply 764.30456 by 75.74063; 80096.208034 by 6.00007.
4. $.\dot{3}4\dot{5} \times 46$; $89.6\dot{3}\dot{4} \times 25$; $.0\dot{5}2\dot{3} \times .74$; $.\dot{6}93\dot{5} \times 2.36$.
5. Multiply $362.\dot{4}0\dot{5}$ by $.00\dot{3}$; $8457.\dot{3}$ by $87.\dot{5}$; $8.\dot{4}631\dot{4}$ by $5.\dot{0}3206\dot{4}$.

DIVISION OF DECIMALS.

201. We shall consider the three following cases:—
1st, when the dividend is a decimal and the divisor an integer.
2nd, when the dividend is an integer and the divisor a decimal.
3rd, when both divisor and dividend are decimals.

202. First, when the dividend is a decimal and the divisor an integer.

Divide 487.83 by 21.

```
21)487.83(23.23
   42
   ——
    67
    63
    ——
     48
     42
     ——
      63
      63
      ——
```

We find, first, the quotient of 48 tens, and 21, which is 2 tens; secondly, the quotient of 67 units and 21, which is 3 units; thirdly, the quotient of 48 tenths and 21 is 2 tenths; therefore, the decimal point must be placed before the tenths; lastly, the quotient of 63 hundredths and 21 is 3 hundredths.

Therefore, when the dividend alone contains decimals, proceed as in common division, setting the decimal point after the tenths' place of the dividend has been brought down.

203. Secondly, when the dividend is integer, and the divisor a decimal.

Divide 951 by 4.5.

$$951 \div 4.5 = \frac{951}{4.5} = \frac{9510}{45} = 211.\dot{3}.$$

Also, divide 19 by .0024.

$$19 \div .0024 = \frac{19}{\frac{24}{10000}} = \frac{190000}{24} = 7916.\dot{6}.$$

Therefore, if the divisor only is a decimal quantity, annex to the right of the dividend as many ciphers as there are decimals in the divisor, and divide as in common division; to the last remainder ciphers are added to get decimals, and the process is carried on to the approximation required.

204. Thirdly, when both divisor and dividend are decimal.

$12.5816 \div 4.7 = \frac{12.5816}{4.7} = \frac{125816}{47000} = 2.6769$; or thus: $12.58160 \div 4.7 = 2.6769$.

In the first method we have multiplied both dividend and divisor, by such number as will make the longest decimal integer, and the division is performed as before, annexing ciphers to obtain decimals in the quotient.

Or, by the second method, which is an abbreviation of the first, divide as whole numbers, attaching ciphers to the dividend

when necessary; then the quotient will have a number of decimal places equal to the excess of the number of decimal places of the dividend above that in the divisor.

205. This last law applies generally to every case in the division of decimals.

Ex. $18.43 \div .0768$

```
.0768)18.43(239.97395
      1536
      ----
       3070
       2304
       ----
        7660
        6912
        ----
         7480
         6912
         ----
          5680
          5376
          ----
           3040
           2304
           ----
            7360
            6912
            ----
             4480
             3840
             ----
              640
```

Ex. $4444 \div .072266$.

```
.072266)4444.00(61495.0322
        433596     42 nearly
        ------
        108040
         72266
        ------
        357740
        289064
        ------
         686760
         650394
         ------
          363660
          361330
          ------
           233000
           216798
           ------
            162020
            144532
            ------
            174880
            144532
            ------
             303480
             289064
             ------
             144160
             144532
             ------
```

206. If the divisor be a recurring decimal, it will be more convenient to reduce it to a vulgar fraction; the dividend had better be unaltered when a recurring decimal.

Ex. $\dfrac{23.5}{.\dot{4}} = \dfrac{23.5}{\frac{4}{9}} = \dfrac{23.5 \times 9}{4} = 52.875.$

Ex. $\dfrac{3.\dot{3}7\dot{0}}{.7} = 4.81\dot{4}8\dot{1}.$

Ex. $\dfrac{17.\dot{4}\dot{5}}{.1\dot{8}\dot{4}} = \dfrac{17.\dot{4}\dot{5}}{\frac{183}{990}} = \dfrac{17.\dot{4}\dot{5} \times 990}{183} = \dfrac{17280}{183} = 94.42623$ nearly.

207. There is also a method for shortening the labour of the division of two numbers, each consisting of many figures. This

method, which we are going to investigate, depends upon this fact: that every figure of the quotient is obtained by dividing the two or three last figures of the several partial dividends by the two first figures of the divisor. Thus, the quotient will be correct if we operate upon the two or three first figures of every partial dividend. We subjoin both operations. The common method will explain the reason of the contracted method.

208. Let the quotient of 1234.569 and 27.35894 be required, correct to three decimal places.

```
27.35894)1234.56|90(45.124     27.35894)1234.56900000(45.124
          109435|76                ......  109436
          ------|----                      ------
           14021|140                        14020
           13679|470                        13679
           -----|----                       -----
             341|6700                         341
             273|5894                         273
             ---|-----                        ---
              68|08060                         68
              54|71788                         55
              --|------                        --
              13|362720                        13
              10|943576                        11
              --|------                        --
               2|419144                         2
```

The operation on the left hand is performed according to the common method; therefore, we shall not make any remark upon it.

Begin by making the number of decimal figures of the dividend equal to that of the divisor, by adding two ciphers, and as three decimal places are required in the quotient, annex three more ciphers to the dividend, then cut off *six* figures from the right of the dividend, since there are *seven* in the divisor; but as the divisor is not contained in the dividend. two figures of it are omitted, which is expressed by placing dots below them; divide now 123456 by 27358, the quotient is 4; multiply 27358 by 4, and add to the product 4, proceeding from 4 times the last figure cut off, or 36, which is nearer 40 than 30, the remainder is 14020; instead of adding the next figure of the dividend to it, we omit the 8 from the divisor, which is denoted by placing a point below it, and we divide 14020 by 2735, the quotient is 5, multiply 2735 by 5, and add to the product 4, proceeding from 40, the product of the omitted figure and 5, the remainder is 341; we now reject 5 from the

divisor, and place a point below it, divide 341 by 273, the quotient is 1, multiply 273 by 1, and subtract the product from 341, the remainder is 68. Omit the 3 from 273, and divide 68 by 27, the quotient is 2 and the remainder 13. Reject 7 from 27 and divide 13 by 2, the quotient is 4, since $4 \times 2 + 3$ to carry from 28, make 11; the last remainder, 2, is neglected. The quotient is found to be 45124, from which we point out the three decimal places required, and for which we annexed three figures in the beginning; the answer is then 45.124.

In comparing both operations, it is easily perceived that the figures rejected exercise no influence on the figures of the quotient.

Instead of preparing the dividend as we did, we may notice that the number of figures of the divisor used at first, is the same as the number of figures of the quotient; therefore, we may begin by rejecting as many figures to the right of the divisor as its own number of figures exceeds that of the quotient, then find how many times the divisor, thus prepared, will be contained in the first figures of the dividend, and proceed as explained above. By a single observation, it is generally easy to see how many places of whole numbers the quotient will contain. In this example it is seen, by dividing 1234 by 27, to be two places, and there are three decimal places to be found, so together we have to determine five places in the quotient.

Determine the quotient of 229.4703568 and 7.3596 correct to four places.

The quotient will contain two places of integers and four decimal places, together six places; but as the divisor contains only five figures, a cipher is annexed to it, and we proceed as explained before.

```
7.35940)229.4703568(31.1805
  .....   2207820
          -------
            86883
            73594
            -----
            13289
             7359
             ----
             5930
             5887
             ----
               43
               37
               --
                6
```

Divide .625 by $.\dot{4}2857\dot{1}$ correct to six places of decimals.

$$
\begin{array}{r l}
.\dot{4}2857\dot{1}).6250000(1.458335 & \\
4285712 & \\
\hline
1964288 & \\
1714285 & \\
\hline
250003 & \\
214286 & \\
\hline
35717 & \\
34280 & \\
\hline
1437 & \\
1286 & \\
\hline
151 & \\
128 & \\
\hline
23 & \\
21 & \\
\hline
2 &
\end{array}
$$

Divide $.3\dot{4}7\dot{4}$ by $.\dot{6}$, retaining six decimal places.

$$
\begin{array}{r l}
.666666).3\dot{4}74\dot{4}74(.521170 & \\
3333330 & \\
\hline
141144 & \\
133333 & \\
\hline
7811 & \\
6667 & \\
\hline
1144 & \\
667 & \\
\hline
467 & \\
467 & \\
\hline
\end{array}
$$

209. EXERCISES ON THE DIVISION, &c., OF DECIMALS.

1. Required, the quotient of 67.4903 and 8.04; of .3495 and .735; of .026705 and .0834; of .0572 and .0014; of .0594 and 26356; of 1 and 10.1673.

2. Divide 2.295 by $.2\dot{9}\dot{7}$; $2.1453\dot{5}$ by $.0\dot{7}$; $234.\dot{6}$ by $.\dot{7}$; $41.\dot{5}43\dot{2}$ by $34.\dot{4}\dot{8}$; $27.217948\dot{7}$ by $.\dot{1}1534\dot{6}$; $.056\dot{3}21\dot{5}$ by $1.34\dot{5}6\dot{2}$,

3. Find the sum, difference, product, and quotient of $.5\dot{7}3\dot{6}$ and .23.

4. $3.562+16.71-3.05+.394\times.62\div3.45$.

REDUCTION OF DECIMALS.

210. The knowledge which the learner must now have acquired about vulgar and decimal fractions, will very much facilitate the study of this section.

First, we propose to find the value of any decimal of a given quantity.

Example: What is the value of .86875 of £1, or £.86875?

£.86875 = .86875 × 20s. = 17.37500 shillings.

.375s. = .375 × 12d. = 4.500 pence.

.5d. = .5 × 4f. = 2.0 farthings.

Or briefly thus:

$$\begin{array}{r} £.86875 \\ 20 \\ \hline 17.37500 \\ 12 \\ \hline 4.500 \\ 4 \\ \hline 2.0 \end{array}$$

Therefore, £.86875 = 17s. 4½d.

Example: Convert .00213 of a day to positive terms:

Here .00213 day = .00213 × 24 hrs. = .05112 hrs.;

and .05112 hrs. = .05112 × 60 min. = 3.06720 min.;

and .0672 min. = .0672 × 60 sec. = 4.032 sec.

or thus:

$$\begin{array}{r} .00213 \\ 24 \\ \hline .05112 \text{ hrs.} \\ 60 \\ \hline 3.06720 \text{ min.} \\ 60 \\ \hline 4.03200 \end{array}$$

Therefore, .00213 day = 3 min. 4.032 sec.

Example: What is the value of $3.\dot{7}6\dot{5}$ of a cwt?

Here $3.\dot{7}6\dot{5}$ cwt. $=3\frac{85}{111}$ cwt. $=3$ cwt. 3 qrs. 1 lb. 12.25 oz.

From these examples, it follows that, to express by means of known units the value of decimals, multiply the given decimal by the numbers which would reduce it to the lower denominations were it an integer, and the integral parts of the products, pointed off as they occur, will be the required value. If the decimal be expressed as part of more than one denomination, reduce it to one, and then apply the law. In case of recurring decimals, it is generally better to reduce it to a common fraction and to find the value.

211. EXERCISES.

1. Find the value of .64 of £1; .6325 of £1; 1.626 of 1 guinea.
2. Express 32.08 of £5; 2.16506 of £50; .205 of 6s. 8½d.
3. What is the value of 1.265 of 1 ton; of .275 of 1 mile; .025 of 1 yard?
4. Required, the value of $36.\dot{8}$ of 24 tons 2 cwt.; of $.\dot{3}7\dot{5}$ of 6s. 8d.
5. Determine the value of $.\dot{6}$ of 3s. 9d. + $.\dot{0}3\dot{7}$ of 27s.—$.0\dot{8}\dot{1}$ of £3. 5s.

212. Secondly, we have to consider the manner of reducing a given quantity to the decimal of another quantity.

Example: Express 16s. 9½d. as the decimal of £1.

Since ½d. is the $\frac{1}{480}$ of £1;
therefore, 16s. 9½d, or $\frac{403}{2}$d. are $403\times\frac{1}{480}$ of £1, or £$\frac{403}{480}$; o £$.83958\dot{3}$; or thus, ½d. $=.5$d.; 9½d. $=9.5$; 9.5d. reduced to th decimal of 1 shilling is $\frac{9.5}{12}=.791\dot{6}$; and 16.7916s., express as a decimal of £1, is $\frac{16.7916}{20}=.83958\dot{3}$. The operation here exhibited:—

$$\begin{array}{l} 12\underline{)9.5\text{d.}} \\ 20\underline{)16.791\dot{6}\text{s.}} \\ \text{£}.83958\dot{3} \end{array}$$

Example: Reduce 13s. $6\frac{3}{4}$d. to the decimal of 15s. 6d.

Here 15s. 6d. are 186d.;

and 1d. is the $\frac{1}{186}$ of 15s. 6d.;

therefore, 13s. $6\frac{3}{4}$d., or 162.75d. are $162.75 \times \frac{1}{186}$, or $\frac{162.75}{186}$, or ·875 of 15s. 6d.;

thus, 3f. = .75d.

$$\begin{array}{r} 12\overline{)6.75} \\ 15.5\overline{)13.5625} \\ .875 \text{ of 15s. 6d.} \end{array}$$

Hence, to reduce a given quantity to the decimal of another quantity, bring both quantities to the same denomination, and divide the first by the second; or, when convenient, divide the lowest denomination by the number which connects it with the next, and affix to the left of the quotient the number of this denomination; reduce the result to a decimal of the next higher denomination, and so on, till the required denomination is obtained.

213. EXERCISES.

1. Reduce 7s. $6\frac{1}{2}$d. to the decimal of £1.
2. Reduce 17 cwt. 2 qrs. 15 lb. to the decimal of 1 ton.
3. Reduce 6 oz. 5 dwt. to the decimal of 1 lb.
4. Reduce 5 days 15 hrs. 16 min. 5 sec. to the decimal of 1 week.
5. Express 2 r. 16 p. as the decimal of 1 acre.
6. Express 496 yards as the decimal of 1 mile.
7. Express 19s. $11\frac{3}{4}$d. as the decimal of £2. 10s.
8. Express £3. 15s. $6\frac{1}{2}$d. as the fraction of £6.

214. MISCELLANEOUS EXERCISES IN DECIMALS.

We have given here a great variety of examples, in order to familiarize the pupil with the several parts explained before. He has already experienced that some problems are susceptible of several solutions, but there is generally one which is preferable to others; let it, therefore, be his aim to choose the one best adapted to the conditions of the question.

1. A merchant owes a certain sum of money, he pays severally £248.40, £150.60, £545.75, and lastly, a bill of £500, and receives back £147.75. What was the debt?

2. Had I £24.50 more, I should have £100. What money have I ?

3. The smaller of two numbers is 2675.894, and their difference is 11.896. What is the larger number ?

4. What is that number, which being multiplied by .55, the product is 65.8 ?

5. The remainder of a number divided by 25 is 12. Express its decimal value.

6. Express the quotient of 24.589 by 73, correct to five decimal places.

7. By what number must 12.668569 be multiplied so that the product may be 50.674236 ?

8. The gain arising from a speculation is £127.55, which is one-sixteenth of the money received. How much was at stake ?

9. At a dinner, every guest paid 2.875 shillings, and the whole receipts were £3. 17s. $7\frac{1}{2}$d. How many persons partook of the dinner ?

10. What is the half of the fourth part of twenty-one times 224.56 ?

11. 155 yards of cloth are sold for £178.25. How many yards will be sold for £74.75 ?

12. The expenses of 18 persons, half of whom were men, and the other half women, are 130.50 shillings; each man pays 4.50 shillings more than a woman. What does each person pay ?

13. A person promised to pay 6s. to the poor every time his money was doubled ; this was repeated three times, and he had nothing left. How much had he at first ?

14. Bought, 3 cwt. 1 qr. 17 lb. sugar for £$13.3864\dot{3}$, and sold it again at $\frac{17}{18}$ shillings per lb. How much did I gain ?

15. The circumference of every circle is 3.1416 times its diameter. If the circumference of the earth be 24857 miles, how much is its radius ?

16. Find the value of £$.8 + .2635$ shillings $+ .\dot{5}$ guinea.

17. The length of the true year is 365.242264 days. What will be the error in five centuries, if the years are taken at $365\frac{1}{4}$ days ?

18. What decimal, multiplied by 54, expresses the sum of $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{12}$, $\frac{11}{16}$?

19. Divide the difference of 11.229587 and 3.816243 by 274.

20. Air is 770 times lighter than water, and mercury 13.598 times heavier than water. How many times is mercury heavier than air?

21. One degree Fahrenheit is 1.80 degrees centigrades. How many degrees centigrades are 17.5 degrees Fahrenheit?

22. Simplify the expression $4.56+15.037-\frac{4}{7}$; also, $(4.73+15.6)\times(36.1-14.72.)$

23. Determine the value of $123\times(\frac{82.6}{156}-\frac{26.7}{2.67}+36)$.

24. Determine the value of $\frac{7.7}{15.1}+32\times(\frac{7.1}{7.2}-\frac{12}{13}+1)$.

25. Express in positive terms $\dfrac{16.66}{51-16}-\dfrac{14-8+13.6-14}{8\times17-33}$.

26. Reduce $\dfrac{(14.77+1.05)\times(15.04-.899)}{14.77+1.05-15.04+.899}$.

27. Simplify $\dfrac{2\frac{1}{2}}{\frac{14}{15}}\times\dfrac{.1\dot{2}\dot{7}}{1.\dot{2}\dot{7}}\div\dfrac{.\dot{1}7968\dot{7}}{.\dot{5}3846\dot{1}}$ of $\dfrac{\frac{3}{11}}{\frac{9}{22}}$.

28. Reduce $\dfrac{(43+\frac{26}{25})-15\times(\frac{43}{12}-1)}{36+.\frac{14}{01}+.\frac{14}{001}\times(81-34}$.

29. Find the value of $.208\dot{3}$ of $.\dot{3}42857\dot{1}$ of 1 cwt.

30. Required, the value of $.\dot{8}4615\dot{3}$ of $.0\dot{8}\dot{1}$ of £6.50.

31. How much money must a person distribute who wishes to bestow £.375 a piece on 248 poor persons?

32. Add together $\frac{5}{8}$, $\frac{7}{16}$, $\frac{3}{4}$, .09375 and 2.40 by vulgar fractions and by decimals.

33. Subtract .6495 of 1 guinea from .8735 of £1.

34. Reduce to a decimal, accurate to 7 places: $1+\dfrac{1}{1}+\dfrac{1}{1\times2}+\dfrac{1}{1\times2\times3}+\dfrac{1}{1\times2\times3\times4}$, &c.

35. Reduce £24. 16s. 4½d. and £167. 10s. 6¼d. ½f. to decimals of pounds, and find the quotient of the first divided by the latter.

36. The solar year consists of 365.242264 days, and the sidereal year of 365.256383 days. Express the difference of these years in minutes, seconds, &c.

37. Required, in decimals and in common fractions, the sum of the following quantities: $\frac{3}{5}$, $\frac{7}{9}$, $\frac{3}{7}$, $\frac{1}{4}$, $\frac{2}{3}$, $\frac{5}{6}$; and find, also, the product of both results.

38. A bar of iron expands 1.472 inches for every yard, when brought to a certain temperature. How much would a bar 7 yds. 2 ft. 10.34 inches expand at the same temperature?

PART V.

TABLES OF MONEY, WEIGHTS AND MEASURES. COMPOUND QUANTITIES.

215. In the preceding parts it has been shown that when it is necessary to consider quantities smaller than unity, this unity is supposed to be divided into a certain number of equal parts, which are treated as other units. But in order to facilitate dealings, it has been found convenient to have large units and small units; for instance, when speaking of long distances, we take a mile as the unit, and where it is required to express smaller distances, we take the yard, foot, or inch as unity; thus, if instead of dividing the mile into a great number of equal parts, and saying $\frac{1}{1364}$ of a mile, or $\frac{1}{1460}$ of a mile, the difference between which would be difficult to appreciate, I were to say, this road is four yards broad, and that five yards, the mind would at once form an idea of the real dimensions.

What strikes every one, when looking at the tables of money, weights, measures, &c., is the confusion, the want of system, which exists almost throughout. They are the offspring of unscientific minds, of different epochs; they are like patchworks placed in the way of youths to puzzle them and retard their progress. It is astonishing, that in a country so enlightened as England is, some steps are not taken to improve this extraordinary labyrinth. This improvement appears the easier since some continental nations, and scientific men in this country, have already adopted the whole or part of the decimal system, which is the only rational one.

TABLES OF MONEY, WEIGHTS, AND MEASURES.

216. TABLE OF MONEY.

2 farthings	=	1 halfpenny	½d.
4 farthings	=	1 penny ...	1d.
12 pence	=	1 shilling...	1s.
20 shillings	=	1 pound ...	£1.

4f. = 1d.
48f. = 12d. = 1s.
960f. = 240d. = 20s. = £1

The coins not in circulation, but frequently mentioned, are the groat = 4d.; tester = 6d.; noble = 6s. 8d.; angel = 10s.; half-guinea = 10s. 6d.; mark = 13s. 4d; guinea = 21s.; Carolus = 23s.; Jacobus = 25s.; moidore = 27s.

The pound is generally called *pound sterling*, to distinguish it from the weight called a pound; also from foreign coins, and from stocks, shares, &c.

The standard gold coin of this kingdom is made of 22 parts of pure gold and two parts of alloy (copper). From a pound Troy of this metal are coined $46\frac{29}{40}$ sovereigns = £46. 14s. 6d.; so that the weight of each is 5 dwts. $3\frac{171}{623}$ grs. = 123.274 grs.; and the value per ounce of the *standard* gold is £3. 17s. $10\frac{1}{2}$d., whilst the value per ounce of *pure* gold, at the Mint, is £4. 4s. $11\frac{5}{11}$d.

The standard silver coin consists of 37 parts of pure silver and three parts of alloy (copper). From a pound Troy of this metal are coined 66 shillings, so that the weight of a shilling is 3 dwts. $15\frac{3}{11}$ grs.; and at the Mint, the price of standard silver is 5s. 6d. per oz., and of pure silver 5s. $11\frac{13}{37}$d. Silver coins are not legal tender for more than 40s.

24 pence are coined from a pound of copper avoirdupois, so that a penny weighs $10\frac{2}{3}$ dwts. avoirdupois, or $291\frac{2}{3}$ grs. Troy. Copper coins are not legal tender for more than 12d.

217. TABLE OF TROY WEIGHT.

Grains are	marked	gr.
24 grains make 1 pennyweight	,,	dwt.
20 pennyweights 1 ounce	,,	oz.
12 ounces 1 pound	,,	lb.

gr. dwt.
24 = 1 oz.
480 = 20 = 1 lb.
5760 = 240 = 12 = 1

This weight, which was formerly used for weighing articles of every kind, is now applied to gold, silver, jewels, liquors, and in philosophical experiments.

A grain of wheat is said to be the original of all weights used in England. It was gathered out of the middle of the ear, and being well dried 32 of them were to make one pennyweight;

but in later times it was thought sufficient to divide the same pennyweight into 24 equal parts, still called grains, though really a third part greater than the original grain. The penny-weight was so called because it was the weight of the silver penny then in use. Ounce and pound are derived from the Latin.

Troy weight was introduced into France about the time of the Crusades, from Cairo, in Egypt. It was first adopted in Troyes, where great fairs were held, and whence it has its name. It is also believed that the word Troy has been derived from the monkish name given to London. The legend avers that Brute, a direct descendant of Æneas, founded the city of London, in 2855 A.M., and called it Trinovantum, corrupted afterwards into Tronevant, or Troynovant.

218. TABLE OF APOTHECARIES' WEIGHT.

20 grains	make	1 scruple.........	sc. or ℈
3 scruples	,,	1 dram	dr. or ʒ
8 drams	,,	1 ounce	oz. or ℥
12 ounces	,,	1 pound	lb. or ℔

gr.	sc.			
20=	1	dr.		
60=	3=	1	oz.	
480=	24=	8=	1	lb.
5760=	288=	96=	12=	1

The gr., oz., and lb. are the same as in Troy weight.

This weight is employed by apothecaries in mixing up medical prescriptions, though they buy and sell their drugs by avoirdupois weight.

219. AVOIRDUPOIS WEIGHT.

Drams			marked	dr.
16 drams	make	1 ounce	,,	oz.
16 ounces	,,	1 pound	,,	lb.
14 pounds	,,	1 stone	,,	st.
2 stones, or 28 lbs.	,,	1 quarter.........	,,	qr.
4 quarters	,,	1 hundredweight	,,	cwt.
20 hundredweight	,,	1 ton	,,	ton.

dr.	oz.				
16=	1	lb.			
256=	16=	1	qr.		
7168=	448=	28=	1	cwt.	
28672=	1792=	112=	4=	1	ton.
573440=	35840=	2240=	80=	20=	1

The lb. avoirdupois contains 7000 grains, or 14 oz. 11 dwts. 16 grs. Troy, or $\frac{175}{144}$ lb. The lb. Troy $= \frac{144}{175}$ lb. av. $=$ 13 oz. $2\frac{114}{175}$ drs.

By this weight all heavy and coarse goods are weighed; the imperial pound avoirdupois is defined in an Act of Parliament, which came into operation on the 1st of January, 1826, to be the weight of one-tenth of an imperial gallon, or of 27.7274 cubic inches of distilled water, at the temperature of 62° Fahr., when the barometer is at 30°.

The name avoirdupois is derived from *avoirs*, meaning, in old Norman, *goods* or *chattels*, and *poids*, signifying *weight*.

220. WOOL WEIGHT.

7	pounds	are	1 clove	marked	cl.
2	cloves	,,	1 stone	,,	st.
2	stones	,,	1 tod	,,	tod.
$6\frac{1}{2}$	tods	,,	1 wey	,,	wey.
2	weys	,,	1 sack	,,	sack.
12	sacks	,,	1 last	,,	last.
20	pounds	,,	1 score	,,	score.
12	scores	,,	1 pack	,,	pack.

221. TABLE OF LINEAL MEASURE.

3	barleycorns	make	1 inch	marked	in.
12	inches	,,	1 foot	,,	ft.
3	feet	,,	1 yard	,,	yd.
6	feet	,,	1 fathom	,,	fth.
$5\frac{1}{2}$	yards	,,	1 pole, rod, or perch	,,	pl.
40	poles	,,	1 furlong	,,	fur.
8	furlongs	,,	1 mile	,,	mile.
3	miles	,,	1 league	,,	lea.
$69\frac{1}{2}$	miles nearly	,,	1 degree	,,	deg. or °.

in.	ft.				
12 =	1	yd.			
36 =	3 =	1	pl.		
198 =	$16\frac{1}{2}$ =	$5\frac{1}{2}$ =	1	fur.	
7920 =	660 =	220 =	40 =	1	mile.
63360 =	5280 =	1760 =	320 =	8 =	1

By the act above-mentioned, it was enacted that the yard should henceforth be the standard of length, which is equivalent to 3 feet, or 36 inches. The inch appears to have been originally obtained by putting together 3 grains of barley, and the

yard from the length of the arm of Henry I. of England. But in order to have an invariable standard, it has been ascertained that in the latitude of Greenwich the pendulum vibrating seconds is 39.1393. inches

222. CLOTH MEASURE.

2¼ inches	make	1 nail	.........marked	nl.
4 nails	,,	1 quarter	,,	qr.
3 quarters	,,	1 ell Flemish	,,	e. Fl.
4 quarters	,,	1 yard	,,	yd.
5 quarters	,,	1 ell English	,,	e. E.
6 quarters	,,	1 ell French...	,,	e. Fr.

Other lineal measures sometimes met with are :—

A line $=\frac{1}{12}$ in.
A palm $=3$ in.
A hand $=4$ in.
A span $=9$ in.
A cubit $=18$ in.
A pace $=5$ ft.
A link $=7\frac{92}{100}$ in.
A chain $=100$ links.

223. TABLE OF SUPERFICIAL OR SQUARE MEASURE.

144	square inches ....make	1 square foot	marked	ft.
9	square feet............... ,,	1 square yard	,,	yd.
30¼	square yards .... ,,	1 square pole	,,	pl.
40	square poles .... ,,	1 rood	,,	rd.
4	roods, or 10 sq. chains ,,	1 acre	,,	acre.
640	acres .... ,,	1 square mile	,,	mile.

sq. in.		sq. ft.						
144	=	1		sq. yd.				
1296	=	9	=	1	sq. pole.			
39204	=	272¼	=	30¼	= 1 rd.			
1568160	=	10890	=	1210	= 40	= 1 acr.		
6272640	=	43560	=	4840	= 160	= 4	= 1	

224. LAND MEASURE.

40 poles make 1 rood, marked ro.
4 roods ,, 1 acre ,, ac.
Also, 1210 square yards, or 25000 links = 1 rood.
4840 square yards, or 100000 links = 1 acre.

225. CUBIC OR SOLID MEASURE.

1728	cubic inches	are	1 cubic foot.
27	cubic feet	„	1 cubic yard.
40	cubic feet	„	1 load of rough timber.
227.274	cubic inches	„	1 imperial gallon.

A cubic foot of water, at a temperature of 60°, weighs very nearly 1000 ounces.

226. DRY AND CORN MEASURE.

2 pints	.............make	1 quart	......marked	qt.
2 quarts	.... „	1 pottle	„	pot.
2 pottles	.... „	1 gallon......	„	gal.
2 gallons	.... „	1 peck	„	pec.
4 pecks	.... „	1 bushel ...	„	bu.
2 bushels	.... „	1 strike......	„	str.
2 strikes or 4 bushels	„	1 coom	„	coom.
2 cooms or 8 bushels	„	1 quarter ...	„	qr.
5 quarters	.... „	1 wey or load	„	wey.
2 weys	.... „	1 last	„	last.

pts.	gal.					
8 =	1	pec.				
16 =	2 =	1	bu.			
64 =	8 =	4 =	1	qr.		
512 =	64 =	32 =	8 =	1	wey.	
2560 =	320 =	160 =	40 =	5 =	1	last.
5120 =	640 =	320 =	80 =	10 =	2 =	1

Corn, seeds, roots, fruits, salt, sand, oysters, and such dry goods as are not usually heaped up above the measure of capacity are sold by the above measure. The act states that a gallon contains 277.274 cubic inches. 10 lbs. av. of distilled water, weighed in air at 62° Fahr., the barometer at 30°, will just fill this space; also the imperial bushel contains 2218.192 cubic inches. The measure is to be a cylinder, the internal diameter of which is 18.789 inches, and depth 8 inches.

227. COAL MEASURE.

4	pecks	...... are	1 bushel.
3	bushels	... „	1 sack.
36	bushels	... „	1 chaldron.
21	chaldrons	... „	1 score.

Since coals are sold by weight, this table is of little use. The act directs that the *heaped imperial bushel* shall contain 2815.4887 cubic inches.

228. TABLE OF ALE AND BEER MEASURE.

2	pints	make	1 quart	marked	qt.
2	quarts	„	1 gallon	„	gal.
42	gallons	„	1 tierce	„	tier.
1½	tierces, or 63 gallons	„	1 hogshead	„	hhd.
2	tierces	„	1 puncheon	„	pun.
2	hogsheads	„	1 pipe or butt	„	pi.
2	pipes	„	1 tun	„	tun.

pts.	qt.						
2=	1	gal.					
8=	4=	1	tier.				
336=	168=	42=	1	hhd.			
504=	252=	63=	1½=	1	pun.		
672=	336=	84=	2 =	1½=	1	pi.	
1008=	504=	126=	3 =	2 =	1½=	1	ton.
2016=	1008=	252=	6 =	4 =	3 =	2=	1

This table is used to measure wines, spirits, cyder, mead, perry, vinegar, oil, honey, &c.

229. TABLE OF MEASURE OF TIME.

A second		1 sec. or 1″.
60 seconds	make	1 minute, or 1′.
60 minutes	„	1 hour, or 1 hr.
24 hours	„	1 day, or 1 day.
7 days	„	1 week, or 1 wk.
4 weeks	„	1 month, or 1 mo.
13 months, 1 day, 6 hours, or 365 days, 6 hours		1 Julian year, or 1 yr.

sec.	min.					
60=	1	hr.				
3600=	60=	1	day.			
86400=	1440=	24=	1	wk.		
604800=	10080=	168=	7 =	1	mo.	
2419200=	40320=	672=	28 =	4 =	1	yr.
31557600=	525960=	8766=	365¼ =	$52\frac{5}{28}$ =	$13\frac{5}{112}$ =	1

The month mentioned here is the lunar month, but a common year consists of 12 calendar months, of $30\frac{5}{12}$ days nearly.

The true solar year is 365 days 5 hrs. 48 min. 48 sec. (which is 11 min. 12 sec. less than the Julian year), but as it was most convenient to begin the year on the commencement of a day, it was agreed, in the time of Julius Cæsar, that the year should, during 3 successive times, consist of 365 days, called a *common* year, and of a year of 366 days called a *leap* year; and it is so managed that whenever the number of years is divisible by 4 the corresponding year is a leap year, the month of February having 29 days instead of 28.

But as the true solar year is 365.242264, and not 365.25, the correction is too much by 0.007736 day; by finding how many times this is contained in 1 day, we shall know in how many years the error amounts to 1 day, which is every 129.2657 years nearly; or in every 400 years the error is very nearly 3.0944 days. So it was ordained that whenever the number expressing the centuries is not divisible by 4, the corresponding year shall not be a leap year; thus 1600 was a leap year, because 16 is divisibe by 4; but 1700 and 1800 are not, since 17 and 18 are not divisible by 4. This correction is too great, but the error only amounts to 28 hours in 5000 years.

The calendar, thus rectified, is called the Gregorian calendar, and is used throughout Europe, with the exception of Russia and Greece, where the Julian calendar is still employed. The first was promulgated by Pope Gregory, in 1582; and England adopted it on the 2nd of September, 1752. The error then amounted to 11 days, which were omitted, and the 3rd of September was called the 14th. This calendar is called the *new style*, the Julian being the *old style*.

REMARKABLE CHRONOLOGICAL ERAS.

The account of time from any particular date or epoch is called an *era*.

The Christian, vulgar, or Dionysian era dates from January 1st, 4004 years after the creation of the world.

The era of the deluge dates from February 18th, 2358 years before Christ.

The era of Abraham dates from October 1st, 1996 years before Christ.

The era of the Olympiads dates from July 1st, 776 years before Christ.

The era of the building of Rome dates from April 22nd, 753 years before Christ.

The era of Nabonassar dates from 747 years before Christ.

The era of Constantinople, used by the Greeks, dates from the creation of the world, and the year 5509 corresponds to the 1st of September, before the Christian era.

The era of the Seleucidæ dates from October 1st, 312 years before Christ.

The Spanish era dates from January 1st, 45 years before Christ.

The era of Diocletian dates from August 29th, 284 years after Christ.

The era of the Hejira dates from July 15th, 622 years after Christ.

The last day of the old style (catholic nations), October 4th, 1582.

The last day of the old style in England, September 2nd, 1752.

New style in catholic nations, October 15th, 1582.

New style in England, September 14th, 1752.

230. TABLE OF UNITS, &c.

12 units	are	1 dozen.
12 dozens	,,	1 gross.
20 units	,,	1 score.
24 sheets of paper	,,	1 quire.
20 quires	,,	1 ream.

231. TABLE OF ANGULAR MEASURE.

A second		....	marked	1″.
60 seconds	make	1 minute......	,,	1′.
60 minutes	,,	1 degree	,,	1°.
90 degrees	,,	1 right angle	,,	90°.
360 degrees	,,	4 right angles, or circumference.		

REDUCTION.

232. A quantity expressed in different units, such as £3. 11s. 6d. or 15 cwt. 3 qr. 16 lbs., is called a *compound quantity;* and by the tables, it is evident that any compound quantity can be expressed in several ways. For instance, £4. 9s. 6d. is the same as 89s. 6d., or 1074 pence, or 4296 farthings.

When a quantity is expressed in one or more denominations, *reduction* shows the method of converting it into one or more others. Since quantities may be either reduced from a high denomination to a lower one, or raised from a lower one to a higher, then there are two kinds of reduction, *reduction descending* and *reduction ascending.*

233. First, reduce £24. 8s $7\frac{3}{4}$d. into farthings.

Since £1. = 20s., £24. = 24×20s. = 480s.; therefore, £24. 8s = 480 + 8 = 488s. Since 1s. = 12d., 488s. = 488×12d. = 5856d.; therefore, £24. 8s. 7d. = 5856d. + 7d. = 5863d. And since 1d. = 4 farthings, 5863 pence = 5863×4f. = 23452f.; and, there fore, £24. 8s. $7\frac{3}{4}$d. = 23452f. + 3f. = 23455 farthings.

The operation is performed thus in practice :—

$$\begin{array}{r}
£24.\ 8s.\ 7\tfrac{3}{4}d. \\
20 \\
\hline
488 \\
12 \\
\hline
5863 \\
4 \\
\hline
23455f.
\end{array}$$

234. Secondly, reduce 37849 farthings to pounds, shillings, and pence

Since 4f. = 1d., 1f. = $\frac{1}{4}$d.; therefore, 37849f. = $\frac{37849 \times 1}{4}$d. = $9462\frac{1}{4}$d.; also, since 12d. = 1s., 1d. = $\frac{1}{12}$s.; therefore, 9462d. = $\frac{9462 \times 1}{12}$ = 788s. 6d.; and also, since 20s. = £1, 1s. = £$\frac{1}{20}$, therefore, 788s. = $788 \times$ £$\frac{1}{20}$ = £39. 8s. Hence, 37849f. = $9462\frac{1}{4}$d. = 788s. $6\frac{1}{4}$d. = £39. 8s. $6\frac{1}{4}$d., which process is thus written :—

$$\begin{array}{l}
4)\underline{37849} \\
12)\underline{9462.\ 1f.} \\
\ \ 20)\underline{788.\ 6\tfrac{1}{4}d.} \\
\qquad £39.\ 8s.\ 6\tfrac{1}{4}d.
\end{array}$$

As a proof of the correctness of these results, operate in a converse way.

235. From what has been said in (§ 233 and 234), the operations in reduction are evident; it is only required to refer to the tables.

236. EXERCISES.

1. Reduce £164. 19s. 7¼d. to farthings, and prove the operation.
2. In £764. 11s. 11¼d. how many farthings? Verify the result.
3. What number of pounds, &c., are contained in 3769473 farthings? and prove the correctness of the result.
4. Change 48493 pence to pounds, &c., and conversely.
5. Find the number of farthings in £100. 10s. 10½d., and prove it.
6. Required, the number of pence in 564 guineas, and verify the operation.
7. Convert 112546 grains into pounds Troy, and prove the result.
8. Bring 54 lbs. 10 oz. 16 dwts. 19 grs. to grains.
9. In 24 cwt. 2 qrs. 14 lbs., how many pounds?
10. How many drams are there in 3 tons 17 cwt. 2 qrs. 24 lbs. 15 oz. 3 drs.?
11. Bring 24895 lbs. to cwts., &c.
12. Change 24 lasts 1 wey 4 tods 1 stone 0 cl. 5 lbs. to pounds.
13. Find the number of miles, &c., in 463972 inches.
14. Bring 5 mi. 4 fur. 3½ pl. 4 yds. 1 ft. 7 in. to inches.
15. Reduce 14 yds. 3 qrs. 2 nl. 2 in. to inches.
16. How many inches are there in 54 E. e. 4 qrs. 3 nl. 1 in.?
17. Change 714 inches to Flemish ells.
18. Convert 1476 French ells to inches.
19. Reduce 3 ac. 3 rd. 33 pl. 27 yds. 7 ft. 15 in. to inches
20. In 964893 yds., how many acres?
21. Bring 174 cubic yds. 24 ft. 362 in. to cubic inches.

22. What number of cubic yards, &c., are equivalent to 7945673 cubic inches?

23. Convert 64596 pints to lasts, &c.

24. Reduce 6 lasts 0 wey 3 qrs. 6 bu. 3 pk. 1 gal. 7 pts. to pints.

25. In 37496 pecks how many chaldrons?

26. In 7 scores 20 ch. 35 bu. 3 pk. how many pecks?

27. Bring 434 hhds. 48 gal. 1 qt. of ale to pints.

28. 76465 pints are equivalent to how many butts, &c.

29. How many tuns, &c., are there in 264054 pints of wine?

30. How many pints are there in 9 hhds. 55 gals. 3 qts. 1 pt. of wine?

31. Convert 365 days 5 hrs. 48 min. 48 sec. to seconds.

32. Convert 4673360 seconds to years, &c.

33. Reduce 24 reams 16 quires 20 sheets of paper to sheets.

34. Convert 1649750 sheets of paper to reams, &c.

35. Find the number of seconds in $164° 45' 34''$.

36. Find the number of degrees, &c., in $6849''$.

37. How many barleycorns will reach round the earth, which is 25000 miles in circumference?

38. The wheel of a railway carriage makes 369600 revolutions in 700 miles. What is the circumference of the wheel?

39. The distance between Dover and Calais is 21 miles. How many arches, each of 75 feet span, would a bridge between the two places have?

40. The hind wheel of a carriage is 15 ft. 8 in. in circumference, and the fore wheel 12 ft. 3 in. How many revolutions will the latter make more than the former between London and Edinburgh, the distance being 389 mi. 6 fur. 20 p.?

41. An equal number of moidores, guineas, pounds, shillings, and pence make £389. 8s. 4d. How many are there of each?

42. A sack of flour weighs 22 stones 12 lbs. How many loaves of 4 lbs., $3\frac{1}{2}$ lbs., 3 lbs. $2\frac{1}{2}$ lbs., and 1 lb. can be made from it?

43. How many seconds have elapsed from the Christian era to the beginning of 1853?

44. In what time would sound travel from the earth to the moon, the distance being 240000 miles, and sound moving about 1143 feet per second?

45. How long would it take a cannon ball, at the velocity of 1960 feet per second, to travel from the earth to the sun, the distance being 95 millions of miles?

46. The piston of a steam engine moves at the rate of 240 feet per minute. What is the rate in miles, per day of 16 hours?

47. If 7 yds. 3 qrs. of cloth make a suit of clothes, how many suits can be made from 24 pieces, each of 45 yards?

48. How many times does a clock, which strikes the hours and quarters, strike, between noon on the 20th of June, and midnight on the 31st of December, of the same year?

49. An indolent youth loses 8 minutes per hour. How much time will he lose from August 12th to December 17th, reckoning 10 hours work per day?

50. A wheel is 5 yards in circumference. How long is that part of the circumference which measures 48° 24′?

51. If a person counts 80 sovereigns per minute for 15 hours each day, in how many days will he count a million?

52. How many canisters of tea can be filled out of 8 cwt. 2 qrs., the canisters holding respectively 2 lbs., ½ lb., ¼ lb., and there being the same number of each?

ADDITION OF COMPOUND QUANTITIES.

237. Little need be said upon the principles of performing the fundamental operations of compound quantities; references to the corresponding operations of simple quantities will suffice. The former differ from the latter in this, that the subdivisions of the unit are not uniform, as they do not follow the decimal system.

238. A owes to B £564. 2s. 7¾d.; to C, £324. 16s. 11½d.;

to D, £216. 14s. 10¼d.; and to E, £949. 15s. 6¼d. How much does he owe in all?

£.	s.	d.
564	2	7¾
324	16	11½
216	14	10¼
949	15	6¼
£2055	9	11¾

Having written the numbers under each other, taking care that the units of the same kind are under each other, begin by adding together the farthings, which amount to 7f., or 1¾d., set down the ¾ under the farthings; the sum of the pence with the 1 penny resulting from the addition of the farthings, is 35 pence, or 2s. 11d., set down 11d. under the pence; the sum of the shillings, with the 2s. resulting from the addition of pence, is 49 shillings, or £2. 9s., the 9s. are set down under the shillings, and the £2 carried to pounds, the sum of which is £2055.

The same method of proof may be employed as for simple addition.

When the quantities are of any other sort, the same method is followed. The tables render the process easy.

239. EXERCISES.

1. Find the sum of £3489. 11s. 6¾d.; £267. 7s. 7¼d; £5671. 14s. 10½d.; £367. 19s. 11½d.; and £1304. 19s. 11½d.
2. The expenses of building a house were: surveyor, £340; bricklayer, £5696 17s. 4d.; mason, £2740. 16s. 7d.; carpenter, £4169. 17s. 0d.; plumber, £1565. 15s. 3d.; glazier, £473. 10s. 6d.; painter, £375. 18s. 6d.; paper-hanger, £124. 1s. 4d.; and locksmith, £275. 19s. 8d. What did the house cost?
3. The distance from A to B is 24 mi. 7 fur. 20 p.; from B to C, 35 mi. 4 fur. 28 po. 3 yds.; from C to D, 47 mi. 6 fur. 31 po. 4 yds.; from D to E, 15 mi. 3 fur. 16 po. 2 yds. What is the distance between A and E?
4. Of five pieces of timber, the first contains 3 cub. yds. 24 ft 416 in.; the second, 5 cub. yds. 16 ft. 94 in.; the third, 4 cub. yds. 18 ft. 1014 in.; the fourth, 2 cub. yds. 12 ft. 96 in.; and the fifth, 3 cub. yds. 10 ft. 184 in. How many cubic yards, &c., are there in all?

5. What is the weight of 6 bales of cotton, weighing respectively 5 cwt. 3 qrs. 19 lbs.; 7 cwt. 0 qrs. 16 lbs.; 3 cwt. 2 qrs. 24 lbs.; 5 cwt. 2 qrs. 18 lbs.; 6 cwt. 1 qr. 10 lbs.; and 4 cwt. 3 qrs. 25 lbs.?

6. How many yards are contained in 3 pieces of cloth, the first of which contains 542 yds. 3 qrs. 3 nls.; the second, 643 yds. 3 qrs. 2 nls.; and the third, 576 yds. 1 qr. 3 nls.?

7. An apothecary weighs several parcels of drugs: one weighs 17 lbs. 7 oz. 2 scr. 17 grs.; another, 14 lbs. 10 oz. 8 dr. 1 scr. 15 grs.; another, 11 lbs. 9 oz. 5 dr. 1 scr. 19 grs.; another, 21 lbs. 10 oz. 6 dr. 2 scr. 10 grs.; and another, 16 lbs. 8 oz. 6 drs. 2 scr. 16 grs. What is the weight of the whole?

8. A was born on the 24th of November, 1845, and died when 8 yrs. 6 mo. 11 days old. What was the exact date of his death?

9. B was born at 58 minutes after 5 p.m., April 14th, 1820, and lived 27 yrs. 10 mo. 22 days 22 hrs. 46 min. When did he die?

10. A farmer has sown 23 bu. 1 pk. 4 gals. 4 pts. of corn; 13 bu. 0 pk. 6 gals. 3 pts. of oats; 12 bu. 1 pk. 5 gals. 6 pts. of rye. How much corn has he sown altogether?

11. A jeweller used 1 lb. 10 oz. 18 dwts. 21 grs. of silver for the manufacture of a piece of plate; for a second piece, 1 lb. 8 oz. 16 dwts.; and for a third, 1 lb. 9 oz. 14 dwts. 20 grs. How much silver was used?

12. A landowner, possessing 216 ac. 3 ro. 32 po. of land, bought two fields, one of which contained 15 ac. 2 ro. 31 po., and the other 19 ac. 3 ro. 32 po. How much land has he at present?

SUBTRACTION OF COMPOUND QUANTITIES.

240. The subtraction of compound quantities is performed by subtracting the units of the same kind from each other; and the principle laid down in simple subtraction is equally true here.

From £576. 16s. 9¼d. subtract £267. 18s. 10¾d.

These quantities are written under one another, thus:—

£.	s.	d.
576	16	$9\frac{1}{4}$
267	18	$10\frac{3}{4}$
£308	17	$10\frac{1}{2}$

As $\frac{3}{4}$ cannot be taken from $\frac{1}{4}$, add 1d. to both quantities, it does not alter the difference, and we have $\frac{3}{4}$ from $\frac{5}{4}$ leaves $\frac{1}{2}$, which is set down under the farthings; 1d.+10 make 11d., now since 11d. cannot be subtracted from 9d., to each quantity 1s. is added, and we take 11d. from 9d.+12d., or 21d., the difference is 10d., which is set down under the pence; 1s.+18s., or 19s., subtracted from 16s., cannot be effected, £1 is added to each quantity, and 19s. taken from 16s.+20s., or 36s., leaves 17s., which is set down under the shillings; now, £1+£7, or £8, taken from £6 cannot be done, but by adding 1 tens to each quantity, we have £8 from £6+£10, or £16 leaves £8; and 6 tens +1 tens, or 7 tens, leaves 0 tens; 2 hundreds taken from 5 hundreds leaves 3 hundreds. Thus, the whole difference is £308. 17s. $10\frac{1}{2}$d.

The operation explained is represented below:—

h.	t.	u.		
5+	7+	16	36	$21\frac{5}{4}$
2+	7+	8	19	$11\frac{3}{4}$
£3	0	8	17	$10\frac{1}{2}$

The proof is the same as in simple subtraction.

241. The same method is applied for the subtraction of quantities in the tables.

	£.	s.	d.
1. From	16	7	$4\frac{1}{2}$
take	11	9	$2\frac{1}{4}$

	£.	s.	d.
From	2464	17	$10\frac{1}{2}$
take	1378	17	$11\frac{3}{4}$

	£.	s.	d.
2. From	3070	14	$9\frac{1}{4}$
take	1346	17	$9\frac{3}{4}$

	£.	s.	d.
From	9004	0	$1\frac{3}{4}$
take	5236	9	$8\frac{1}{4}$

	ton.	cwt.	qr.	lb.	oz.
3. From	34	16	3	20	14
take	19	9	2	24	15

	ton.	cwt.	qr.	lb.	oz.	dr.
From	134	8	1	16	12	13
take	106	17	3	25	14	15

	lb.	oz.	dr.	scr.	gr.
4. From	234	8	7	2	17
take	176	10	6	1	19

	la.	sa.	wey.	td.	st.	cl.	lb.
From	18	10	1	4	1	0	4
take	13	7	1	5	1	1	5

	mi.	fur.	po.	yd.	ft.	in.
5. From	246	6	27	3	1	8
take	196	7	33	2	2	11

From	185°	36′	45″
take	107	41	54

6. From a sugar loaf, weighing 5 stones 11 lbs. 13 oz., a grocer sold 2 stones 12 lbs. 14 oz. How much had he left?

7. I am 54 yrs. 7 mo. 18 days old, and my son is 28 yrs. 8 mo. 26 days old. What was my age at my son's birth?

8. A man is 6 ft. 2¾ in. in height; another is 8¾ in. shorter. What is the height of the latter?

9. Charles XII. was born January 27th, 1682, and died Dec. 11th, 1718. How old was he at his death?

10. B died 35 min. after 5 o'clock p.m., on March 7th, 1853, when he was 17 yrs. 10 mo. 21 days 43 min. old. When was he born?

11. A man received the following sums: £369. 14s. 7d., £564. 17s. 8d., £273. 19s. 6d.; and paid the following debts: £176. 18s. 2d., and £144. 13s. 8d. How much has he left?

12. Four towns are represented by A, B, C, and D respectively; a man travels from A to B in 7 hrs. 20 min. 30 sec.; from B to C in 11 hrs. 14 min. 38 sec.; from A to D in 34 hrs. 36 min. 40 sec. How long will it take him to go from B to D, and from C to D?

13. A man being asked the age of his daughter, said, I am 52 yrs. 3 mo. old; my wife is 45 yrs. 5 mo. 15 days old; and my daughter's age is 77 yrs. 5 mo. 8 days less than the sum of her parents'. Find her age.

14. A landowner has two farms, one of 740 ac. 3 ro. 30 pl. and the other of 675 ac. 2 ro. 24 pl.; from the first he sells 245 ac. 3 ro. 36 pl., and from the second 396 ac. 3 ro. 30 pl. How much land has he left?

15. A bankrupt owes to A £135. 17s. 8d., to B £196. 11s. 6d., to C £214. 16s. 8d., to D £144. 17s. 8d. At the time, he has in cash, £123. 9s. 6d.; in wares, £53. 11s. 4d.; in

household furniture, £143. 19s. 7d.; and in recoverable debts, £246. 17s. 7d. What will the creditors lose?

16. I bought two saddles, a black horse and a white horse; the first saddle cost £16. 10s. 8d.; the second, £11. 16s. 10d.; the black horse, £89. 19s. 6d. When the first saddle is put on the black horse, and the second saddle on the white horse, the black horse is of the same value as the white. Find the price of the latter.

17. In 1850, the spring lasted 92 days 21 hrs. 16′ 15″; the summer, 93 days 13 hrs. 52′ 8″; autumn, 89 days 17 hrs. 8′ 3″; and winter 89 days 1 hr. 31′ 2″. What was the length of the year, and how much longer were the two first seasons than the two last?

MULTIPLICATION OF COMPOUND QUANTITIES.

242. We shall here make three general cases:

1st, to multiply a compound quantity by an integer.

2nd, to multiply a compound quantity by a fraction, or by a mixed quantity.

3rd, to multiply a compound quantity by another.

243. First case: To multiply a compound quantity by an integer.

How much will 9 yards of cloth cost, at £2. 17s. 6d. per yard?

£.	s.	d.
2	17	6
		9
£25	17	6

It is evident that 9 yards will cost 9 times as much as 1 yard, that is to say, $9 \times$ 6d., $9 \times$ 17s., and $9 \times$ £2. Hence we see that every part of the multiplicand must be multiplied by the multiplier. It is indifferent by which unit we begin, but it is preferable to commence with the least, thus: $9 \times$ 6d. $=$ 54d. $=$ 4s. 6d., the 6d. are set down under the pence, and the 4s. carried to the shillings; $9 \times$ 17s. $+$ 4s. $=$ 157s., or £7. 17s., the 17s. are set down under the shillings, and the £7 are carried to the pounds; $9 \times$ £2 $+$ £7 $=$ £25. Thus, the whole cost of 9 yards is £25. 17s. 6d.

EXERCISES.

1. What will 12 loads of corn cost, at £16. 12s. 8d. per load?

2. If one yard cost 18s. 8¼d., what will 16 yards cost?

3. How many acres are there in 18 farms, each of 56 acs. 3 rds. 28 po. 16 yds.?

4. What is the duty on 24 gallons of rum, at 10s. 10d. per gallon?

5. A person bought of a grocer 12 lbs. of coffee, at 1s. 3d. per lb.; 14 lbs. of tea, at 4s. 5d. per lb.; 18 lbs. of lump sugar, at 8¼d. per lb.; 20 lbs. of brown sugar, 4¾d. per lb.; 8 lbs. of cocoa, at 1s. 2d. per lb.; 28 lbs. of soap, at 9d. per lb.; 10 lbs. of mould candles, at 1s. 1d. per lb.; 16 lbs. of dip candles, at 6¾d per lb. What is the amount of the bill?

6. A bought of B 10 pairs of worsted stockings, at 3s. 8d. per pair; 8 pairs of silk, at 12s. 6d. per pair; 24 pairs of cotton, at 2s. 7d. per pair; 18 pairs of gloves, at 4s. 2d. per pair; 30 yards of flannel, at 1s. 8¼d. per yard; 24 ells of diaper, at 2s. 6½d. per ell; 36 yards of Irish linen, at 2s. 8d. per yard. What is the cost of the whole?

244. The student will have observed that this process becomes laborious when the multiplier exceeds 20; but any of the following methods will facilitate his work.

1st method: To multiply a compound quantity by 36 is the same as to multiply it first by 6, and then the product by 6, for $36=6\times 6$.

Likewise, 56 times a quantity is the same as $7\times$ the quantity, and 8 times the product, for $56=7\times 8$.

Since $38=6\times 6+2$, we multiply a quantity by 38 if we multiply it by 6 first, then the product by 6, and add twice the original quantity to the last product; or since $38=4\times 10-2$, multiply the quantity by 4, then the product by 10, and subtract twice the original quantity from the last product.

Since $112=4\times 4\times 7$, to multiply a quantity by 112 we should first multiply it by 4, then the product by 4, and this second result by 7.

Since $325=4\times 9\times 9+1$, to multiply a quantity by 325, we might proceed to multiply it by 4, the result by 9, and the second result by 9, and to this product add the original quantity.

For instance, in this question : what is the value of 273 cwt. of goods, at £2. 16s. $4\frac{1}{2}$d. per cwt. ?

Since 1 cwt. costs £2. 16s. $4\frac{1}{2}$d., evidently 273 cwt. will come to $273 \times$ £2. 16s. $4\frac{1}{2}$d., but $273 = 3 \times 7 \times 13$. We shall then multiply the price of 1 cwt. by 13, this product by 7, and the last product by 3, thus :—

$$\begin{array}{rrl}
\text{£.} & \text{s.} & \text{d.} \\
2 & 16 & 4\tfrac{1}{2} = \text{price of 1 cwt.} \\
 & & 13 \\
\hline
\text{£}36 & 12 & 10\tfrac{1}{2} = \text{price of 13 cwt.} \\
 & & 7 \\
\hline
\text{£}256 & 10 & 1\tfrac{1}{2} = \text{price of } 13 \times 7 \text{ cwt.} \\
 & & 3 \\
\hline
\text{£}769 & 10 & 4\tfrac{1}{2} = \text{price of } 43 \times 7 \times 3 \text{ cwt.}
\end{array}$$

245. EXERCISES.

1. Find the price of 288 yards of cloth, at £1. 11s. 7d. per yard.
2. Multiply 12 days 15 hrs. 32 min. by 176.
3. What distance will an engine go in 16 days, working 8 hours per day, and going at the rate of 28 miles 6 fur. 19 po. 3 yds. per hour ?
4. A chest of tea weighs 1 cwt. 3 qrs. 24 lbs. How much will 96 chests weigh ?
5. Required, the value of 29 quarters of corn, at £2. 5s. $3\frac{1}{4}$d. per quarter.
6. If I spend £1. 17s. $5\frac{1}{2}$d. a day, how much is that a year of 365 days ?

246. 2nd method : Another way of performing questions of this kind would be to reduce the multiplicand to the lowest denomination contained in it, multiply this result by the multiplier, and then reduce the product back again to higher denominations.

Thus, if one person receives £2. 14s. $6\frac{1}{2}$d. per day, how much will he receive in 164 days ?

Now, £2. 14s. $6\frac{1}{2}$d. $=1309$ halfpence, the sum received in 1 day;

And 164×1309 h.p. $= 214676$ h.p. $=$ £447. 4s. 10d., the sum received in 164 days.

247. EXERCISES.

1. How much ditching will a man do in 42 days, at the rate of 3 po. 4 yds. 2 ft per day?
2. A ship's crew, consisting of 450 men, received £16. 17s. 9½d. each. How many pounds were distributed?
3. In a town, the inhabitants consume daily 324 cwt. 2 qrs. 17 lbs. 13 oz. of bread. How much will they use in one year?
4. One piece of cloth measures 28 yds. 3 qrs. How many yards are there in 97 pieces?
5. If 1 bale of cotton weighed 1 cwt. 1 qr. 13 lbs., how much would 272 bales weigh?
6. How much beer would a garrison of 2640 men drink in 97 weeks, allowing each man 1 pint 1 gill per day?

248. 3rd method: The following questions will illustrate this third method:—

Ex. Find the value of 346 tons, at £7. 10s. per ton.

Since 1 ton cost £7. 10s., 346 tons will cost $346 \times$ £7. 10s.

$$\begin{array}{lr} & \text{£.} \\ \text{Now, } 346 \times \text{£}7 \ldots\ldots\ldots\ldots\ldots\ldots = & 2422 \\ \text{And } 346 \times 10\text{s.} = 346 \times \text{£}\tfrac{1}{2} = \dfrac{\text{£}346}{2} = & 173 \\ \hline \text{Therefore, } 346 \times \text{£}7.\ 10\text{s.} = & \text{£}2595 \end{array}$$

The operation is usually arranged in the following way:—

$$\begin{array}{l|l|l} & & \quad\ \text{£}\ \ \text{s.} \\ & & \quad\ \ 7\ \ 10 \\ 10\text{s.} & \tfrac{1}{2} & \ldots\ldots\ldots 346 \\ \cline{3-3} & & \quad 2422 = \text{price of } 346 \text{ tons, at £}7 \text{ per ton.} \\ & & \quad\ \ 173 = \quad ,, \quad 346 \quad ,, \quad 10\text{s.} \quad ,, \\ \hline & & \ \text{£}2595 = \quad ,, \quad 346 \quad ,, \quad \text{£}7.\ 10\text{s.} \ ,, \end{array}$$

Ex. What is the value of 109 cwt., at £2. 17s. 9d. per cwt.?

Now, 109 cwt. will cost $109 \times$ the price of 1 cwt., or $109 \times$ £2. 17s. 9d.

We may observe, first, that 17s. 9d. = 10s. + 5s. + 2s. 6d. + 3d.

	£	s.	d.
Then $109 \times £2 \ldots\ldots = $	218	0	0
$109 \times 10s. = 109 \times £\frac{1}{2} = \frac{£109}{2} \ldots\ldots =$	54	10	0
$109 \times 5s. = \frac{1}{2} \times 109 \times 10s. = \frac{1}{2} \times £54.\ 10s. = \frac{£54.\ 10s.}{2} \ldots\ldots =$	27	5	0
$109 \times 2s.\ 6d. = \frac{1}{2} \times 109 \times 5s. = \frac{1}{2} \times £27.\ 5s. = \frac{£27.\ 5s.}{2} \ldots\ldots =$	13	12	6
$109 \times 3d. = \frac{1}{10} \times 109 \times 2s. 6d. = \frac{1}{10} £13.\ 12s. 6d. = \frac{£13.\ 12s.\ 6d.}{10} \ldots\ldots =$	1	7	3
	£314	14	9

which process is written down as follows :—

			£	s.	d.	
			2	17	9	
10s.	$\frac{1}{2}$	...	109			
			218			price of 109 cwt., at £2 per cwt.
5s.	$\frac{1}{2}$	...	54	10		" " 10s. "
2s. 6d.	$\frac{1}{2}$	...	27	5		" " 5s. "
3d.	$\frac{1}{10}$	...	13	12	6	" " 2s. 6d. "
			1	7	3	" " 3d. "
			£314	14	9	" " £2. 17s. 9d. per cwt.

249. This convenient method of performing compound multiplication is called *Practice*, from its daily use amongst merchants and tradesmen. It consists in decomposing the units of the subdivisions into *aliquot parts* or into fractions, whose numerator is unity, either of the highest denomination or of any other higher. A knowledge of fractions will soon make the pupil expert in finding the aliquot parts in any case. But the following observations will be found useful :—

1st. Since $10s. = £\frac{1}{2}$, to multiply a number by 10s. we have to take the half of the number, and the result is the answer in pounds. Example, $48 \times 10s. = 48 \times £\frac{1}{2} = £\frac{48}{2} = £24$.

2nd. Since $6s.\ 8d. = £\frac{1}{3}$, therefore $100 \times 6s.\ 8d. = 100 \times £\frac{1}{3} = £\frac{100}{3} =$ £33. 6s. 8d.

3rd. Since 5s. $= £\frac{1}{4}$, therefore $264 \times 5s. = 264 \times £\frac{1}{4} = \frac{264}{4} =$ £66. 0s. 0d.

4th. Since 4s. $= £\frac{1}{5}$, therefore $136 \times 4s. = 136 \times £\frac{1}{5} = £\frac{136}{5} =$ £27. 4s. 0d.

5th. Since 3s. 4d. $= £\frac{1}{6}$, therefore $124 \times 3s.\ 4d. = 124 \times £\frac{1}{6} =$ $£\frac{124}{6} =$ £20. 13s. 4d.

6th. Since 2s. 6d. $= £\frac{1}{8}$, therefore $526 \times 2s.\ 6d. = 526 \times £\frac{1}{8} =$ £65. 15s. 0d.

7th. Since 1s. 8d $= £\frac{1}{12}$, therefore $324 \times 1s.\ 8d. = 324 \times £\frac{1}{12} =$ £27. 0s. 0d.

8th. Since 1s. 3d. $= £\frac{1}{16}$, therefore $158 \times 1s.\ 3d. = 158 \times £\frac{1}{16} =$ £9. 17s. 6d.

9th. Since 1s. $= £\frac{1}{20}$, therefore $445 \times 1s. = 445 \times £\frac{1}{20} =$ £22. 5s.

10th. Since 6d. $= \frac{1}{2}$ shilling, therefore $764 \times 6d. = 764 \times \frac{1}{2}s. = 382$ shillings $=$ £19. 2s. 0d.

11th Since 4d. $= \frac{1}{3}$ shilling, therefore $172 \times 4d. = 172 \times \frac{1}{3}$ shilling $=$ 57s. 4d $=$ £2. 17s. 4d.

12th. Since 3d. $= \frac{1}{4}$ shilling, therefore $154 \times 3d. = 154 \times \frac{1}{4}$ shilling $=$ 38s. 6d. $=$ £1. 18s. 6d.

13th. Since 2d. $= \frac{1}{6}$ shilling, therefore $672 \times 2d. = 672 \times \frac{1}{6}$ shilling $= 112$ shillings $=$ £5. 12s. 0d.

14th. Since $1\frac{1}{2}$d. $= \frac{1}{8}$ shilling, therefore $464 \times 1\frac{1}{2}d. = 464 \times \frac{1}{8}$ shilling $=$ 58s. $=$ £2. 18s. 0d.

15th. Since 1d. $= \frac{1}{12}$ shilling, therefore $964 \times 1d. = 964 \times \frac{1}{12}$ shilling $=$ 80s. 4d. $=$ £4. 0s. 4d.

250. EXERCISES.

1. Find the cost of 65 cwt., at £7. 18s. 8d. per cwt.
2. Required, the value of 26 yards, at 7s. $6\frac{1}{2}$d. per yard.
3. If 1 gallon cost 17s. $9\frac{3}{4}$d., required the price of 156 gallons.
4. Determine the price of 25 cwt., at £4. 3s. $9\frac{1}{2}$d.
5. Find the value of 36 acres, at £2. 18s. 11d. per acre.
6. What is the value of 219 cwt., at £12. 7s. 6d.?

251. 2nd case : To multiply a compound quantity by a fraction or by a mixed number.

The process is usually written down as follows :—

			£.	s.	d.	
2 qrs.	$\frac{1}{2}$	....	2	14	6	
			8 cwt.	3 qrs.	22 lbs.	
			21	16	0	
1 qr.	$\frac{1}{2}$	....	1	7	3	
14 lbs.	$\frac{1}{2}$	....	0	13	$7\frac{1}{2}$	
7 lbs.	$\frac{1}{2}$	....	0	6	$9\frac{3}{4}$	
1 lb.	$\frac{1}{7}$	....	0	3	$4\frac{3}{4}$	$\frac{1}{2}$
			1	0	$5\frac{3}{4}$	$\frac{5}{14}$
			£24	7	$6\frac{3}{4}$	$\frac{6}{7}$

Determine the value of 3 cwt. 3 qrs. 27 lbs. 15 oz. 12 drs., at £7 per cwt.

The previous example will be a guide for this, and the process will present no difficulty :—

2 qrs.	$\frac{1}{2}$	....	£7		
			3 cwt. 3 qrs. 27 lbs. 15 oz. 12 drs.		
			21		
1 qr.	$\frac{1}{2}$	....	3	10	
14 lbs.	$\frac{1}{2}$	....	1	15	
7 lbs.	$\frac{1}{2}$	....	0	17	6
1 lb.	$\frac{1}{7}$	....	0	8	9
5 lbs. ×	1	}	0	1	3
8 oz.	$\frac{1}{2}$	}	0	6	3
4 oz.	$\frac{1}{2}$	....	0	0	7.5
2 oz.	$\frac{1}{2}$	....	0	0	3.75
1 oz.	$\frac{1}{2}$	....	0	0	1.875
8 drs.	$\frac{1}{2}$	....	0	0	.9375
4 drs.	$\frac{1}{2}$	....	0	0	.46875
					.234375
			£27	19	11.765725 $=\frac{3}{4}$ nearly.

If the expenses of making 69 yds. 2 ft. 11 in. of ditching be £1, how many yards would £25. 19s. 5d. pay for?

Since for £1 69 yds. 2 ft. 11 in. are done, for £25. 19s 5d. we shall have made £25. 19s. 5d. × 69 yds. 2 ft. 11 in., which operation can be performed as the preceding.

			yds.	ft.	in.	
10s.	½		69 yds.	2 ft.	11 in.	
			£25	19	5	
			349 yds.	2 ft.	7 in.	= work performed for £5.
					5	
			1749	0	11	= work performed for £25.
5	½		34	2	11.5	
4	⅕	of £1.	17	1	5.75	
5d.	$\frac{1}{12}$	of 5s.	13	2	$11.\dot{8}$	
			1	0	$5.98\dot{3}$	
			1816 yds.	2 ft.	$10.03\dot{3}$ in.	

Sufficient has been said to show the principle of this method, and the pupil will acquire readiness and dexterity in it by solving the following problems :—

253. MISCELLANEOUS EXERCISES.

1. A father being asked his age, answered, my son is 9 yrs. 5 mo. 23 days, and I am five times as old. Find his age?
2. Find the value of 11 cwt. 2 qrs. 17 lbs., at £2. 18s. 9d. per cwt.
3. 65 persons divided a sum of money amongst themselves, and each received £14. 11s. 6d. What sum was divided?
4. A pedestrian walks 66 steps, each 2 ft. 10¾ in., in 1 minute. How many yards, feet, and inches will he go over in 2 hrs. 54 min.?
5. In a nursery garden, there are 3440 young trees, worth 14s. 8¾d. each. The value of the whole number of trees is required.
6. At £1. 14s. 2d. per yard, what will be the price of 63⅝ yards?
7. What is the rent of 500 ac. 3 ro. 25 pl. of land, at £1. 7s. 8d. per acre?
8. A bankrupt paid 13s. 4½d. in the pound, and his debts are £9748. How much did the creditors receive?
9. What is the value of a gold snuff box, weighing 9 oz. 8 dwts. 19 grs., at the rate of £3. 19s. 10d. per oz.?
10. 4 ac. 3 ro. 24 pl. of land will keep a horse 1 year. How much will be required to keep 84 horses the same time?

11. 25 masons built a house in 28 days, and each man's wages were 3s. $8\frac{3}{4}$d. per day. What is the whole pay?

12. The rent of some lodgings is £47. 18s. 9d. yearly. What will the rent amount to in 7 years 7 months?

13. The lead of an old tank, which weighs 7 cwt. 2 qrs. 17 lbs., is sold at £1. 18s. 9d. per cwt.; and a new one, which weighs 11 cwt. 1 qr. 24 lbs., is worth £2. 1s. 1d. per cwt. What is the value of each tank, and the expense of the one more than the other?

14. Required, the duty on 143 cwt. 3 qrs. 12 lbs. of coffee, at £5. 4s. 8d. per cwt.?

15. What is the cost of 224 ac. 2 ro. 35 pl., at £12. 16s. 9d. per acre?

16. My daily expenses are £1. 5s. 5d. How much do I save yearly out of £600?

17. Compute the value of $501\frac{1}{12}$ yards, at £1. 1s. $1\frac{1}{3}$d. per yard?

18. Determine the expense of travelling 276 mi. 5 fur. 120 yds., at $11\frac{1}{2}$d. per mile.

19. A man, 60 years old, weighs 2 cwt. 1 qr. 24 lbs. What was his weight 30 years ago, supposing it to have increased by $\frac{3}{8}$ what it was then?

20. If a square yard of marble cost 15s. 9d., what will be the value of a slab, 24 square yds. 7 ft. 100 in.?

21. If a gallon of a mixture cost 14s. $7\frac{3}{4}$d., it is required to ascertain the price of 53 gals. 2 qts. $1\frac{1}{7}$ pts.

22. Find the rent of $24\frac{1}{12}$ acres of land, at £2. 14s. 6d. per acre.

23. Three plots of land were sold,

the 1st	containing	$346\frac{1}{2}$ acres, at £14. 17s. 10d.	per acre.
2nd	,,	272 ac. 3 ro. 18 pl., at £17. 12s. 8d.	,,
3rd	,,	$648\frac{5}{8}$ acres, at £15. 16s. 6d.	,,

The value of the whole is required.

24. What is the amount of the following bill:—

$257\frac{1}{3}$ lbs. of green tea,	at	4s. 4d.	per lb.
$424\frac{3}{4}$ lbs. of hyson	,,	5s. $7\frac{1}{2}$d.	,,
$214\frac{5}{6}$ lbs. of bohea	,,	5s. $10\frac{3}{4}$d.	,,
372 lbs. of black	,,	3s. 9d.	,,
186 lbs. of coffee,	at	1s. $4\frac{1}{2}$d.	,,

25. Mr. John Dufour, Worksop, 8th October, 1853.

Bought of Charles Clovis,

346 yards	fine white linen,	at	5s. 7½d. per yd.
145 ,,	cambric	,,	14s. 9d. ,,
684 ,,	muslin	,,	3s. 10½d. ,,
172 ,,	calico	,,	1s. 4d. ,,
78 ,,	diaper	,,	1s. 7¼d. ,,
256¾ ,,	Holland	,,	1s. 10¾d. ,,

26. Sheffield, May 1st, 1853.

Thomas Harrison buys from John Rogers,

Silver castors,	weight	52 oz. 4 dwts. 15 grs.,	at	7s. 6d. per oz.
Silver cup	,,	8 oz. 11 dwts. 10 grs.	,,	6s. 4d. ,,
Silver teapot	,,	18 oz. 8 dwts. 14 grs.	,,	6s. 8d. ,,
Silver urn	,,	7 lbs. 6 oz. 10 dwts.	,,	5s. 7½d. ,,
A dozen silver spoons		2 lbs. 7 oz. 15 dwts.	,,	6s. 2¾d. ,,

Required, the whole amount.

DIVISION OF COMPOUND QUANTITIES.

255. In the division of compound quantities three cases present themselves:

1st, to divide a compound number by a simple number.

2nd, to divide a compound number by a mixed one, or by a fraction.

3rd, to divide a compound quantity by another.

256. First case: To divide a compound quantity by a simple number.

If 8 tons cost £21. 16s. 4d., what is the price of 1 ton?

Since 8 tons cost £21. 16s. 4d., 1 ton must evidently cost one-eighth part of that money. Now, $\frac{1}{8}$ of £21 = £2, and £5 over, which, reduced into shillings, and adding the 16s., amount to 116s., the $\frac{1}{8}$ part of 116s. = 14s., with 4s. over; 4s. = 48d. and 4d. are 52d., and $\frac{1}{8}$ part of 52d. is 6d., and there remains 4d.; again, 4d. are 16 farthings, and the $\frac{1}{8}$ part of 16f. is 2f. The price of a ton is, therefore, £2. 14s. 6½d. The process is written down as follows:—

	£.	s.	d.
8)	21	16	4
	2	14	6½

Thus we have taken one-eighth part of £21. 16s. 4d., or divided £21. 16s. 4d. by 8. Had the divisor been larger, the operation would have been performed by long division, as in the following example :—

If the clothing of 748 soldiers amounts to £3153. 5s. 9d., what is the cost per man ?

Since 748 soldiers cost £3153. 5s. 9d. in clothing, therefore 1 soldier costs $\frac{1}{748}$ of £3153. 5s. 9d., or $\frac{\text{£}3153.\ 5\text{s}.\ 9\text{d}.}{748}$ in clothing, which quotient is found by the following process :—

```
        £.   s. d. £.  s. d.
    748)3153 5  9(4  4  3¾, cost of each
        2992           soldier's clothing.
        ----
         161
          20
        ----
        3225
        2992
        ----
         233
          12
        ----
        2805
        2244
        ----
         561
           4
        ----
        2244
        2244
        ----
        ....
```

The division may be effected by a process analogous to the one explained in (§ 245), by reducing the dividend to the lowest denomination, and dividing as in common division; and this quotient is brought back again into higher denominations. But this process is too laborious to be useful in practice.

257. EXERCISES.

1. If 1 cwt. cost £137. 18s., what is the price per lb. ?
2. A wheel makes 514 revolutions in passing over 1 mi. 467 yds. 1 ft. What is its circumference ?
3. How much will 1 piece of cloth cost, if 99 pieces cost £841. 10s. 10d.?
4. What are raisins per cwt., if 145 cwt. cost £167. 19s. 2d. ?

5. If 40 bales of cotton weigh 52 cwt. 3 qrs. 12 lbs., what is the average weight of a bale?

6. The duty on a pipe of port wine, containing 138 gallons, is £52. 6s. 6d. How much is the duty per gallon?

258. Second case: To divide a compound number by a mixed number, or by a fraction.

Ex. A piece of cloth, containing $64\frac{5}{8}$ yards, costs £75. 10s. $7\frac{1}{4}$d. $\frac{1}{4}$f. What is the price per yard?

It has been proved before that the quotient is not altered, if both the dividend and divisor be multiplied by the same number; therefore, before dividing £75. 10s. $7\frac{1}{4}$d. $\frac{1}{4}$f. by $64\frac{5}{8}$, we shall multiply these two numbers by 8, and then proceed to divide as in case 1.

$$
\begin{array}{rrrll}
 & \text{£.} & \text{s.} & \text{d.}\ \ \text{f.} & \\
64\frac{5}{8}) & 75 & 10 & 7\frac{1}{4}\ \ \frac{1}{4} & \\
8 & & & 8 & \text{£.}\ \ \text{s.}\ \ \text{d.} \\
\hline
517) & 604 & 4 & 10\frac{1}{2} & (1\ \ 3\ \ 4\frac{1}{2} \\
 & 517 & & & \\
\cline{2-2}
 & 87 & & & \\
 & 20 & & & \\
\cline{2-2}
 & 1744 & & & \\
 & 1551 & & & \\
\cline{2-2}
 & 193 & & & \\
 & 12 & & & \\
\cline{2-2}
 & 2326 & & & \\
 & 2068 & & & \\
\cline{2-2}
 & 258 & & & \\
 & 4 & & & \\
\cline{2-2}
 & 1034 & & & \\
 & 1034 & & & \\
\cline{2-2}
 & \cdots & & &
\end{array}
$$

The same result would have been obtained had we multiplied the divisor alone by 8; and after having performed the division, multiplied the quotient by 8.

If the divisor were a fraction, the same process would be followed, for we should multiply both divisor and dividend by the denominator of the divisor, and proceed to divide as in case 1.

Ex. Suppose it is required to divide £243. 4s. 9d. by $\frac{11}{12}$.

$$
\begin{array}{rrrr}
 & \text{£.} & \text{s.} & \text{d.} \\
\frac{11}{12}) & 243 & 4 & 9 \\
12 & & & 12 \\
\hline
11) & 2918 & 17 & 0 \\
\hline
 & \text{£}265 & 7 & 0
\end{array}
$$

259. EXERCISES.

1. What quantity of hops will £100. 10s. 10d. purchase, at 15¾d. per lb.?

2. An estate of 630¼ acres is let for £1543. 10s. 6d. What is the rent per acre?

3. 38⅝ chests of tea weigh 18 cwt. 3 qrs. 24 lbs. What is the average weight of 1 chest?

4. How many yards of cotton can be made for £1, if £$\frac{5}{7}$ pay for the making of 45 yds. 2 ft. 8 in.?

5. If 7043⅔ lbs. of tea cost £908. 10s. 10d., what is the price of 1 lb.?

6. If a railway train goes 254 miles 3 fur. in 14$\frac{7}{9}$ hours, at what rate does it travel per hour, including stoppages?

260. Third case: To divide a compound quantity by another.

Here we shall have to consider whether the quotient is of different or of the same nature as the dividend.

First, when the quotient is *not* of the *same kind* as the dividend.

Ex. The making of a road cost £47. 19s. 5d. per mile. How many miles can be made for £2728. 17s. 10d.?

Since £47. 19s. 5d. is the cost of a mile, as many times as £47. 19s. 5d. is contained in £2928. 17s. 10d., so many miles will be performed. Here, then, we must observe that the quotient will express a number of miles, furlongs, &c. Therefore, the lower denominations of the dividend cannot be adopted to determine the lower denominations of the quotient. On that account, in cases of this kind, the two quantities are reduced to the same denomination, and then the division is performed thus: £47. 19s. 5d. = 11513d., and £2728. 17s. 10d. = 654934d.

```
                mi.   fur.  pl.  yds.
11513)654934(56    7     3    3  8036½/11513 or 16073/23026
      57565
      -----
       79284
       69078
       -----
       10206
           8
       -----
       81648
       80591
       -----
        1057
          40
       -----
       42280
       34539
       -----
        7741
          5½
       -----
       38705
        3870½
       ------
       42575½
       34539
       ------
        8036½
```

Secondly, when the quotient is of the *same kind*.

Ex. The wages of a man for 23 yrs. 5 mo. 10 days are £224. 18s. 8d. What is the yearly amount?

Here, by the question, we are led to divide £224. 18s. 8d. by 23 yrs. 5 mo. 10 days. But 23 yrs. 5 mo. 10 days = 8440 days, or $\frac{8440}{12 \times 30}$ or $\frac{8440}{360}$ of 1 year. Therefore, the question resolves itself into dividing £224. 18s. 8d. by $\frac{8440}{360}$, which operation is performed as shown in (§ 257).

```
yrs.  mo. days  £.    s.  d.
 23    5   10)224   18   8
 12                      12
----        -------------
281              2699   4   0
 30                         30
----        -----------------
8440   8440)80976   0   0(£9. 11s. 10 134/211 d.
            75960
            -----
             5016
               20
            ------
           100320
             8440
           ------
            15920
             8440
            -----
             7480
               12
            -----
            89760
            8440
            -----
             5360
```

[Note.—The pupil should never neglect to shorten his labour, when possible. It will have been noticed that $\frac{8440}{360} = \frac{211}{9}$, and the work consists in multiplying the dividend by 9, and dividing the product by 211.]

261. MISCELLANEOUS EXERCISES.

1. In 3 hrs. 25 min. a ropemaker made a rope 76 yds. 2 ft. 9 in. in length. How much of it was made per hour?
2. If 2 cwt. 3 qrs. 1 lb. cost £150. 13s. 10d., how much does 1 lb. cost?
3. Bought 14¼ yards of velvet for £19. 8s. 8d. What was the cost per yard?
4. Twelve men and a boy have earned £45. 17s. 10½d.; the boy is to receive 9 half-crowns. What is each man's share?
5. £300 was paid for 200000 bricks. How much is that per score?
6. If 112 ingots of gold are worth £77878. 5s. 4d., what is the value of 1 ingot?
7. If 150 reams of paper cost £75, what is that per sheet?
8. Divide £10416. 13s. 4d. among one million of persons. What will each receive?

9. The daily wages of a man are 3s. 6d. In how many days will he earn £3. 19s. 6d.?

10. The length of the equatorial circumference is 24899 miles. It is divided into 360°, like every circle. 1st, find the length of 1°, and also of 1′ in yards. 2nd, since the earth rotates in 24 hours, at what speed per hour are objects on the equator carried by that motion?

11. The wheel of an engine makes 1000000 revolutions in 7 hours. What is its rate, in degrees, &c., per hour?

12. If in 1 minute every part of the circumference of a wheel moves 3° 20′, in what time will 100° be described?

13. The circumference of a wheel is $4\frac{1}{3}$ yards. What is the length of 1°, and also of 35° 20′?

14. A schoolmaster receives £8. 7s. 6d. weekly, and each pupil pays 2s. 6d. weekly. How many pupils has he?

15. A mother who was asked the age of her daughter, replied, I am 34 yrs. 7 mo. 8 days 9 hrs. old; my husband's age is 31 yrs. 9 days. Now, if from one-half the sum of our ages 19 yrs. 11 mo. 29 days 13 hrs. be subtracted, the remainder is my daughter's age. Find her age.

16. A merchant sold 13 hhds. 56 gals. of brandy, 9 hhds. 54 gals. of gin, and 10 hhds. 36 gals. of rum in 24 days. How much did he sell each day?

17. A railway carriage went over 346 miles in 14 hrs. 34 min. 45 sec. In what time was each mile performed?

18. At the Great Exhibition, besides other refreshments, there were consumed 870000 plain buns, at 1d. each; 930000 Bath buns, at 2d. each; and 1090000 bottles of ginger beer, &c., at 4d. per bottle. How much money was received for them?

19. If a steam packet sailed 1020 miles in 4 days 18 hrs. 33 min. 36 sec., what was the average rate per hour, and also in what time was each mile performed?

20. Divide £550. 3s. $1\frac{1}{2}$d. among 4 men, 6 women, and 8 children, giving to each man double a woman's share, and to each woman triple a child's.

21. In common marching, soldiers take 75 steps a minute; in quick marching, 108. How far would a regiment advance in 8 hours, going 5 hours common, and 3 hours quick marching, reckoning 2 ft. 10 in. for each step?

22. At the Great Exhibition, there were, on a shilling day, 76473 persons admitted; on the next day, when the admission was 2s. 6d., the number amounted to 39582. On which day was the most money received, and how much?

23. A draper bought several pieces of cloth, at the rate of £10. 2s. $9\frac{3}{4}$d. for $5\frac{1}{2}$ yards, and sold them again at the rate of £20. 18s. $9\frac{1}{4}$d. for $9\frac{1}{5}$ yards. On the whole, he gained £7. 12s. 6d. How many yards were there?

REDUCTION OF CONCRETE QUANTITIES AS FRACTIONS OF OTHERS.

262. All that has been said upon fractions relates to generalities; they have been considered more as parts of abstract units. We shall now treat more particularly of the application of fractions to concrete quantities.

Our first case shall be to express a given quantity in terms of or as the fraction of another given quantity.

Ex. Express 5s. 6d. as the fraction of £1.

As there are 240 pence in £1, therefore 1d. is $\frac{1}{240}$ of £1.

But 5s. 6d. = 66d., hence 66d. are $66 \times \frac{1}{240}$ of £1., or $\frac{66}{240}$ of £1, or $\frac{11}{40}$ of £1.

From which we infer this law: reducing both proposed quantities to the same denomination, the result of the first quantity is the numerator, and the result of the other quantity the denominator of the fraction required.

Ex. What fraction of a ton is 2 cwt. 2 qrs. 14 lbs.?

Since 1 lb. is $\frac{1}{2240}$ of a ton,

therefore 2 cwt. 2 qrs. 14 lbs., or 294 lbs. are $\frac{294}{2240}$ of 1 ton, or $\frac{21}{160}$ of 1 ton.

Or thus: since 14 lbs. are $\frac{1}{160}$ of 1 ton,

therefore 2 cwt. 2 qrs. 14 lbs., or 21 × 14 lbs. are $\frac{21}{160}$ of 1 ton.

Ex. Reduce £3. 14s. $6\frac{1}{2}$d. to the fraction of £5. 14s. 9d.

Here £3. 14s. $6\frac{1}{2}$d. = 1789 halfpence,

and £5. 14s. 9d. = 2754 halfpence.

Since 1 halfpenny is $\frac{1}{2754}$ of £5. 14s. 9d.,

therefore 1789 halfpence are $\frac{1789}{2754}$ of £5. 14s. 9d.

Ex. Express 1 cwt. 6 lbs. as the fraction of 12 lbs.

Since 1 lb. is $\frac{1}{12}$ of 12 lbs.,

therefore 1 cwt. 6 lbs., or 118 lbs., are $\frac{118}{12}$, or $\frac{59}{6}$ of 12 lbs.

263. EXERCISES.

1. Reduce 11s. $6\frac{3}{4}$d. to the fraction of £1.
2. Reduce £1. 10s. $6\frac{1}{4}$d. to the fraction of 6s. $9\frac{1}{4}$d.
3. Express 16 hrs. 45 min. 45 sec. as the fraction of 1 day.
4. Express 11 oz. 6 drs. as the fraction of 1 lb.
5. What fraction of £1. 16s. $8\frac{5}{8}$d. are 11 crowns 2s. 2d.?
6. What fraction of 6 ac. 2 ro. are 3 ac. 0 ro. 28 pl.?
7. Express 6 ft. $6\frac{1}{16}$ in. in terms of 3 yds. 9 in.
8. Express $16\frac{2}{3}$ lbs. in terms of 3 qrs. 18 lbs.
9. Reduce 3 wks. 5 days 14 hrs. 24 min. to the fraction of a lunar month.
10. Express 3 hhds. 38 gals. 1 pt. in terms of 1 ton.
11. Express 8 quires 18 sheets in terms of 1 ream.

264. Secondly, we shall determine a method of expressing a fraction of one given quantity as the fraction of another

Ex. Express $\frac{5}{6}$s. in terms of £1.

Since 1s. $=\frac{1}{20}$ of £1,

then $\frac{1}{6}$s. $=\frac{1}{120}$,

and $\frac{5}{6}$s. $=\frac{5}{120}$, or $\frac{1}{24}$.

From which it follows that we must multiply the given fraction by that fraction which shows what part the lower denomination is of the higher.

Ex. Reduce $\frac{6}{7}$ in. to the fraction of 1 yard.

Here 1 inch is $\frac{1}{36}$ yd.,

then $\frac{1}{7}$ inch is $\dfrac{1}{7\times 36}$ yd.;

therefore $\frac{6}{7}$ inch is $\dfrac{6}{7\times 36}=\frac{1}{42}$ yd.

Ex. Express $\frac{7}{9}$ of 1 guinea as the fraction of 10s.

Here 1s. $=\frac{1}{10}$ of 10s.,

and 1 guinea, or 21s., are $\frac{21}{10}$ of 10s.;

therefore $\frac{7}{9}$ guinea are $\dfrac{7\times 21}{9\times 10}$ or $\frac{49}{30}$ of 10s.

265. EXERCISES.

1. Reduce $\frac{3}{4}$ of a quarter of a guinea to the fraction of £1.

2. Reduce $5\frac{2}{5}$ of a half-crown to the fraction of 16s.

3. Express $\frac{8}{9}$ of £1 as the fraction of £2. 10s.

4. Express $\frac{11}{56}$ of £2. 4s. 6d. as the fraction of £10.

5. Find the fraction of 1 cwt. which expresses $\frac{13}{14}$ oz.

6. Find the fraction of 1 day which expresses $\frac{5}{18}$ of a week.

7. What is the fraction of a mile which expresses $\frac{7}{8}$ of 648 yards?

8. Express $\frac{5}{6}$ of $\frac{2}{3}$ of 16 lbs. as the fraction of 1 ton; and $4\frac{1}{5}$ of $3\frac{2}{3}$ of a square inch as the fraction of a square yard.

9. Reduce $\dfrac{6\frac{1}{3}}{9\frac{1}{12}}$ of 3960 seconds to the fraction of 1 week.

10. Find the sum and difference of $\frac{5}{6}$ of £1, and $\frac{5}{9}$ of 1 guinea.

$$\text{Here } \tfrac{5}{6} \text{ of } £1 = \tfrac{5}{6} \text{ of } 20\text{s.} = \frac{5 \times \overset{10}{\cancel{20}}}{\underset{3}{\cancel{6}}} = 16\text{s. } 8\text{d.},$$

$$\text{and } \tfrac{5}{9} \text{ of } 1 \text{ guinea} = \tfrac{5}{9} \text{ of } 21\text{s.} = \frac{5 \times \overset{7}{\cancel{21}}}{\underset{3}{\cancel{9}}} = 11\text{s. } 8\text{d.};$$

therefore the sum $= 16$s. 8d. $+ 11$s. 8d. $= £1$. 8s. 4d.
and the difference $= 16$s. 8d.-11s. 8d. $= 5$s.;
or thus: 1s. is $\frac{1}{20}$ of £1.
1 guinea, or 21s., are $\frac{21}{20}$ of £1.

$$\tfrac{5}{9} \text{ guinea are } \frac{\cancel{5} \times \overset{7}{\cancel{21}}}{\underset{3}{\cancel{9}} \times \underset{4}{\cancel{20}}} = \tfrac{7}{12} \text{ of } £1.$$

Therefore the sum $= \frac{5}{6} + \frac{7}{12} = £\frac{17}{12} = £1$. 8s. 4d.
The difference $= \frac{5}{6} - \frac{7}{12} = £\frac{1}{4} = 5$s.

11. $\frac{3}{5}$ hour $+ \frac{3}{4}$ week $+ \frac{2}{3}$ day, to be expressed in positive terms.

12. $\frac{7}{8}$ of £3. 16s. $+ \frac{2}{3}$ of $\frac{4}{5}$ of £1. 15s. $3\frac{3}{4}$d. $- \frac{7}{9}$ of £1. 6s. $8\frac{1}{4}$d. to be expressed in positive terms.

13. Find the result of $\frac{6}{7}$ of £2. 3s. 9d.; $\frac{4}{5}$ of 6s. 8d.; $\frac{10}{11}$ of £9. 8s. 10d.

14. Find the result of $\dfrac{7\frac{4}{5}}{3\frac{5}{6}}$ of $12\frac{3}{4}$ lbs.; of $\frac{2}{3}$ of £$70\frac{5}{24}$; and of $2\frac{59}{84}$ guineas.

15. Express the result of $\frac{7}{8}$ of 1 ton 4 cwt.—$\frac{3}{8}$ of 1 cwt. 3 qrs. 9 lbs.—$\frac{3}{8}$ of 1 ton 1 cwt. 1 qr. 1 lb.

16. Express $3\frac{1}{3}\frac{7}{8}$ of £5. 18s. 8d.$+\frac{3}{5}$ of £1. 2s. 6d.$-\frac{3}{8}$ of £6. 14s. $7\frac{1}{3}$d. as the fraction of £10. 10s. 10d.

17. Reduce $\frac{3}{8}$ of 5 yds. 2 ft. 9 in.—$\dfrac{2\frac{1}{3}}{6\frac{1}{3}}$ of $1\frac{1}{3}$ of 4 yds. 1 ft. 6 in. to the fraction of 12 yds.

266. MISCELLANEOUS EXERCISES IN FRACTIONS.

1. The hands of a watch are together at twelve o'clock. When will that occur again for the first time?

Since the minute hand moves round the dial in 1 hour, or 60 minutes, whilst the hour hands moves the 5 minutes' distance, then the minute hand gains 55 min. in 60 minutes' time over the hour hand. But after one hour, or at one o'clock, the hour hand points at one, and the minute hand at twelve; then the minute hand has 5 minutes' distance to gain to overtake the hour hand,

and $\because$ 55 min. are gained in 60 min. time.

1 min. is ,, $\frac{60}{55}$ or $\frac{12}{11}$ min.

$\therefore$ 5 min. are ,, $\dfrac{5\times 12}{11}=\frac{60}{11}=5$ min. $27\frac{3}{11}$ sec.

Thus they will be again over each other in 1 hr. 5 min. $27\frac{3}{11}$ sec.

2. Both hands of a watch are together between six and seven o'clock. The exact time is required.

At six o'clock, the minute hand was 30 minutes' distance from the hour hand, then the minute hand had 30 minutes to gain to overtake the hour hand.

But 55 min. are gained in 60 min. time,

and 1 min. is ,, $\frac{12}{11}$,,

$\therefore$ 30 min. are ,, $\dfrac{30\times 12}{11}$,, or 32 min. $43\frac{7}{11}$ sec.

Therefore the time is 32 min. $43\frac{7}{11}$ sec. after six o'clock.

3. A watch, which loses 4 minutes in a day, is set right at 12 o'clock on May 10th. What will be the true time on May 17th, when the hands of that watch point to 12 o'clock?

In 7 days the watch loses 7×4 min.$=28$ min.

But the day has 24×60 or 1440 minutes, and since it loses 4 minutes, the minute hand moves only $1440-4=1436$ minutes;

therefore, 1436 min. correspond to 1440 min.

$$1 \text{ min.} \quad ,, \quad \tfrac{1440}{1436} = \tfrac{360}{359}.$$

$$28 \text{ min.} \quad ,, \quad \frac{28 \times 360}{359} = 28 \text{ min. } 4\tfrac{244}{359} \text{ sec.}$$

Thus, on May 17th, when it is 12 o'clock by that watch, the true time will be 28 min. $4\frac{244}{359}$ sec. after 12 o'clock.

4. A reservoir is filled by one pipe in 6 hours, by a second in $5\frac{1}{4}$ hours, and by a third in $4\frac{2}{3}$ hours. In what time will it be filled by the three pipes, all running together?

$\because$ first pipe in 1 hour fills $\frac{1}{6}$,

and second ,, $\dfrac{1}{5\frac{1}{4}} = \frac{4}{21}$.

and third ,, $\dfrac{1}{4\frac{2}{3}}$ or $\frac{3}{14}$.

$\therefore$ they fill together in 1 hour $\frac{1}{6} + \frac{4}{21} + \frac{3}{14} = \frac{24}{42} = \frac{4}{7}$ of reservoir.

Now $\because$ $\frac{4}{7}$ are filled in 1 hour,

$\frac{1}{7}$ is ,, $\frac{1}{4}$ hour,

and $\frac{7}{7}$, or the whole, is filled in $\frac{7}{4}$, or $1\frac{3}{4}$ hours.

5. Three men, A, B, and C, perform a work in 24 days; A and B can perform it in 32 days, and A and C in 36 days. What part of the work can B and C perform in $18\frac{2}{7}$ days?

$\because$ A+B+C do in 1 day $\frac{1}{24}$ of the work,

and A+B ,, $\frac{1}{32}$,,

$$\therefore \text{ C does in 1 day } \tfrac{1}{24} - \tfrac{1}{32} = \frac{4-3}{96} = \tfrac{1}{96};$$

but A+C do in 1 day $\frac{1}{36}$,

$$\therefore \text{ B does in 1 day } \tfrac{1}{24} - \tfrac{1}{36} = \frac{3-2}{72} = \tfrac{1}{72};$$

$$\text{and B+C in 1 day do } \tfrac{1}{72} + \tfrac{1}{96} = \frac{4+3}{288} = \tfrac{7}{288};$$

$$\text{and} \therefore \text{B+C in } 18\tfrac{2}{7} \text{ days do } \frac{18\frac{2}{7} \times 7}{288} = \frac{\overset{4}{\cancel{128}} \times \cancel{7}}{\underset{9}{\cancel{288}} \times \cancel{7}} = \tfrac{4}{9}.$$

6. A general, detaching $\frac{4}{11}$ of his army to occupy a certain height, and $\frac{7}{11}$ of the remainder to watch the enemy's motions, had only 700 men left. Required, the whole number of his troops.

Let the whole army be represented by 1, the unit, and we have :—

$\frac{4}{11}$ of the army on the height, and there remains $1-\frac{4}{11}=\frac{7}{11}$.

$\frac{7}{11}$ of $\frac{7}{11}$, or $\frac{49}{121}$ of the army watching the enemy;

then soldiers remaining with general $=\frac{7}{11}-\frac{49}{121}=\frac{77-49}{121}=\frac{28}{121}$ of the army.

Therefore, $\frac{28}{121}$ of the army amount to 700 men,

and $\frac{1}{121}$,, ,, $\frac{700}{28}$ or 25 men;

therefore, the whole, or $1, =121\times 25$, or 3025 men.

7. A man, carrying peaches, sold one-half of them to A, one quarter of the remainder to B, and had 6 left. How many had he at first?

Let what he had be represented by the unit, or 1; having sold $\frac{1}{2}$ to A, he had left $\frac{1}{2}$ of his fruit; to B he sold $\frac{1}{4}$ of $\frac{1}{2}$, or $\frac{1}{8}$ of the whole. $\therefore$ he sold $\frac{1}{2}+\frac{1}{8}$, or $\frac{5}{8}$.

So he had left $1-\frac{5}{8}$, or $\frac{3}{8}$.

$\therefore$ $\frac{3}{8}$ of his fruit $=6$,

and $\frac{1}{8}$ of his fruit $=\frac{6}{3}=2$;

$\therefore$ the whole, or $1=8\times 2=16$ peaches.

8. I am the owner of $\frac{5}{8}$ of a house. I sell $\frac{2}{3}$ of $\frac{5}{6}$ of my share for £825. 10s. 5d. What is the value of the house?

The part sold is $\frac{2}{3}$ of $\frac{5}{6}$ of $\frac{5}{8}$, or $\frac{25}{72}$ of the house;

then $\frac{25}{72}$ of the house are worth £825. 10s. 5d.

and $\frac{1}{72}$ of the house is worth $\frac{\text{£825. 10s. 5d.}}{25}$, or £33. 0s. 5d.,

The house, or $\frac{72}{72}$, is worth $72\times$ £33. 0s. 5d., or £2477. 10s. 0d.

9. A watch, which goes 3 minutes too fast every day, is set right at 12 o'clock. What will be the exact time when the hands of the watch point at 12 minutes after 7?

10. Both hands of a watch point at twelve. It is required to ascertain how many times they will be together from noon to midnight, and at what time will each meeting take place?

11. If $\frac{1}{12}$ of a ship cost £68249. 18s. $5\frac{3}{4}$d., what must $\frac{3}{4}$ of her be sold for to gain £116. 14s. 6d.?

12. A does a work in $4\frac{1}{5}$ days, B in $5\frac{1}{11}$ days, C in $3\frac{7}{8}$ days, and D in $4\frac{2}{3}$ days. How much will each man do in 1 day, and how long will they be performing the work together?

13. I spent first $\frac{3}{8}$ of the contents of my purse, then $\frac{5}{6}$ of the remainder; now $\frac{3}{4}$ of what I have left amounts to £1. 11s. $4\frac{1}{2}$d. How much had I at first?

14. Find the sum, the difference, the product, and the quotient of $\frac{3}{4}$ of £16. 18s. 10d., and $\frac{2}{3}$ of £2. 17s. 4d.

15. Divide the sum of $4\frac{1}{3}$, $5\frac{3}{4}$, $6\frac{4}{5}$ by the difference of $8\frac{7}{8}$ and $5\frac{3}{4}$, and multiply the quotient by the sum of $16\frac{1}{6}$ and $11\frac{5}{8}$.

16. Add together $\frac{3}{4}$ of a guinea, £$\frac{5}{6}$, $\frac{5}{8}$s., $\frac{2}{3}$d., $\frac{8}{9}$ cr., $\frac{1}{1\frac{1}{2}}$ moi., and $\frac{1}{2}$ half-cr.

17. If $\frac{4}{5}$ of a ship be worth £496. 17s. $2\frac{1}{2}$d., what part of her is worth £649. 17s. 6d.?

18. In a plantation, $\frac{1}{5}$ of the trees are oak, $\frac{1}{6}$ chesnut, $\frac{1}{3}$ larch, $\frac{1}{8}$ Scotch fir, and the remaining 497 are beech trees. How many trees are there in the plantation?

19. If $\frac{5}{9}$ of a lottery ticket cost £1. 10s., what is the price of $\dfrac{3\frac{1}{4}}{5\frac{1}{8}}$ of a ticket?

20. A horse and saddle are worth £75. The saddle is $\frac{2}{7}$ of the value of the horse. Find the worth of each.

21. If 10 men or 12 women perform a piece of work in 24 days, in what time can 8 men and 8 women do the same?

22. If A accomplishes a piece of work in 12 days, B three times as much in 16 days, and C four times as much in 25 days, in what time can A, B, and C together do five times the same quantity of work?

23. A piece of linen, containing $25\frac{5}{7}$ yards, weighs $12\frac{2}{3}$ lbs. What is the weight of 1 yard, and how many yards are there in 1 lb. weight?

24. A pedestrian left Worksop for London on the 1st of January, at $8\frac{3}{4}$ o'clock in the morning. His rate of travelling, including stoppages, was $5\frac{2}{3}$ miles in $2\frac{1}{4}$ hours. Another pedestrian left London for Worksop on the 2nd of January, at $1\frac{1}{4}$ o'clock in the morning, and his rate of travelling, including stoppages, was $1\frac{1}{2}$ miles in $\frac{3}{4}$ hour. The distance between London and Worksop is 144 miles. It is required to answer the following questions:—

1st, what is the rate per hour of the first man?
2nd, in what time does he walk 1 mile?
3rd, on what day, and at what time, did he reach London?
4th, how far had he gone when the other man left town?
5th, at what rate per hour did the second man travel?
6th, in what time does he walk 1 mile?

7th, on what day, and at what time, did he reach Worksop?
8th, on what day, and at what time, did they meet on the road?
9th, at what distance from each place did the meeting take place?

25. To make 1 lb. of dough, $\frac{13}{35}$ lb. of water and $\frac{22}{35}$ lb. of flour are required. How much water and how much flour will it take to make 513 lbs. of bread, knowing that $1\frac{1}{6}$ lbs. of dough are reduced by baking to 1 lb. of bread?

26. If $1\frac{4}{5}$ oz. of silver make a wire $2352\frac{1}{4}$ yards long, how long will the wire be that is drawn out of 1 oz. of silver?

27. A thief, pursued by a constable, is 4 miles in advance. In what time will he be caught, if he goes 4 miles per hour, and the constable 1 mile in 8 minutes?

28. What is the difference between $\frac{3}{4}$ of 1 bushel and $\frac{7}{12}$ of 1 peck?

29. A person having $\frac{4}{5}$ of a copper mine sells $\frac{4}{9}$ of his share for £572. 12s. How much is the whole mine worth?

30. Divide £1764. 18s. 6d. into $19\frac{1}{5}$ shares.

31. Express $\frac{5}{7}$ of a guinea—$\frac{5}{8}$ of £1 as a fraction of 10s. 6d.

32. Reduce $\frac{7}{8}$ of a yard to the fraction of 1 English ell?

33. 16 cwt. 3 qrs. 15 lbs. 8 oz. are what part of 1 ton?

34. In a farm, $\frac{1}{5}$ of the land is meadow, $\frac{5}{8}$ arable, and the remainder contains 64 ac. 3 ro. 28 pl. Find the quantities of meadow and arable land, and also the number of acres the farm contains.

35. A person owes each of 5 creditors £1. 10s.; to A he pays $\frac{1}{4}$ of his debt, to B $\frac{2}{3}$, to C $\frac{3}{8}$, to D $\frac{5}{12}$, and to E $\frac{7}{16}$. How much does he owe yet?

36. Express 24 days 15 hrs. 24 min. 16 sec. as the fraction of $365\frac{1}{4}$ days.

37. Find the result of $6\frac{1}{2} \times \frac{1\frac{1}{4}}{16\frac{1}{3}} \div 4\frac{3}{4} \times \frac{8\frac{2}{3}}{9\frac{3}{5}}$.

38. If I gain 24s. 6d. on £18. 18s., how much do I gain per £1?

39. What fraction of £12. 10s., together with £4 4s. 4d., will make £10. 10s.?

40. A young lady's portion was £2400, which was $\frac{2}{3}$ of $\frac{5}{8}$ of her brother's fortune. How much was his fortune ?

41. A bankrupt's assets are $\frac{5}{12}$ of his debts. How much will he pay in the £1 ?

42. If I had as much money as I have, half as much, and one-fourth as much more, I should have £198. How much have I ?

43. What part of $6\frac{1}{2}$d. is $\frac{5}{8}$ of £1. 1s. $1\frac{1}{2}$d. ?

44. What is the difference between $\frac{3}{5}$ of $\frac{5}{6}$ of a crown, and $\frac{3}{5}$ of $\frac{9}{10}$ of 1 guinea ?

45. A younger son received £133. 6s. 8d., which was $\frac{2}{3}$ of $\frac{1}{2}$ of his elder brother's portion, and $3\frac{1}{2}$ times the elder brother's was $1\frac{1}{4}$ times the father's property. How much was the father worth ?

46. A party has a bill of £12. 7s. $1\frac{1}{2}$d. to pay, and one of them pays for himself and three friends £5. 9s. 10d. How many were there ?

47. The $\frac{5}{6}$ of £19. 16s. 9d. are $\frac{5}{51}$ of what ?

48. A man who was asked the time, answered, it is $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{7}{12}$ of $\frac{6}{7}$ of 24 hours. What time was it ?

49. A person being asked the hour of the day, said that the time past noon was $\frac{4}{5}$ of the time till midnight. What was the hour ?

50. A father left to his eldest son $\frac{4}{7}\frac{4}{9}$ of his property, to his second $\frac{3}{7}\frac{5}{9}$ of the remainder, and to his third son what was left. What was the share of each, if the shares of the first and second differed by £1200 ?

51. A gets £4 of a legacy for B's £9, and C £5 for B's £12. A's share is £2000. What is the whole legacy ?

52. If A, B, and C can build a wall in 24 days ; B, C, and D in 27 days ; C, D, and A in 32 days ; and D, A, and B in 36 days, in what time would it be done by all of them together, and by each of them singly ?

53. If 12 apples be worth as much as 21 pears, and 3 pears cost 1d., what is the price of 100 apples ?

54. A cistern is filled by two pipes in 3 and 4 hours respectively, and a tap empties it in 1 hour. If these pipes be opened in

order, at 1, 2, and 3 o'clock, prove that the cistern will be emptied at 12 minutes past five.

55. |The consumption of coal in steam vessels is usually averaged at 7 lbs. per horse-power per hour. How many bushels of coal does an American steamer consume during her passage from Liverpool to New York, it being performed in 10 days 15 hours? The engines are two, of 250 horse-power each. About 82 lbs. of coal make 1 bushel.

56. A hare, pursued by a greyhound, is 50 leaps in advance; the greyhound takes 5 leaps whilst the hare takes 6, but 9 leaps of the hare make only 7 of the greyhound. How many leaps will the hare take before she is caught?

57. What number is that, $\frac{5}{8}$ of which exceeds $\frac{2}{7}$ of it by 144?

58. The pipe A fills a cistern in 5 hours, and B in 4 hours; C empties it in 2 hours. Suppose the cistern filled, and the three pipes left open, how much time will it take to empty it?

59. A poor woman bought some pears; for one-half she paid at the rate of a penny for 2, and for the other half a penny for 3. She sold them again at 5 for 2d., and was astonished to find she had lost 1d. by her bargain. How many had she?

60. Four persons divided £100 amongst themselves; the first received one-half of what the second got, which was two-thirds of what the third received, and the fourth had two-thirds as much as the other three. The amount of each person's share is required.

PART VI.

RULE OF THREE, INTEREST, DISCOUNT, STOCKS, ANNUITIES, PARTNERSHIP, &c.

267. We have now arrived at the examination of questions which are usually solved by proportion; but since the properties of proportion depend more on algebraical than arithmetical principles, we propose postponing the demonstration of those principles, and to make use of a method which is superior, on account of its simplicity, conciseness, elegance, and rationality. It is from long experience in teaching that we prefer this plan to the ordinary one.

Ex. 1. If 108 cwt. of sugar cost £448, what would 65 cwt. cost, at the same rate?

$\because$ 108 cwt. cost £448,

$\therefore$ 1 cwt. will cost $\frac{1}{108}$ of £448, or £$\frac{448}{108}$, or £$\frac{112}{27}$;

and $\therefore$ 65 cwt. „ $65 \times$ £$\frac{112}{27}$, or $\frac{£65 \times 112}{27} =$ £269. 12s. $7\frac{1}{9}$d.

Ex. 2. A pole, 8 feet long, casts a shadow of 7 feet. What is the height of a tower, which at the same time projects a shadow of 203 feet?

$\because$ 7 feet of shadow are cast by a pole of 8 feet,

$\therefore$ 1 foot „ is „ „ $\frac{1}{7}$ of 8 ft., or $\frac{8}{7}$ ft.;

and $\therefore$ 203 feet „ are „ by a tower of $\frac{203 \times 8}{7}$ or 232 ft.

Ex. 3. A person owes £3074. 6s. 8d., but can pay only £1921. 9s. 2d. What will be the dividend, and how much shall I receive for a debt of £553. 3s.?

$\because$ £3074. 6s. 8d. are paid by £1921. 9s. 2d.,

$\therefore$ £1 is paid by $\frac{\text{£1921. 9s. 2d.}}{\text{£3074. 6s. 8d.}} = \frac{3 \times \text{£1921. 9s. 2d.}}{3 \times \text{£3074. 6s. 8d.}} = \frac{\text{£5764. 7s. 6d.}}{9223}$

= 12s. 6d. = the dividend;

and $\therefore$ £553. 3s. are paid by $553\frac{3}{20} \times$ 12s. 6d., or £345. 14s. $4\frac{1}{2}$d.

Ex. 4. If 12 men can do a work in 28 days, how many days will 20 men be doing the same?

$\because$ 12 men perform a work in 28 days,

$\therefore$ 1 man performs the same in 12×28 days;

and $\therefore$ 20 men perform the same in $\frac{1}{20}$ of 12×28 days, or $\dfrac{12 \times \cancel{28}^{7}}{\cancel{20}_{5}}$ or $16\frac{4}{5}$ days.

Ex. 5. How much in length that is 7 feet 9 inches broad will be equivalent to what is 75 feet 6 inches long, and 7 feet 6 inches broad?

$\because$ 7 ft. 6 in., or 90 in. broad, require a length of 75 ft. 6 in.

$\therefore$ 1 in. " " 90×75 ft. 6 in.

and $\therefore$ 7 ft. 9 in., or 93 in. " " $\frac{1}{93}$ of 90×75 ft. 6 in., or $\dfrac{\cancel{90}^{30} \times 75 \text{ ft. } 6 \text{ in.}}{\cancel{93}_{31}} = 73$ ft. $0\frac{24}{31}$ in.

Ex. 6. The provisions in a garrison were sufficient to last 1800 men for 12 months; but at the end of 3 months the garrison was reinforced by 600 men, and two months after that a second reinforcement of 400 men was sent in. How long did the provisions last for the whole?

$\because$ 1800 men remain in the garrison 3 months, there is left 9 months' provisions for 1800 men, when the first reinforcement comes;

then $\because$ 1800 men have provisions for 9 months,

$\therefore$ 1 man has provisions for 1800×9 mo., or 16200 mo.; but $1800 + 600$, or 2400 men remain 2 months before the second reinforcement arrives, which is the same as 1 man remaining 2×2400, or 4800 months, and $\because$ the provisions would keep 1 man 16200 months; take away 4800 months and the difference expresses how many months' provisions would keep 1 man, when the second reinforcement was sent.

$\therefore$ 1 man has provisions for 16200—4800, or 11400 months;

$\therefore$ $2400 + 400$, or 2800 men will have provisions for $\frac{1}{2800}$ of 11400 mo., or $4\frac{1}{14}$ months, and hence the provisions lasted $3 + 2 + 4\frac{1}{14} = 9\frac{1}{14}$ months.

268. Examples of this kind belong to that great variety of questions known under the name of Rule of Three, so called because three quantities being given, a fourth is found by means of them. In the three first examples, we may observe, that the answers *increase* and *decrease* respectively, as the quantities upon

which they depend *increase* and *decrease;* whilst in the three last questions the answers *increase* and *decrease* as the quantities upon which they depend *decrease* and *increase.* On this account, some authors make two divisions, *Rule of Three Direct* and *Rule of Three Inverse.* But since all questions classed under those two heads are solved by the same method, we shall not make that distinction.

269. Another class of questions, differing from those in (§ 267), in having more than three quantities given are known under the name of Double Rule of Three, but which might more correctly be named Rule of Five, &c., since five, &c., quantities are given, and a sixth, &c., is to be found. The following is an instance:—

Ex. 1. If 36 acres of land be mown by 6 men in 5 days, how many acres can be mowed by 40 men in 16 days?

$\because$ 6 men in 5 days mow 36 acres,

$\therefore$ 1 man in 5 days mows $\frac{36}{6}$, or 6 acres,

$\therefore$ 1 man in 1 day mows $\frac{6}{5}$ acre,

$\therefore$ 40 men in 1 day mow $\dfrac{\overset{8}{\cancel{40}}\times 6}{\underset{1}{\cancel{5}}}$, or 48 acres,

$\therefore$ 40 men in 16 days mow 16×48, or 768 acres.

Ex. 2. If 25 men, in 12 days of 9 hours each, can dig a trench 50 yards long, 4 feet broad, and 6 feet deep, how many men would be required to dig a trench 100 yards long, 3 feet broad, and 4 feet deep, in 18 days of 10 hours each?

	days.	hrs.		yds.	feet.	feet.		
$\because$ in	12	of 9	a trench	50 lg.	4 bd.	6 dp.	is dug by	25 men.
$\therefore$	1	,, 9	,,	50	,, 4	,, 6	,,	12×25 or 300.
$\therefore$	1	,, 1	,,	50	,, 4	,, 6	,,	9×300 or 2700.
$\therefore$	1	,, 1	,,	1	,, 4	,, 6	,,	$\frac{2700}{50}$ or 54.
$\therefore$	1	,, 1	,,	1	,, 1	,, 6	,,	$\frac{54}{4}$ or $13\frac{1}{2}$.
$\therefore$	1	,, 1	,,	1	,, 1	,, 1	,,	$\dfrac{13\frac{1}{2}}{6}$ or $2\frac{1}{4}$.
$\therefore$	1	,, 1	,,	100	,, 1	,, 1	,,	$100\times 2\frac{1}{4}$ or 225.
$\therefore$	1	,, 1	,,	100	,, 3	,, 1	,,	3×225 or 675.
$\therefore$	1	,, 1	,,	100	,, 3	,, 4	,,	4×675 or 2700.
$\therefore$	18	,, 1	,,	100	,, 3	,, 4	,,	$\frac{2700}{18}$ or 150.
$\therefore$	18	,, 10	,,	100	,, 3	,, 4	,,	$\frac{150}{10}$ or 15 men.

This very simple and natural process is easily condensed into the following form, when the pupil has acquired practice. It is

observed that the last term is of the same kind as the quantity required.

$$\begin{array}{llllllll} & \text{days.} & \text{hrs.} & & \text{yds.} & \text{feet.} & \text{feet.} & \\ \because \text{ in} & 12 & \text{of } 9 & \text{a trench} & 50 \text{ lg.} & 4 \text{ bd.} & 6 \text{ dp. is dug by} & 25 \text{ men.} \\ \therefore & 1 & ,,\ 1 & ,, & 1 & ,,\ 1 & ,,\ 1 & ,, \quad \dfrac{12\times 9\times 25}{50\times 4\times 6}. \\ \therefore & 18 & ,,\ 10 & ,, & 100 & ,,\ 3 & ,,\ 4 & ,, \quad \dfrac{\cancel{12}\times\cancel{9}\times\cancel{25}\times\cancel{100}\times 3\times\cancel{4}}{\cancel{50}\times\cancel{4}\times\cancel{6}\times\cancel{18}\times\cancel{10}} \end{array}$$

$=3\times 5$, or 15 men.

Ex. 3. If the sixpenny loaf weighs $5\frac{1}{2}$ lbs. when wheat is at 5.75s. per bushel, what must be paid for $52\frac{1}{4}$ lbs. of bread, when wheat is at 18.5s. per bushel?

$$\begin{array}{lllll} \because \text{ when wheat is} & 5.75\text{s. per bushel,} & 5\frac{1}{2} \text{ lbs. of bread cost} & 6\text{d.} & \\ \therefore \quad ,, & 1 \quad ,, & 1 \text{ lb.} \quad ,, & \dfrac{6\text{d.}}{5.75\times 5\frac{1}{2}}. & \\ \therefore \quad ,, & 18.5\text{s.} \quad ,, & 52\frac{1}{4} \text{ lbs.} \quad ,, & \dfrac{\overset{3.7}{\cancel{18.5}}\times 52\frac{1}{4}\times 6\text{d.}}{\underset{1.15}{\cancel{5.75}}\times 5\frac{1}{2}} & \end{array}$$

$=183\frac{9}{23}$d. $=15$s. $3\frac{9}{23}$d.

270. MISCELLANEOUS EXERCISES IN THE RULE OF THREE.

1. If 17 workmen perform 119 yards of work, how much will 12 workmen perform in the same time?
2. If I lend a friend £500 for 9 months, how long ought he to lend me £300 to requite my kindness?
3. How many yards of paper, 27 inches wide, will hang a room that measures 50 feet round and 9 feet high?
4. What is the value of 144 cwt. of cheese, if 9 cwt. be worth £23. 6s. 6d.?
5. How many tons of iron can I procure for £121. 5s. 4d., at the rate of £12. 4s. 4d. per ton?
6. If 6 cwt. 3 qrs. 12 lbs. of flour cost £9, what would 4 cwt. 2 qrs. cost.
7. If 5 acres are rented for £4. 13s. 4d., how much of like land may be rented for £70. 10s. 6d.?
8. If the penny loaf weighed 14 ounces when wheat was 4s. per bushel, what must it weigh when wheat is 6s. 9d. per bushel?

9. If 14 horses eat 56 bushels of oats in 16 days, how many bushels will serve 20 horses 24 days?

10. What must be the breadth of a piece of ground which is $14\frac{3}{8}$ yards long, so that it may be as large as a piece $40\frac{1}{2}$ yards long and $4\frac{3}{4}$ yards broad?

11. An iron beam, 24 feet long, $2\frac{3}{4}$ feet broad, and 13 inches thick, weighs 28800 lbs. What must be the length of a beam whose breadth is $3\frac{1}{2}$ feet, thickness $8\frac{1}{4}$ inches, and weight 12428 lbs.?

12. If 4 steam engines of 8-horse power, when employed 5 days a week, and 12 hours a day, raise through a given height 128 four-bushel sacks of wheat, weighing 64 lbs. per bushel, in what time will 9 engines of 10-horse power, when employed 3 days in a week, and 10 hours per day, raise through 8 times the former elevation 196 three-bushel sacks of wheat, weighing 65 lbs. a bushel?

13. If 8 horses can plough, in 5 days, 56 acres, how many acres will 35 horses plough in $8\frac{1}{2}$ days?

14. A captain of a ship has 10 months' provisions for 40 men. How long would those provisions last 32 men?

15. If the shilling loaf weighs 3 lb. 6 oz. when flour sells at £1. 13s. 6d. per quarter, how much should it weigh when flour sells at £1. 7s. 6d. per quarter?

16. Provisions in a fortified town are found to last 10000 soldiers 6 months; but a reinforcement being required, so that the provisions will last only two months, the number of soldiers to be added to the garrison is required.

17. If £1. 7s. be the wages of 4 men for 7 days, what will be the wages of 14 men for 10 days?

18. If 5 compositors, in 16 days of 14 hours long, can compose 20 sheets, of 24 pages in each sheet, 50 lines in a page, and 40 letters in a line, in how many days of 7 hours each can 10 compositors compose 40 sheets, 16 pages in a sheet, 60 lines in a page, and 50 letters in a line?

19. If the carriage of $4\frac{1}{2}$ cwt. of goods, for 50 miles, cost £1. 2s. 9d., how far can I have $14\frac{1}{4}$ cwt. carried, for the same money?

20. If 30 men perform a piece of work in 11 days, how many will accomplish another piece, six times as large, in a fourth of the time?

21. If 540 tiles, each 12 in. long and 12 in. broad, will pave a floor, how many will the same require when the tiles are 10 in. long and 8 in. broad?

22. The length of a tower, which casts a shadow 160 feet in length is required, knowing that at the same time a post 4 ft. 6 in. casts a shadow of 15 ft. 8 in.?

23. A coat requires in making $2\frac{3}{4}$ yards of cloth, $\frac{3}{4}$ yard broad. How much would be required if the cloth was $1\frac{1}{8}$ yards broad?

24. A company of gold seekers extracted 475 lbs. 8 oz. 12 dwts. 16 grs. of gold in 9 yrs. 207 days 7 hrs. 50 min., when working 11 hours per day. How much will be extracted by the same company in 7 yrs. 67 days 9 hrs. 36 min., working 12 hrs. a day?

25. A garrison of 6000 men has provisions for 6 months, allowing every man 18 oz. per day; a reinforcement of 1500 men is sent, and the provisions are to last 10 months. What must be each man's allowance?

26. Two pumping engines, in 8 days of 11 hours each, have lowered the level of a lake 6 feet. How many hours a day would three engines, similar to the first, be at work to produce the same effect in 5 days?

27. What is the cost of 13 packs of wool, each 2 cwt. 1 qr. 19 lbs., at the rate of £21. 10s. for 3 cwt. 17 lbs.?

28. A person completes a journey of 160 miles in 3 days, travelling 11 hours a day. In how many days would he complete 1000 miles, travelling 15 hours a day, at the same rate?

29. If 9 masons can erect a wall, in $20\frac{4}{5}$ days of $9\frac{2}{3}$ hours each, how long would it take 3 masons to do $3\frac{3}{4}$ as much, reckoning $12\frac{1}{3}$ hours to the day?

30. If the rent of 46 ac. 3 ro. 14 p. be £100, what will be the rent of 35 ac. 2 ro. 10 p.?

31. Suppose a greyhound makes 27 springs while a hare makes 25, and the springs are of equal length. Now, if the hare

be 50 springs before the hound, in how many springs will he overtake her?

32. If the value of 1 lb. avoirdupois be £4. 0s. $2\frac{1}{2}$d., how many shillings must be paid for 1 lb. Troy?

33. If a canal, 1500 yards long, 6 yards wide, and 3 yards deep, be dug in 30 days, of 12 hours each, by 280 men, how many men would be required to dig another canal, 3 miles long, 4 yards wide, and 2 yards deep, in 125 days of 10 hours each?

34. If 14 lbs. of sugar be sold for 9s. 4d., when the cost of 1 cwt. is £3. 7s. 6d., what would be the cost of 1 cwt. when 11 lbs. are sold for 7s. $1\frac{1}{4}$d.?

35. If 4 artillerymen can fire a gun 48 times, and 5 men 52 times in an hour, how much more time will be required for firing 21216 shots from 26 guns, when there are 4 men to a gun than when there are 5 men?

36. In what time would the wind move from the pole to the equator, at the rate of $2\frac{3}{4}$ miles per hour, the distance being 6214 miles?

37. If 63 lbs. of tea cost £20. 10s. 6d., what will 70 lbs. cost of a different quality, 9 lbs. of the former being equal in value to 10 lbs. of the latter?

38. If the circumference of the driving wheel of a locomotive be $16\frac{1}{2}$ feet, how many revolutions will it make between Bristol and Exeter, the distance being $75\frac{1}{2}$ miles?

39. If $135\frac{6}{7}$ lbs. of thread are woven into a piece of linen, $86\frac{3}{4}$ yards long and $\frac{4}{5}$ yard broad, what would be the length of another piece, woven out of $116\frac{3}{4}$ lbs. of thread, and which is $1\frac{1}{3}$ yards broad?

40. If $\frac{3}{8}$ yard of velvet cost £$\frac{2}{5}$, what will $\frac{5}{16}$ yard cost?

41. The making of 43 yds. 1 qr. 8 in. of cloth cost £48. 16s. 9d. What will be the cost of 77 yds. 2 qrs. 9 in., at the same rate?

42. To clothe 25000 men, it takes 125000 yards of cloth, $\frac{5}{4}$ yard wide. How many yards of cloth, $\frac{7}{8}$ yard wide, are wanted to clothe 3840 men?

INTEREST.

271. The payment made for the use of money lent for any length of time is called *Interest*, and is usually reckoned at so much for £100 during a year; if the interest of £100 for a year be £5, the money is said to be lent at 5 per cent. per annum, and 5 is called the *rate* of interest. The money lent is termed the *principal*, and the sum of the interest and principal is called the *amount*.

272. The rate is a conventional agreement between the lender and the borrower of the principal. There is, however, a limit, beyond which the rate is *illegal*. Usurers are those who lend money at a higher rate than the law permits.

273. The learner must remember that 4 per cent. does not signify only that the interest of £100 is £4, but it means as well that the interest of 100s. is 4s., or that the interest of 100 farthings, is 4 farthings; in fact, it implies that the interest of 100 units is 4 units of the same kind.

274. The interest depends, then, on the principal, the time that the principal is lent, and the rate.

275. Interest is of two kinds; if the borrower pays the interest at a specified period, either yearly, half-yearly, or quarterly, according to agreement, it is called *Simple Interest;* but when at the end of any stated time, as a year, &c., the interest is added to the principal for the second year; and again, the interest accruing being added to the last principal, &c., it is called *Compound Interest.*

276. There are five quantities concerned in interest, the principal, the rate, the time, the interest, and the amount, any three of these, (except the principal, the interest, and the amount) being given, the others can be found.

277. Therefore, we have to consider the four following cases:

1st, to find the interest of a given principal, for a given time, at a given rate.

2nd, to find the rate, when the principal, the interest, and the time are given.

3rd, to find the time when the rate, the principal, and the interest are given.

4th, to find the principal, when the time, the interest, and the rate are known.

278. 1st case: Find the simple interest of £240. 10s. for 1 year, and also for 6 years, at 4 per cent.

∵ 100 in 1 year gains 4.

∴ 1 „ 1 „ $\frac{4}{100}$ or .04.

∴ £240.5 „ 1 „ 240.5 × £.04, or £9. 12s. $4\frac{3}{4}$d. Ans. 1.

£240.5 „ 6 years gains 240.5 × £.04 × 6 or £57. 14s. $4\frac{1}{2}$d. „ 2.

Operation:
```
 240.5
   .04
 -----
 9.620 = £9. 12s. 4¾d.
     6
 -----
57.720 = £57. 14s. 4½d.
```

Ex. 2. What is the interest of £500. 13s. 4d. for $2\frac{3}{4}$ years, at $2\frac{3}{4}$ per cent?

Here 100 in 1 year gains 2.75.

∴ 1 in 1 year gains .0275.

∴ £500$\frac{2}{3}$ in $2\frac{3}{4}$ yrs. gains 500$\frac{2}{3}$ × 2.75 × £.0275 = £37. 17s. $3\frac{1}{10}$d.

Operation:
```
 .0275
  2.75
 ------
  1375
 1925
 550
 --------
.075625
     500 ⅔
 ---------
37812500
   25208 ⅓
   25208 ⅓
 ----------------
37.862916 13 4
        20
 ---------
17.258333
       12
 ---------
 3.100000
```

Answer, £37. 17s. $3\frac{1}{10}$d,

Ex. 3. Required, the interest of £365. 4s. $10\frac{1}{2}$d. for 2 years $4\frac{1}{2}$ months, at $5\frac{5}{8}$ per cent.

$\because$ 100 in 1 year gains $5\frac{5}{8}$,

$\therefore$ £365. 4s. $10\frac{1}{2}$d. in 2 yrs. $4\frac{1}{2}$ mo. gains $\dfrac{\overset{\text{yrs. mo.}}{2\ \ 4\frac{1}{2}} \times \overset{\text{£.}}{5\frac{5}{8}} \times \overset{\text{£.}\quad \text{s.}\quad \text{d.}}{365.\ \ 4\ \ 10\frac{1}{2}}}{100}$

or £48. 15s. $10\frac{1}{2}$d.

	£.	s.	d.
Operation:	365	4	$10\frac{1}{2}$
			$5\frac{5}{8}$
$\frac{5}{8} = \frac{1}{8}$ of	1826	4	$4\frac{1}{2}$
	228	5	$6\frac{9}{16}$
4 mo. $= \frac{1}{3}$ yr.	2054	9	$11\frac{1}{16}$
			2 yrs. $4\frac{1}{2}$ mo.
	4108	19	$10\frac{1}{8}$
$\frac{1}{2}$ mo. $= \frac{1}{8}$ of 4 mo.	684	16	$7\frac{11}{16}$
	85	12	$0\frac{123}{128}$
100)	48.79	8	$6\frac{99}{128} = \frac{3}{4}$ nearly
	20		
	15.88		
	12		
	10.62		
	4		
	2.51		

Ex. 4. Find the interest of £140. 10s. for 76 days, at 5 per cent.

$\because$ 100 in 365 days gains 5,

$\therefore$ 1 in 1 day gains $\dfrac{5}{100 \times 365} = \dfrac{10}{100 \times 730}$,

$\therefore$ £140. 10s. in 76 days gains $\dfrac{\overset{\text{£.}\quad\text{s.}}{140\ \ 10} \times 76 \times 10}{100 \times 730} = \dfrac{\overset{\text{£.}\ \text{s.}}{14\ \ 1} \times 76 \times \overset{\text{£.}}{1}}{10 \times 73}$

$= £1.\ 9s.\ 3\frac{21}{365}d.$

	£.	s.
Operation :	14	1
1s. = $\frac{1}{20}$	76	
	84	
	98	
	3	16
730)	1067	16(£1. 9s. $3\frac{21}{365}$d.
	730	
	337	
	20	
	6756	
	6570	
	186	
	12	
	2232	
	2190	
	42	

These questions will afford many opportunities of abbreviating the work, but for that the pupil must be well grounded in fractions; he must be able to detect at a glance that by transforming an expression he may often obtain another more convenient to use. In the first and second examples we have introduced some decimals, which make the process very simple. The third example has been worked by practice; and in the second line of the solution of the last example, we have adopted a transformation, in order to facilitate the division.

279. 2nd case. Ex. 1. At what rate per cent. must £102. 10s. be lent, so that the interest may be £12. 13s. $8\frac{1}{4}$d. in $2\frac{1}{4}$ years?

$\because$ £102. 10s. in 2.25 years gains £12. 13s. $8\frac{1}{4}$d.,

$$\therefore \text{£}100 \text{ in 1 year gains } \frac{\overset{4}{\cancel{100}} \times \text{£}12.\ 13\text{s}.\ 8\tfrac{1}{4}\text{d}.}{\underset{4\cdot1}{\cancel{102.5}} \times 2.25} = 5\tfrac{1}{2}.$$

```
                £.  s.  d.
Operation :    12  13  8¼
                        4
              ───────────
  2.25         50  14  9
   4.1         20
  ────        ────
   225        1014
  900           12
──────────────
9.225)12177(1.32 × 1000 = 1320d. = 5½
       9225
       ─────
       29520
       27675
       ─────
       18450
       18450
       ─────
       .....
```

Ex. 2. If £1 amounts to £1. 2s. 9d. in $3\frac{1}{4}$ years, at what rate per cent. must it have been lent ?

Here interest = £1. 2s. 9d. — £1 = 2s. 9d.

$\because$ £1 in $3\frac{1}{4}$ years gains 2s. 9d., or 33d.,

$\therefore$ £100 in 1 year gains $\dfrac{100\times 33\text{d.}}{3.25}=\dfrac{\overset{1}{\cancel{\overset{2}{\cancel{100}}}}\times \pounds\overset{11}{\cancel{33}}}{\underset{\cdot 65}{\cancel{3.25}}\times \underset{4}{\cancel{\underset{12}{\cancel{240}}}}}=4\frac{3}{13}$.

280. 3rd case. Ex. 1. In what time will £300 amount to £350. 12s. 6d., at $3\frac{3}{4}$ per cent. per annum ?

Here interest = £350. 12s. 6d — £300 = £50. 12s. 6d.

$\because 3\frac{3}{4}$ is gained by 100 in 1 year,

$\therefore 1$ is gained by 1 in $\dfrac{100\times 1}{3\frac{3}{4}}$,

$\therefore$ £50. 12s. 6d. is gained by 300 in $\dfrac{1\cancel{00}\times \overset{\text{yr.}}{1}\times 50\frac{5}{8}}{3\frac{3}{4}\times 3\cancel{00}}=4\frac{5}{18}$ years.

Ex. 2. In what time will £818. 18s. 4d. amount to £1245. 5s., at $3\frac{3}{4}$ per cent. ?

Here interest = £1245. 5s. — £818. 18s. 4d. = £426. 6s. 8d.

$\because 3\frac{3}{4}$ is gained by 100 in 1 year,

$\therefore 1$ is gained by 1 in $\dfrac{1\times 100}{3\frac{3}{4}}$,

$\therefore$ £426. 6s. 8d. is gained by 818. 18s. 4d. in $\dfrac{\overset{\text{yr.}}{1}\times 100\times 426\frac{1}{3}}{3\frac{3}{4}\times 818\frac{11}{12}}$ $=13\frac{8}{9}$ yrs. nearly.

$$
\begin{array}{rrl}
 & 818\frac{11}{12} & \\
 & 3\frac{3}{4} & \\
\hline
\frac{3}{4}=\frac{1}{4} \text{ of } & 2456\frac{3}{4} & 426\frac{1}{3} \\
 & 614\frac{3}{16} & 100 \\
\hline
 & 3070\frac{15}{16} & 42633\frac{1}{3} \\
 & 16 & 16 \\
\hline
 & 49135\ ) & 682133\frac{1}{3}(13.88 \\
 & & 49135 \\
\hline
 & & 190783 \\
 & & 147405 \\
\hline
 & & 433783 \\
 & & 393080 \\
\hline
 & & 407033 \\
 & & 393080 \\
\hline
\end{array}
$$

281. 4th case. Ex. 1. What principal will produce £121. 15s. 5d. in 2 yrs. 1 mo., the interest at $5\frac{3}{4}$ per cent.?

$\because 5\frac{3}{4}$ interest is gained in 12 mo. by 100,

$\therefore 1$ „ „ 1 „ $\dfrac{100\times 12}{5\frac{3}{4}}$,

$\therefore$£121. 15s. 5d. „ 25 „ $\dfrac{\overset{4}{1\cancel{00}}\times 12\times \text{£}121.\ 15\text{s}.\ 5\text{d}.}{5\frac{3}{4}\times \underset{1}{\cancel{25}}}=$ £1016. 10s. $5\frac{5}{23}$d.

Ex. 2. What principal, lent from the 24th of March, 1852, till the 17th November, 1853, at $4\frac{1}{2}$ per cent. per annum, will gain £1200?

From March 24th, 1852, to November 17th, 1853, are $1\frac{238}{365}$ yrs., or 603 days.

$\because 4\frac{1}{2}$ is gained in 365 days by 100,

$\therefore 1$ is gained in 1 day by $\dfrac{100\times 365}{4\frac{1}{2}}$,

$\therefore$£1200 is gained in 603 days by $\dfrac{\overset{400}{\cancel{1200}}\times 100\times 365}{\underset{201}{\cancel{603}}\times 4\frac{1}{2}}=$ £16141. 10s. $3\frac{1}{2}$d. nearly.

282. EXERCISES.

1. What is the interest of £862. 14s. 2d. for 5 years, at $4\frac{1}{2}$ per cent.?
2. Find the interest of £535 for 117 days, at $4\frac{3}{4}$ per cent.

3. What principal will amount to £1292. 6s. 4½d. in 9⅜ years, at 5⅞ per cent. per annum ?

4. In what time will £350 amount to £402. 10s., at 3 per cent. per annum ?

5. At what rate per cent. will £1500 amount to £1850 in 4⅓ years ?

6. Find the amount of £1440. 15s. for 1 year 73 days, at 5 per cent. per annum.

7. In what time will any principal double itself, at 4½ per cent. per annum ?

8. If the interest on £261. 11s. 8d. for 15 days be 8s. 8d., what is the rate per annum ?

9. At whatrate per cent. per annum will any principal triple itself in 20 years ?

10. Find the amount of £4000 for 3 years 10 weeks, at 3¼ per cent. per annum ?

11. Find the interest of £3996. 15s. for 4 years 225 days, at 3⅔ per cent. per annum.

12. What principal will bring £1312. 13s. interest in 27 months, at 1½ per cent. per month ?

13. Borrowed, on March 12th, 1849, the sum of £2456. 12s. 6d., at 5 per cent. ; on February the 2nd, 1851, I paid on account £932. 15s. 8d. ; on August 13th, 1852, I paid £442. 18s. 6d. ; and on May 18th, 1854, I shall pay £925. 10s. 9d. What will be the balance for me to pay on the 31st of December, 1855 ?

14. At what rate per cent. will £827. 10s. amount to £986. 15s. 10½d. in 5½ years ?

15. What principal will amount to £1383. 15s. 11¼d. in 5¼ years, at 3⅔ per cent. per annum ?

16. Find the interest and amount of £1000 for 2 yrs. 9 mo. 25 days, at 4¾ per cent. per annum.

17. The interest of £319. 6s. for 5¾ years was £68. 14s. 9½d. Find the rate per cent. per annum ?

18. Required, the simple interest on £1064. 18s. 6d., from May 1st to October 8th, at 3⅛ per cent. per annum ?

19. How long will £670 be in amounting to £840. 10s., at $6\frac{1}{4}$ per cent. per annum ?

20. A's income is derived from two equal principals, one being lent at 4, and the other at 5 per cent. per annum. If his annual income be £650, what is the value of each principal ?

21. A persons daily income is £$3\frac{61}{219}$; $\frac{1}{5}$ of the principal is at $3\frac{1}{3}$ per cent., $\frac{1}{3}$ at $3\frac{2}{3}$ per cent., $\frac{7}{15}$ at $4\frac{1}{2}$ per cent. per annum. How much is each principal, and also the whole fortune ?

COMPOUND INTEREST.

283. When the interest is added yearly to the principal, and we consider the sum accruing as a new principal, the interest is compound.

Ex. Required, the compound interest of £1200, at 5 per cent. per annum, for 4 years 6 months.

There are several methods of solving questions of this kind.

1st method : Find the interest, year after year, adding it each time to the last principal.

For instance, the interest of 1st year's principal, or £1200, is $\frac{£1200\times5}{100}=£60$.

$\therefore$ the interest of 2nd year's principal, or £1260, is $\frac{£1260}{20}=£63$.

$\therefore$ the interest of 3rd year's principal, or £1323, is $\frac{£1323}{20}=£66.15$

$\therefore$ the interest of 4th year's principal, or £1389.15, is $\frac{£1389.15}{20}=£69.4575$.

$\therefore$ the interest of next $\frac{1}{2}$ year's principal, or £1458.6075 is $\frac{£1458.6075}{20\times2}=£36.4652$.

$\therefore$ the amount$=£1458.6075+36.4652+£1495.0727$.

Hence C.I.$=£1495.0727-£1200=£295.0727=£295$. 1s. $5\frac{1}{4}$d. nearly.

2nd method: Find the amount of £1 for the given time, and multiply it by the given principal.

$\because$ 100 bring 5, $\therefore$ 1 brings .05.

Then, at the end of 1st year, £1 amounts to £1.05;

2nd year, £1 amounts to $1.05 \times .05 + 1.05 = 1.1025$;

3rd year, £1 amounts to $1.1025 \times .05 + 1.1025 = 1.157625$;

4th year, £1 amounts to 1.215506;

the next ½ year £1 amounts to 1.245894.

And $\because$ £1 is worth, at the end of 4½ years, £1.245894,

$\therefore$ £1200 is worth, at the end of 4½ years $1200 \times$ £1.245894, or £1495.0728.

To avoid the tediousness of this process, tables have been constructed on this principle, giving the amount of £1 for different numbers of years, at different rates of interest. The following concise table may be found useful:—

TABLE OF THE AMOUNT OF £1 FOR ANY NUMBER OF YEARS UP TO TWENTY.

Yrs.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
1	1.03	1.035	1.04	1.045	1.05
2	1.0609	1.071225	1.0816	1.092025	1.1025
3	1.092727	1.1087179	1.124864	1.1411661	1.157625
4	1.1255088	1.1475230	1.1698586	1.1925186	1.2155062
5	1.1592741	1.1876863	1.2166529	1.2461819	1.2762816
6	1.1940523	1.2292553	1.2653190	1.3022601	1.3400956
7	1.2298739	1.2722793	1.3159318	1.3608618	1.4071004
8	1.2667701	1.3168090	1.3685690	1.4221006	1.4774554
9	1.3047732	1.3628974	1.4233118	1.4860951	1.5513282
10	1.3439164	1.4105988	1.4802443	1.5529694	1.6288946
11	1.3842339	1.4599697	1.5394541	1.6228530	1.7103394
12	1.4257609	1.5110687	1.6010322	1.6958814	1.7958563
13	1.4685337	1.5639561	1.6650735	1.7721961	1.8856491
14	1.5125897	1.6186945	1.7316764	1.8519449	1.9799316
15	1.5579674	1.6753488	1.8009435	1.9352824	2.0789282
16	1.6047064	1.7339860	1.8729812	2.0223702	2.1828746
17	1.6528476	1.7946756	1.9479005	2.1133768	2.2920183
18	1.7024331	1.8574892	2.0258165	2.2084788	2.4066192
19	1.7535060	1.9225013	2.1068492	2.3078603	2.5269502
20	1.8061112	1.9897889	2.1911231	2.4117140	2.6532977

3rd method: $\because$ 100 are worth in 1 year 105,

$\therefore$ £1200 are worth in 1 year $\frac{1200 \times £105}{100}$ = amount of £1200 at the end of 1st year.

$\because$ 100 are worth in 1 year 105,

$\therefore \dfrac{1200 \times £105}{100}$ are worth in 1 year $\dfrac{1200 \times 105 \times £105}{100 \times 100} =$ amount of £1200 at the end of 2nd year.

$\therefore \dfrac{1200 \times 105 \times £105}{100 \times 100}$ are worth in 1 year $\dfrac{1200 \times 105 \times 105 \times £105}{100 \times 100 \times 100}$ $=$amount of £1200 at the end of 3rd year.

Likewise, $\therefore$ $\dfrac{1200 \times 105 \times 105 \times £105}{100 \times 100 \times 100}$ are worth in 1 year $\dfrac{1200 \times 105 \times 105 \times 105 \times £105}{100 \times 100 \times 100 \times 100} =$ amount of £1200 at the end of 4th year.

And $\therefore$ $\dfrac{1200 \times 105 \times 105 \times 105 \times £105}{100 \times 100 \times 100 \times 100}$ are worth in 1 year

$$\frac{\overset{3}{\cancel{1200}} \times \overset{21}{\cancel{105}} \times \overset{21}{\cancel{105}} \times \overset{21}{\cancel{105}} \times \overset{21}{\cancel{105}} \times £\overset{\overset{4.1}{\cancel{20.5}}}{\cancel{102.5}}}{\underset{\underset{1}{\cancel{20}}}{\cancel{100}} \times \underset{\underset{1}{\cancel{20}}}{\cancel{100}} \times \underset{20}{\cancel{100}} \times \underset{20}{\cancel{100}} \times \underset{\underset{4}{\cancel{20}}}{\cancel{100}}} = \text{amount of £1200 at the}$$

end of the next $\frac{1}{2}$ year.

Performing the work, we find as before, £1495.0727.

Hence, we must multiply the principal by the amount of 100 for 1 year as many times as there are years in the question, and divide that product by 100, multiplied by itself as many times as there are years. The quotient will be the amount.

4th method depends upon the raising of quantities to different powers, and for that reason we postpone the explanation of it to a later part of the work.

Ex. 2. Determine the amount of £725 in 4 years, at 5 per cent. C.I.

	£.
1st year's principal	725
Interest at 5 per cent.	36.25
2nd year's principal	761.25
Interest at 5 per cent.	38.0625
3rd year's principal	799.3125
Interest at 5 per cent.	39.9656
4th year's principal	839.2781
Interest at 5 per cent.	41.9639
	£881.2420 = £881. 4s. $10\frac{2}{25}$d.

Ex. 3. What is the difference between the simple interest of £10000 for 1 year, at 6 per cent. per annum, and the C.I. of the same sum for a year, taken quarterly?

The simple interest of £1000 = 100 × 6 = £600.

$$\text{Amount(C.I.)of £10000} = \frac{1\cancel{0000} \times \overset{20\cdot3}{\cancel{101.5}} \times \overset{20\cdot3}{\cancel{101.5}} \times \overset{20\cdot3}{\cancel{101.5}} \times \overset{20\cdot3}{\cancel{101.5}}}{1\cancel{00} \times 1\cancel{00} \times \underset{\underset{4}{\cancel{20}}}{\cancel{100}} \times \underset{\underset{4}{\cancel{20}}}{\cancel{100}}}$$

= £10613. 12s. 8¼d.

C.I. = £10613. 12s. 8¼d.—10000 = £613. 12s. 8¼d.
Difference £613. 12s. 8¼d.—£600 = £13. 12s. 8¼d.

284. EXERCISES IN C.I.

1. Find the C.I. of £640 for 5 years, at 3 per cent. per annum.
2. Which is the most advantageous to receive for the value of a house, £5000 ready money, or £5750 in 3 years, supposing the interest compound, and the rate 5 per cent. per annum?
3. An engineer, whose salary is £150 a year, puts in the Savings Bank every year one-tenth of his stipend. How much will he have in 6 years, the rate of interest being 3½ per cent. per annum?
4. Required, the C.I. of £9375, at 10 per cent. per annum, for 3 years 8 months.
5. What is the rate per cent. per annum on a principal lent at 1¼ per cent. quarterly, C.I.
6. Find the amount of £819. 4s. in 4 years, at 12½ per cent. per annum, C.I.
7. Required, the difference between the simple and compound interest on £1460. 18s. 9d. for 5 years, at 4 per cent. per annum.
8. What will be the amount of 6 payments of £1200, put yearly into a bank for 6 years, at 10 per cent. per annum?
9. How much must be put to C.I., at 4 per cent. per annum, to make the sum of £33955. 13s. in 5 years?
10. What principal, lent at C.I., at 6 per cent. per annum, will amount to £2050. 10s. 8d. in 3 years?
11. If a boy, 14 years old, has a legacy of £2000 left to him. How much will he have to receive at the age of 21, the

legacy increasing, at C.I., at the rate of 5 per cent. per annum ?

12. Determine the C.I. of £15000, at 4 per cent. per annum, for $3\frac{1}{4}$ years ?

13. During $5\frac{1}{2}$ years a person puts £1000 each year, at C.I., at $4\frac{1}{2}$ per cent. per annum. How much will he have to receive, at the end of that time ?

DISCOUNT.

285. Discount is an allowance made upon the payment of money before it is due. To explain this, let us suppose that a bill of £3000, which is to be paid in 1 year, is taken to a banker in order to have it discounted. It is required to find what he will deduct, and what he will pay to the holder of the bill, if the rate of interest be 6 per cent.

The holder of a bill is evidently entitled to receive a sum, which being added to its interest for 1 year, would amount to £3000, the value of the bill.

But since £100, at 6 per cent., amounts to £106 in 1 year, it follows that £106, due 1 year hence, is of the same value as £100 ready money.

$\therefore$ if 106 are worth now 100,

1 is worth now $\frac{100}{106}$.

$\therefore$ £3000 are worth now $\frac{3000 \times 100}{106}$ = £2830. 3s. 9d. nearly.

Also, if for 106 the banker deducts 6,

$\therefore$ for 1 the banker deducts $\frac{6}{106}$;

$\therefore$ for £3000 the banker deducts $\frac{3000 \times 6}{106}$ or £169. 16s. 3d. nearly.

The present worth of the bill is, therefore, £2830. 3s. 9d., and the discount £169. 16s. 3d.

We have, then, the present worth = the given sum — the discount;

And the discount = the given sum — the present worth.

286. This method of calculating discount, though the true one, is not that employed by bankers and merchants; it is customary to charge as discount the *interest* of the sum for

the given time, which gives the discount too large, and, consequently, the present worth too little, by the interest of the true discount. Thus, if a banker discounted a bill of £3000, at 6 per cent., he would deduct the interest of that sum for 1 year, which is £180, exceeding the true discount by £10. 3s. 9d., which is the interest of £169. 16s. 3d., and the present worth is £3000—£180=£2820. Therefore, by this transaction, the holder would lose £10. 3s. 9d., which sum the banker would gain.

287. The bankers' method is supposed to be preferred, as involving less intricate calculations; and when the time is short, and the sum small, as it is generally the case in real business, the difference between the results found by the two methods is but trifling, and the easier method, though false in principle, may be adopted; but when the time is long, and the sum large, the error is too considerable to be allowed for the sake of easy calculation.

288. Discount is mostly applied to the payment of bills, which are stamped documents, given by buyers to sellers, by debtors to creditors, and under many other circumstances, both as security for money due, and as a means of obtaining the immediate use of cash, not payable till a given time. If such a bill be presented to a banker, before the time fixed for payment, and he accepts the security of the person promising payment, or of the holder of the bill, he will discount it, or pay its present value at once, deducting the interest for the time it has to run.

289. When a bill is drawn to run a certain time, three days are allowed after the expiration of the term before the bill is presented for payment. Thus, a three months' bill, dated May 1st, would be nominally due on the 1st of August, but legally on the 4th of August. These extra days are called *days of grace.*

290. When articles are paid for at the time of sale, or shortly afterwards, tradesmen allow a discount of 5 per cent., which is the same as 1s. per £1 from the amount. For example, in settling an account of £24. 14s. 8d., most tradesmen would allow £1. 4s. 6d., viz., 24s. for the £24, and 6d. for the 14s. 8d. The discount may consist of odd shillings and pence over a certain sum in pounds, but it is generally in the shape of a

percentage. This allowance is the interest of the *debt*, and not of the *present value*, and will be to the payer's advantage: it thus differs in principle from the bankers' discount.

Ex. 1. Required, the present worth of a bill of £486. 18s. 8d., drawn March 25th, at 10 months, and discounted on June 19th, at 5 per cent. Here, from June 19th to January 25th, are 220 days; with the three days of grace, 223 days.

By bankers' discount:

∵ the interest of 100 for 365 days is 5,

∴ the interest of 100 for 223 days is $\dfrac{223\times\cancel{5}}{\underset{73}{\cancel{365}}}$ or $3\frac{4}{73}$.

∴ the interest of £486. 18s. 8d., the amount of the bill, for 223 days is $\dfrac{£486.\ 18s.\ 8d.\times 3\frac{4}{73}}{100}$ = or £14. 17s. 6d.

∴ present worth = £486. 18s. 8d. — £14. 17s. 6d. = £472. 1s. 2d. nearly.

By true discount:

∵ $103\frac{4}{73}$ are worth now 100,

∴ the present value of £486. 18s. 8d. is $\dfrac{£486.\ 18s.\ 8d.\times £100}{103\frac{4}{73}}$ = £472. 10s. nearly.

Ex. 2. What is the present worth and discount of £550. 10s. for 9 months, at 5 per cent. per annum?

Bankers' discount:

∵ 100 in 12 mo. give 5 discount,

∴ £550. 10s. in 9 mo. give $\dfrac{£55\cancel{0}.\ 1\cancel{0}s.\times\overset{3}{\cancel{9}}\times\cancel{5}}{\underset{2}{\cancel{100}}\times\underset{4}{\cancel{12}}}$ = £20. 12s. 10½d. discount.

True discount:

∵ 103.75 are worth now 100,

∴ £550. 10s. „ $\dfrac{£\overset{110}{\cancel{550}}\ \overset{2}{\cancel{10}}s.\times\overset{4}{\cancel{100}}}{\begin{array}{c}\cancel{103.75}\\ \cancel{20.75}\\ .83\end{array}}$ = £530. 12s. 0½d. = present worth.

∴ £550. 10s, — £530. 12s. 0½d, = £19. 17s. 11½d. = discount.

Ex. 3. What would a banker gain by discounting, on July 8th, a bill of £447. 12s. 6d., dated June 23rd, at 6 months, at 5¾ per cent. per annum?

Counting 6 months and 3 days from the 23rd of June, we find the bill to be due on the 26th of December: from the 8th of July to this date there are 171 days.

Then, by bankers' discount:

100 in 365 days give $5\frac{3}{4}$ discount,

$$\therefore £447.\ 12s.\ 6d. \text{ in 171 days give } \frac{171 \times £447.\ 12s.\ 6d \times £\overset{11\frac{1}{2}}{\cancel{5}}\frac{\overset{3}{\cancel{3}}}{\cancel{4}}}{100 \times \underset{730}{\cancel{365}}}$$

$$\begin{array}{rrrr}
 & £. & s. & d. \\
\text{Operation}: & 447 & 12 & 6 \\
10s. = £\frac{1}{2}\ \ldots & 171 & & \\
\hline
 & 447 & & \\
 & 3129 & & \\
 & 447 & & \\
2s.\ 6d. = £\frac{1}{4}\ \ldots & 85 & 10 & \\
 & 21 & 7 & 6 \\
\hline
 & 76543 & 17 & 6 \\
 & & 11\frac{1}{2} & \\
\hline
 & 841982 & 12 & 6 \\
 & 38271 & 18 & 9 \\
\hline
73000) & 880254 & 11 & 3(£12.\ 1s.\ 2d. \text{ nearly.} \\
 & 73 & & \\
\hline
 & 150 & & \\
 & 146 & & \\
\hline
 & 4254 & & \\
 & 20 & & \\
\hline
 & 85091 & & \\
 & 73 & & \\
\hline
 & 12091 & & \\
 & 12 & & \\
\hline
 & 145095 & & \\
 & 146000 & & \\
\hline
\end{array}$$

$\therefore$ present worth $= £447.\ 12s.\ 6d. - £12.\ 1s.\ 2d. = £435.\ 11s.\ 4d.$

By true discount:

$\because 102\frac{1013}{1460}$ are worth now 100,

$\therefore £447.\ 12s.\ 6d.$ are worth now $\dfrac{£447.\ 12s.\ 6d. \times 100}{102\frac{1013}{1460}} = £435.$ 17s. 8d. nearly.

```
                              £.  s. d.
               Operation:   447  12  6
                                    100
                            -----------
   102 1013/1460            44762  10
   1460     10s.=£½            1460
   ----                     -------
   6120                     2685720
   408                     179048
  102                      44762
   1013                       730
  ------                  --------
  149933 )                 65353250(£435. 17s. 8d. nearly
                           599732
                           ------
                            538005
                            449799
                            ------
                             882060
                             749665
                             ------
                             132395
                                 20
                             -------
                             2647900
                             149933
                             -------
                             1148570
                             1049531
                             -------
                               99039
                                  12
                             -------
                             1188468
                             1199464
```

∴ bankers' gain = £435. 17s. 8d. — £435. 11s. 4d. = 6s. 4d.

291. EXERCISES.

1. What is the discount of £690. 3s. 9d., due 9 months hence, at 3 per cent. per annum?

2. Required, the present worth and discount of £1780. 19s. 8d. for $3\frac{3}{4}$ years, at $4\frac{1}{2}$ per cent. per annum.

3. What would a banker gain by discounting a bill of £720. 13s. 8d., drawn on May 18th, at 10 months, and paid on October 30th, at 5 per cent. per annum?

4. If the present worth of £11178 be 10800, what is the discount per cent.?

5. Bought goods to the value of £35. 8s., to be paid 8 months hence. What ready money will pay for them?

6. What ready money will pay a bill of £1000, due 130 days hence, discounting at 6 per cent. per annum ?

292. There are several other instances in which the principle of interest is applied, the examples being computed in the same way, and the rate per cent. is charged as in simple interest.

Commission is an allowance made by one merchant to another, called agent, for buying and selling goods.

Brokerage is a small allowance paid to brokers, for negociating bills, or transacting other money concerns.

Insurance is a security given by a person to another party, to insure the value of property against loss by fire, or other accidents. The rate of insurance is generally 2s. 6d., or $\frac{1}{8}$ per cent. per annum, for houses ; but the percentage is regulated by the nature of the property insured, and the risk to which it is exposed. The deed of agreement in insurance transactions is called a *policy of insurance*, and the annual payment is the *premium*.

Sums of money are also insured on persons' lives, so that an individual paying annually a premium during his life, has a sum insured to be paid to his family at his decease. The percentage is regulated by the age and the health of the person whose life is insured, at the time of the contract.

Insurance companies publish tables, in which are laid down the percentage allowed, according to the kind of property, the dangers to which it is exposed, &c.

Statistics are a class of useful facts relating to the population of countries, the number of births, marriages, deaths, the sanitary condition, the revenue, wealth, and resources, and various other particulars, which enlightened governments find necessary to collect, in order that, by comparison with previous statistics, the increased or decreased prosperity of nations may be traced. The principle of these *statistical calculations* is the same as that laid down in interest.

Ex. 1. What is the commission on £569. 14s. 9d., at $7\frac{1}{2}$ per cent. ?

Here we have : on 100 the commission is $7\frac{1}{2}$,

$\therefore$ on £569. 14s. 9d. the commission is $\dfrac{£569.\ 14s.\ 9d. \times 7\frac{1}{2}}{100}$ or £42. 14s. $7\frac{1}{4}$d.

Ex. 2. What is the brokerage of £879. 18s., at $\frac{5}{8}$ per cent. ?

$\because$ on 100 the brokerage is $\frac{5}{8}$,

$\therefore$ on £879. 18s. the brokerage is $\frac{\text{£}879.\ 18\text{s.} \times 3}{100 \times 8} = \text{£}3.\ 5\text{s.}\ 11\tfrac{3}{4}\text{d.}$

Ex. 3. What is the insurance of an East India ship and cargo, valued at £35727. 17s. 6d., at $17\frac{7}{8}$ per cent.?

$\because$ on 100 the insurance is $17\frac{7}{8}$,

$\therefore$ on £35727. 17s. 6d. the insurance is $\frac{\text{£}35727.\ 17\text{s.}\ 6\text{d.} \times 17\frac{7}{8}}{100}$ $= \text{£}6386.\ 7\text{s.}\ 1\tfrac{3}{4}\text{d.}$

Ex. 4. At $5\frac{1}{2}$ guineas per cent., how much must be insured on goods worth £813. 11s. 3d., so that, in case of loss, the owner may receive the value of the goods and of the premium?

If £100 be insured at $5\frac{1}{2}$ guineas per cent., the owner would receive only £100—$5\frac{1}{2}$ guineas, or £94. 4s. 6d. Thus, £100 cover goods to the value of £94. 4s. 6d., and the £5. 15s. 6d. per cent. which is paid for the insurance.

$\because$ to receive £94. 4s. 6d., you must insure £100,

$\therefore$ to receive £813. 11s. 3d. you must insure $\frac{\text{£}813.\ 11\text{s.}\ 3\text{d.} \times 100}{94\frac{9}{40}}$ $= \text{£}863.\ 8\text{s.}\ 6\text{d.}$

Ex. 5. What is the premium on a policy of insurance for £3548. 12s. 6d. upon the life of a person aged 40, at the rate of £3. 8s. per cent. for that age?

$\because$ £100 is insured for £3. 8s.,

$\therefore$ £3548. 12s. 6d. is insured for $\frac{\text{£}3548.\ 12\text{s.}\ 6\text{d.} \times 3\frac{2}{5}}{100} =$ £120. 13s. $0\frac{3}{4}$d.

Ex. 6. In 1833, the population of England and Wales was estimated at 15 millions, and the total number of children at school was 276947. What part of the population was at school, and also how much per cent.?

$\because$ in 15000000 there are 1276947 at school,

$\therefore$ in 1 there is $\frac{1276947}{15000000}$, or .08513, or $\frac{17}{200}$ nearly.

$\therefore$ in 100 there are $100 \times .08513$, or 8.5 nearly.

293. EXERCISES.

1. Find the commission on £816. 7s. 2d., at 9s. 9d. per cent.

2. If a broker sells to the value of £1400, what will be the amount of his brokerage, at $\frac{1}{8}$ per cent.?

3. What would be the expense of insuring £18000, at 10s. 6d. per cent., policy 2s. 6d., and duty 3s. per cent., on the value insured?

4. What would be the expense of insuring property worth £5200, at $1\frac{3}{4}$ per cent.?

5. What premium must be paid by a person, 34 years of age, on a policy of £2000, at the rate of £2. 16s. per cent.?

6. For what sum should a cargo, worth £10526 be insured, at $8\frac{3}{5}$ per cent., so that the owner may receive, in case of loss, the value of both cargo and premium?

7. In an hospital, there were admitted, in 1 year, 2340 patients, of whom 360 died, and the remainder were dismissed cured. How many per cent. died?

8. An agent received £240 commission for goods sold. The value of the goods is required, commission being at $6\frac{1}{4}$ per cent.

9. If a broker sells goods to the amount of 1000 guineas, what is his charge, at $\frac{1}{8}$ per cent.?

10. At £3. 4s. 8d. per cent., what will be the cost of insuring goods worth £2640, so that, in case of loss, the owner may be entitled to the value of the goods and the premium?

11. A cargo, worth £60000 is insured at 6 per cent.; during the voyage it is damaged to the value of £1250. What remains for the Insurance Office?

12. Some goods, valued at £264, are insured at $1\frac{1}{2}$ per cent.; whilst being conveyed to their destination they suffer damage to the amount of £66. How much does the Insurance Company lose?

13. The population of a town is 164246 in 1852. What will it be in 1853, if the increase is at the rate of $1\frac{1}{70}$ per cent.?

STOCKS.

294. It sometimes happens that the exigencies of a state demand a larger supply of money than can be obtained by the ordinary process of taxation, and on such occasions the government finds it necessary to contract a loan, and this is effected by giving to the lender a *bond*, acknowledging the debt, and agreeing to pay a certain rate of interest for the money, whilst it reserves to itself the option of the time of paying off the principal.

These bonds are transferable; their amount is called *Stock*, and it is usual to call each bond £100 stock.

Suppose a person is willing to lend £100 to government, he applies to a respectable stock-broker, (an agent who transacts the sales and purchases of stock) who makes the purchases, charging $\frac{1}{8}$ per cent. for the commission. The broker receives a receipt or bond, which he will hand to the person, who becomes *fund-holder*, or owner of government stock, by which he is entitled to a *dividend*, or interest, every half-year, until he chooses to withdraw his money from the hands of government, which may be done by employing the stock-broker to sell the £100 stock, allowing him, as before, $\frac{1}{8}$ per cent. for the commission.

There are many circumstances which are continually altering the price of stock, such as money being scarce or plentiful; political changes at home and abroad, bringing prospect of war or increasing confidence; commercial dulness or commercial prosperity, &c., but the nature of the security remains unimpaired.

Notwithstanding this continual fluctuation, (which may occur even three or four times in one day, according to the news which reach the Stock Exchange,) it is customary to offer shares of nominally £100, for £80, £83, £90 each.

Thus, to buy in the 3½ per cent. at £80, means that the owner of every £100 stock, paid at the time of buying £80 sterling for it, and he gets 3½ per cent. interest. If, when buying, money is scarce, the price of stock is low; on the contrary, if it be plentiful, the price of stock will be high. Now, since during this fluctuation the rate remains the same, it follows that the lower the stock is in value, the larger interest the buyer obtains, and the higher the stock is, the less interest he gets for his money.

The stock is said to be at *par* when the price of £100 stock is £100 sterling; it is at *a premium* if the price be more than £100 sterling, and at a discount when below.

There are various government stocks: the 3 per cent. Consols, which is an abbreviation of consolidated annuities, or of several annuities consolidated together; the 3 per cent. Reduced Annuities, the Bank Annuities, &c.

The money borrowed by government from those who hold bonds constitutes the *National Debt*, and the source from which the interest or dividend is paid is called the *Public Funds;*

but that portion of the public securities, whose dividend is paid out of the Public Funds, is specially termed the *Funded Debt*, and whenever temporary loans have been raised by government by the issue of securities called Exchequer Bills, they constitute the *Unfunded Debt*.

When a person becomes proprietor of stock, the transaction is recorded in the account books of that stock, kept by government, in the Bank of England.

295. The same view is taken of the capital of joint-stock associations; the money is raised by *shares*, among the partners. The shares are fixed at a certain sum: £10 each, £50 each, or any other sum. Now, should the undertaking prove lucrative, and yield a percentage larger than was expected, the shares find a ready market, and rise in price: a £50 share may bring £64, in which case the shares are said to be £14 above par, &c.

Ex. 1. What is the purchase of £816. 15s. 3d., in the 3 per cent. Annuities, at $74\frac{1}{2}$ per cent.?

$\because$ £100 stock cost £$74\frac{1}{2}$ sterling,

$\therefore$ £816. 15s. 3d. cost $\frac{£816.\ 15s.\ 3d. \times £74\frac{1}{2}}{100}$ or £608. 9s. 9d. sterling.

Ex. 2. What stock can be bought for £2418, in the 3 per cent., at 93?

$\because$ £93 sterling buy £100 stock,

$\therefore$ £2418 sterling buy $\frac{2418 \times £100}{93}$ or £2600 stock.

Ex. 3. How much per cent. will be obtained by purchasing $3\frac{1}{2}$ per cent. stock, at 92?

$\because$ £92 sterling gain £$3\frac{1}{2}$,

$\therefore$ £100 sterling gain $\frac{100 \times £3\frac{1}{2}}{92}$ or £$3\frac{37}{46}$.

Ex. 4. If £1170 be invested in the $3\frac{1}{2}$ per cent, at 78, what annual income arises?

$\because$ £78 gain £$3\frac{1}{2}$,

$$\therefore £1170 \text{ gain } \frac{\overset{\overset{15}{\cancel{195}}}{\cancel{1170}} \times £3\frac{1}{2}}{\underset{\underset{1}{\cancel{13}}}{\cancel{78}}} = £52.\ 10s.$$

Ex. 5. What sum must be invested in the $3\frac{1}{2}$ per cent. stock, at 91, to produce an income of £72?

$\because$ to receive £$3\frac{1}{2}$ you must invest £91,

$\therefore$ to receive £72 you must invest $\dfrac{72 \times £\cancel{91}^{13}}{\cancel{3\frac{1}{2}}^{\frac{1}{2}}} = £1872.$

Ex. 6. How much stock, in the 3 per cent. Consols, can be bought for £2600, when the price is $97\frac{3}{8}$, and the commission to the stock broker $\frac{1}{8}$ per cent. ?

$\because$ the commission is $\frac{1}{8}$ for £100 stock,

$\therefore$ £$97\frac{3}{8}+\frac{1}{8}$, or £$97\frac{1}{2}$, is the cost of £100 stock;

$\therefore$ £2600 is the cost of $\dfrac{260000}{97\frac{1}{2}}$ or £2666. 13s. 4d.

Ex. 7. How much stock can be purchased by the transfer of £1000 stock from the 3 per cent. at 72 to the 4 per cent. at 90, and what would be the difference of income?

Income on £1000 stock, at 3 per cent. $= 10 \times$ £3, or £30.

And $\because$ 100 is bought by 72,

$\therefore$ 1000 is bought by $10 \times$ £72 $=$ £720 sterling $=$ money invested.

Now, $\because$ £90 buy £100 stock,

$\therefore$ £720 buy $\dfrac{72 \times £100}{9} =$ £800 stock;

And income on £800 stock, at 4 per cent. $= 8 \times$ £4, or £32.

Difference of income £32—£30 $=$ £2 gain per year.

The second income might have been found without determining the second stock.

Thus, $\because$ 90 brings 4 income,

720 brings $8 \times$ £4, or £32 income.

Ex. 8. In which is it the most advantageous to invest, the $3\frac{1}{2}$ per cent. at $96\frac{1}{2}$ or the 3 per cent at $88\frac{1}{2}$?.

1st, $\because$ $96\frac{1}{2}$ gain $3\frac{1}{2}$,

$\therefore$ 100 gain $\dfrac{100 \times 3\frac{1}{2}}{96\frac{1}{2}} = \frac{700}{193} = 3\frac{121}{193}$.

2nd, $\because$ $88\frac{1}{2}$ gain 3,

$\because$ 100 gain $\dfrac{300}{88\frac{1}{2}} = \frac{600}{177} \div \frac{200}{59} = 3\frac{23}{59}$.

Hence it is most advantageous to invest in the $3\frac{1}{2}$ per cent., at $96\frac{1}{2}$.

296. EXERCISES.

1. What is the purchase of £3260 Bank Stock, at 156 per cent. ?

2. What quantity of stock, in the 3 per cent., at $63\frac{1}{4}$, can be purchased for £780?

3. At what price must a purchase be effected in the 3 per cent., to secure $5\frac{1}{2}$ per cent. interest ?

4. What sum must be invested in the $3\frac{1}{2}$ per cent. stock, at $88\frac{1}{2}$, to produce an income of £420 ?

5. If I buy £840 stock in the 3 per cent., at $80\frac{3}{4}$, and pay $\frac{1}{8}$ per cent. brokerage, what does it cost me?

6. What must be paid for the purchase of £12000 stock, at $105\frac{5}{8}$, and the commission $\frac{1}{8}$ per cent.?

7. What would be the difference in the incomes of two persons, one of whom invests £5650 in the $4\frac{1}{2}$ per cent. stock, at $102\frac{1}{8}$, and the other the same sum in the $3\frac{1}{2}$ per cent., at $93\frac{1}{2}$?

8. A person invests £6800 in the $3\frac{1}{2}$ per cent. stock, at $93\frac{5}{8}$, and soon after sells out, the stock having risen $2\frac{1}{4}$ per cent.; in both buying and selling he pays brokerage at $\frac{1}{8}$ per cent. His gain is required.

9. Bought, some £25 railway shares, when they were at £$13\frac{1}{2}$, and sold them again at £$33\frac{1}{8}$, and thus made a profit of £760. How much was invested, and what was the gain per cent. ?

10. The £100 share in a railway pays a dividend of £5. What must be given per share to receive $7\frac{1}{2}$ per cent. for my money ?

11. Bought £1242 stock, in the $4\frac{1}{2}$ per cent., for £1471. 16s. 8d. What is the price of stock, and what is my income ?

12. What is the value of £1000 India stock, at $115\frac{3}{4}$ per cent. ?

13. What must be the price of $3\frac{1}{2}$ per cent. stock to enable me to receive $4\frac{1}{2}$ per cent. for my money ?

14. Having invested £840 in the 3 per cent. stock, at $85\frac{1}{3}$, I want to know how much I must invest in the 8 per cent., at $151\frac{1}{4}$, to secure an income of £540 ?

15. By selling out of the 3 per cent., at $84\frac{1}{2}$, a person obtains £1784, of which he invests £840 in the $3\frac{1}{2}$ per cent., at $86\frac{1}{4}$, and the remainder in the $7\frac{1}{2}$ per cent., at 162. Find the difference in his income.

16. What is the rate per cent. obtained for money in the $8\frac{1}{2}$ per cent., at 124 ?

17. What would be the difference of income made by the transfer of £10000 stock from the $3\frac{1}{2}$ per cent., at 80, to the $4\frac{1}{2}$, at 98?

18. In which is it the most advantageous to invest money, the $5\frac{1}{2}$ per cent. stock, at $121\frac{1}{4}$, or the 3 per cent., at 84?

19. When the 3 per cent. stock is at 94, what is the corresponding rate in the 5 per cent., and what is the price of £3000 stock of each?

20. If the $3\frac{1}{2}$ per cent. stock falls $2\frac{1}{2}$, what must be the corresponding fall in the 3 per cent.?

ANNUITIES.

297. When a sum of money is to be paid yearly for a certain number of years it is called an *annuity.*

There are two kinds of annuities, those that are for a given number of years, called *annuities certain*, and those that are to be paid so long as one or more individuals shall live, and are, therefore, called *contingent* or *life annuities.*

Questions relating to annuities involve some principles of interest, but they require the assistance of algebraical formulæ, which it would be out of place to explain here.

But, nevertheless, we shall give a few instances, which can be solved without a direct reference to these formulæ.

Ex. 1. What is the amount of an annuity of £60, to continue 5 years, allowing 5 per cent. compound interest?

Suppose the last payment to be £1, the 4th would be £1.05, that is, £1 + its interest for 1 year; then the 3rd would be $(£1.05)\times(1.05)$ or $(1.05)^2$; the 2nd, $(£1.05)\times(1.05)\times(1.05)$ or $(£1.05)^3$; and the 1st, $(£1.05)^4$. Therefore, the amount of an annuity of £1, for 5 years, is $£1+£1.05+(£1.05)^2+(£1.05)^3+(£1.05)^4=£5.52563125$.

And hence the amount of an annuity of £60 is $60\times$ £5.52563125, or £331. 10s. 9d.

Ex. 2. What is the present worth of an annuity of £40, to continue 5 years, compound interest, at 6 per cent. per annum?

$\because$ £1.06 in 1 year is at present worth £1,

$\therefore$ £40 in 1 year are at present worth $\dfrac{£40}{1.06}=£37.735849$.

$\therefore$ £40 in 2 years are at present worth $\frac{£40}{(1.06)^2} = \frac{£40}{1.1216} =$ £35.599857.

$\therefore$ £40 in 3 years are at present worth $\frac{£40}{(1.06)^3} = \frac{£40}{1.191016} =$ £33.584771.

$\therefore$ £40 in 4 years are at present worth $\frac{£40}{(1.06)^4} = \frac{£40}{1.26247696}$ £31.683746.

$\therefore$ £40 in 5 years are at present worth $\frac{£40}{(1.06)^5} = \frac{£40}{1.3382255776}$ $=$ £29.910327.

$\therefore$ £168.51455 or £168. 10s. $3\frac{1}{2}$d. = the present worth of the annuity.

Ex. 3. A freehold estate (or an annuity to continue for ever, or a perpetuity) brings in yearly £75. What would it sell for, allowing the purchaser $5\frac{1}{2}$ per cent., compound interest, for his money?

By the question, we have to find a sum which, if put to interest, will bring the same as the yearly rent.

Then, $\because 5\frac{1}{2}$ are bought by 100,

$\therefore$ £75 are bought by $\frac{£7500}{5\frac{1}{2}}$ or £1365. 12s. $8\frac{3}{4}$d. = value of free hold estate.

Ex. 4. A person will buy a freehold farm, provided that he can get $6\frac{2}{3}$ per cent. for his money. How many years' purchase should he offer?

Since "How many years' purchase" signifies the same as how many years' rent, we must then find in how many years the rent must be paid to make £100.

$\because 6\frac{2}{3}$ are made in 1 year,

$\therefore$ 100 are made in $\frac{100}{6\frac{2}{3}}$ or 15 years' purchase.

Ex. 5. I bought a freehold estate at 16 years' purchase. What rate per cent. have I for my money?

By the last example, the answer is evidently $\frac{100}{16}$, or $6\frac{1}{4}$ per cent.

Ex. 6. What is the present worth of an annuity of £75, to commence 10 years hence and to continue 7 years after, at 6 per cent.?

This is an example of an *annuity in reversion*, or an annuity which does not come into possession till some time has elapsed.

Since the present worth of £75, to continue 7 years, at 6 per cent. per annum, is found to be (by Ex. 1) £418. 13s. 6½d., and this is to be paid in 10 years; by discounting this sum, at compound interest, for 10 years, we find the present value of it to be £238. 11s. 9d. = the present worth of the annuity in reversion.

298. EXERCISES.

1. If an annuity of £100 continues 5 years, what will it amount to, at 6 per cent. compound interest?

2. Required, the present value of an annuity of £100 per annum, to be continued 5 years, interest at 5 per cent.

3. Required, the present worth of a freehold estate of £6000 a year, to continue for ever, the rent being payable half-yearly, interest at 5 per cent. per annum.

4. The annual rent of a freehold estate, which cost £20000, is £1500. In what time will it clear itself?

5. A freehold estate is sold for 18½ years' purchase. What rate of interest is allowed the purchaser?

6. The reversion of a freehold estate of £100, to continue 10 years, but not to commence till the end of 6 years, is to be sold. What is it worth, allowing the purchaser 4 per cent. per annum for his money?

PARTNERSHIP, OR FELLOWSHIP.

299. We often find that two or three persons form an association, in order to carry on some commercial or industrial undertaking, which requires more capital than one person can raise. These associations are called *Partnership* or *Fellowship*. Every *partner* subscribes a certain sum towards the joint-stock, which constitutes his share. After some time, the partners may agree to share the profit or loss among themselves, and it is evident that each partner's portion of the profit or loss will depend upon his share in the concern: if A's share be twice as much as B's, then he will receive twice as much of the profit; if it be half of B's, then his part will be half as much, and so on. The method by which we determine the respective gains or losses of the partners is called Fellowship or Partnership.

Ex. 1. Three merchants, A, B, and C, join in a speculation; A contributes £800; B, £750; and C, £1200. How much is the share of each, in a gain of £1200?

Here the joint-stock is £800+£750+£1200, or £2750.

Then ∵ £2750 gained £1200,

$\therefore$ £1 gained $\frac{£1200}{2750}$ or $\frac{£24}{55}$;

		£.	s.	d.	
$\therefore$ £800 gained $\frac{800 \times £24}{55}$	or	349	1	$9\frac{9}{11}$	= A's share.
$\therefore$ £750 gained $\frac{750 \times £24}{55}$	or	327	5	$5\frac{5}{11}$	= B's share.
$\therefore$ £1200 gained $\frac{£1200 \times 24}{55}$	or	523	12	$8\frac{8}{11}$	= C's share.
		£1200	0	0	= whole gain.

Ex. 2. A, B, and C formed a joint-stock for conducting a business, of which A contributed £3000 for 6 years; B, £4000 for 5 years; and C, £8000 for 9 years; and they gained £33000. What is each man's share of the gain?

A £3000 share for 6 years is the same as $6 \times$ £3000, or £18000 for 1 year.

A £4000 share for 5 years is the same as $5 \times$ £4000, or £20000 for 1 year.

A £8000 share for 9 years is the same as $9 \times$ £8000, or £72000 for 1 year.

The question is now this: A contributed £18000; B, £20000; and C, £72000. What is each man's share of a gain of £3300?

The joint capital is = £18000+£20000+£72000, or £110000.

Then ∵ 110000 produce 33000,

$\therefore$ 1 produces $\frac{33000}{110000}$, or $\frac{3}{10}$;

		£.	
$\therefore$ £18000 produce $\frac{18000 \times £3}{10}$	or	5400	= A's share.
$\therefore$ £20000 produce $\frac{20000 \times £3}{10}$	or	6000	= B's share.
$\therefore$ £72000 produce $\frac{72000 \times £3}{10}$	or	21600	= C's share.
		£33000	= whole gain.

Ex. 3. Divide £36000 among 4 persons, so that the 2nd gets twice as much as the 1st; the 3rd as much as the first two together, and the 4th three times as much as the 3rd.

By the sense of the question, we perceive that if the 1st person's share be taken as 1 part, the 2nd person's is 2 parts;

the 3rd person's share is $1+2$, or 3 parts; and the 4th is 3×3, or 9 parts; therefore, we have to divide £36000 into $1+2+3+9$ parts, or 15 parts.

$\because$ 15 parts = £3600,

$$\therefore\ 1 \text{ part } = \frac{£36000}{15} \ldots\ldots\ldots\ldots\ldots\ldots = \overset{£.}{2400}.$$

$$\therefore\ 2 \text{ parts} = \frac{2\times£36000}{15} = 2\times£2400, \text{ or } \quad 4800.$$

$$\therefore\ 3 \text{ parts} = \frac{3\times£36000}{15} = 3\times£2400, \text{ or } \quad 7200.$$

$$\therefore\ 9 \text{ parts} = \frac{9\times£36000}{15} = 9\times£2400, \text{ or } \underline{21600.}$$

$$£36000$$

Ex. 4. Bought, equal quantities of coffee and sugar for £20; for the coffee I paid 13d. per lb., and for the sugar 11d. How much was bought of each, and what did each cost?

For 13d.+11d., or 24d., or 2s., I buy 2 lbs.

$\therefore$ for £20 I buy $200\times$2lbs., or 400 lbs.,

and as there are equal quantities of each, the coffee weighs 200 lbs., and so does the sugar.

	£.	s.	d.
Now, $\because$ 1 lb. of coffee cost 13d.,			
$\therefore$ 200 lbs. of coffee, at 13d., cost $200\times$13d. =	10	16	8
And $\therefore$ 200 lbs. of sugar at 11d. = $200\times$11d. =	9	3	4
	£20	0	0

Ex. 5. Divide 782 into three parts, which are to one another as the quantities $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$; or in such a manner, that for every $\frac{1}{2}$ share of the 1st, the 2nd gets $\frac{2}{3}$ share, and the 3rd $\frac{3}{4}$ share.

$\because$ for every $\frac{1}{2}$ share of 1st portion, the 2nd has $\frac{2}{3}$, and the 3rd $\frac{3}{4}$;
or „ $\frac{6}{12}$ „ „ $\frac{8}{12}$ „ $\frac{9}{12}$;
or „ 6 „ „ 8 „ 9.

$\therefore$ 782 must be divided into $6+8+9$ shares, or 23 shares.

$\therefore$ 23 shares = 782,
$\therefore$ 1 share = $\frac{782}{23}$, or 34;
$\therefore$ 6 shares = 6×34, or 204;
$\therefore$ 8 shares = 8×35, or 272;
$\therefore$ 9 shares = 9×34, or $\underline{306.}$

782

Ex. 6. Divide £3120 among three partners, so that for every £5 A receives, B gets £4, and also for every £7 A receives, C gets £3.

If A receives £5, B receives £4,
∴ if A „ £1, B „ £$\frac{4}{5}$;
∴ if A „ £7, B „ £$\frac{28}{5}$, or £$5\frac{3}{5}$.

It follows that if A receives £7, B must have £$5\frac{3}{5}$, and C £3;
then if A „ 35, B „ 28, and C 15.

Hence A = 35 parts, B 28, and C 15.

∵ 35 + 28 + 15 parts, or 78 parts = £3120,

$$\therefore 1 \text{ part } = \frac{3120}{78} = £40;$$

	£.
∴ 35 parts = 35 × £40 =	1400;
∴ 28 parts = 28 × £40 =	1120;
∴ 15 parts = 15 × £40 =	600.
	£3120

Ex. 7. Three graziers rented a field for £30. 5s.; X keeps there 50 sheep for $4\frac{1}{2}$ months; Y, 80 for 5 months; and Z, 90 for $6\frac{1}{2}$ months. Find the rent paid by each.

Here we see that 50 sheep grazing $4\frac{1}{2}$ months is the same as $4\frac{1}{2} \times 50$ sheep, or 225 sheep grazing 1 month.

Also, 80 sheep grazing 5 months is the same as 5 × 80 sheep, or 400 sheep grazing 1 month.

And also 90 sheep grazing $6\frac{1}{2}$ months is the same as $6\frac{1}{2} \times 90$ sheep, or 585 sheep grazing 1 month.

∴ 225 + 400 + 585 sheep, or 1210 sheep grazing 1 month cost £30. 5s.

$$\therefore 1 \text{ sheep grazing 1 month cost } \frac{£30.\ 5s.}{1210} = 6d.$$

	£.	s.	d.	
∴ 225 sheep grazing 1 month cost 225 × 6d. =	5	12	6	= X's expenses.
∴ 400 sheep grazing 1 month cost 400 × 6d. =	10	0	0	= Y's expenses.
∴ 585 sheep grazing 1 month cost 585 × 6d. =	14	12	6	= Z's expenses.
	£30	5	0	

300. EXERCISES.

1. A ship's cargo belongs to three merchants, A owns £5600; B, £6000; and C, £6400. The whole cargo was sold for £10800. What is each man's share of it?

2. Four person's speculated: A with £7400 for three months; B, £6700 for four months; C, £3540 for 11 months; and

D, £5000 for 15 months: they gained £1000. What was each man's gain?

3. A ship, worth £2000, was lost; the owners were four persons: N possessed $\frac{1}{2}$; O, $\frac{1}{8}$; P, $\frac{1}{4}$; and Q, $\frac{1}{8}$. She was insured for £2500. What will each receive, and how much will each gain?

4. A commenced business on January 1st, 1852, with a capital of £1000; he took in B as a partner, with a capital of £1600, on the 1st of March following; three months after, they admitted C as a partner, who brought into the firm £2400: having carried on business till the end of the year, they found the gain to be £1846. 15s. What was each man's share of the gain?

5. A person had divided among his three sons £1200, so that A got $\frac{1}{3}$ part; B, $\frac{1}{4}$ part; and C, $\frac{1}{5}$ part. Find the share of each.

6. Three persons remained partners for 2 years: the 1st brought in £4000, which remained the whole time; the 2nd began with £300, and 6 months after put in £300 more; the 3rd began with £200, and 1 year after put in £500 more. The whole gain was £6600. Determine each partner's share of it.

7. Three workmen were employed to perform a job; L worked 6 days of 10 hours each; M, 7 days of 8 hours each; N, 9 days of 6 hours each. They receive for their labour £25. 10s. What is each man's wages?

8. Divide £600 among A, B, C, and D, so that B gets $\frac{1}{3}$ as much as A; C, 4 times as much as B; and D, $\frac{1}{2}$ as much as B. How much will each get?

9. Divide £288 into parts, which are to one another as the numbers 3, 5, 4; or in such a manner, that as often as the first person receives £3, the second shall receive £5, and the third £4. How much will each receive?

10. Four persons divide among themselves £249270, in such a manner that for every £$\frac{3}{4}$ A receives, B has £$\frac{4}{5}$; for every £$\frac{5}{6}$ A receives, C has £$\frac{4}{7}$; and for every £$\frac{7}{8}$ A receives, D has £$\frac{5}{8}$. The share of each man is required.

11. A, B, and C own £800, which are to be divided so that B shall receive £40 more than A, and C £20 more than B. How much is each part?

12. Three persons share £1000: B=2 A—£50, C=2 B+£30. Find the share of each.

13. Divide £4000 among four persons, so that A's share $=\frac{3}{4}$ B +£40, C's=2 B+£70, D's $=\frac{1}{2}$ B—£20. What does each man get?

14. A sum of £91. 10s. was distributed among 15 men, 17 women and 8 children; each woman's share was three times as much as that of a child, and $\frac{13}{21}$ of that of a man. What did each man, each woman, and each child receive?

15. A detachment, consisting of five companies, compose a garrison, in which the duty requires 152 men per day. What number of men must each company furnish, according to their strength; the first company containing 108 men, the second 102 men, the third 96 men, the fourth 78 men, and the fifth 72 men.

16. A general laid a contribution of £140000 on four towns, to be paid according to the number of inhabitants contained in each: the first had 50000, the second 70000, the third 80000, and the fourth 100000 inhabitants. What part must each town pay?

17. In a manufactory, each man receives 16s. 6d. per week, each woman 10s. 6d., and each boy 4s. 6d. The whole wages in a month, of 24 working days, are £1273. 10s., of which the men received £924, and the boys £76. 10s. How many men, women, and boys are working in the manufactory, and what is each person's wages?

18. Three corn factors freighted a ship; the first loaded 360 tons of corn, the second 200 tons, and the third 160 tons; on the voyage, during a storm, the sailors were obliged to throw overboard a part of the cargo, viz., 150 tons of the first factor, 90 of the second, and 30 of the third. How are they to settle accounts, so that the loss be shared according to each factor's load?

PROFIT AND LOSS.

301. In every mercantile transaction, some of the parties engaged may either gain or lose, or neither gain nor lose. Questions in which one of these three alternatives is to be determined, are classed under the head of *Profit and Loss.*

Ex. 1. Bought, 124 yards of cloth, at 15s. 6d., and sold it at 17s. 9d. What did I gain?

∵ 1 yard is bought at 15s. 6d.,

∴ 124 yards are bought for 124×15s. 6d., or £96. 2s.

And ∵ 1 yard is sold for 17s. 9d.,

∴ 124 yards are sold for 124×17s. 9d., or £110. 1s.

Hence the gain = £110. 1s.—£96. 2s. = £13. 19s.

Or thus: the gain per yard is 17s. 9d.—15s. 6d., or 2s. 3d;

∴ the gain on 124 yards is 124×2s. 3d., or £13. 19s. as before.

Ex. 2. If tea be bought at 4s. 6d., and sold at 6s. 2d. per lb, what is the gain per cent.?

∵ on 4s. 6d., or 54d., the gain is 6s. 2d.—4s. 6d., or 1s. 8d., or 20d.,

∴ on 1 the gain is $\frac{20}{54}$.

And ∴ on 100 the gain is $\frac{2000}{54}$, or $37\frac{1}{27}$.

Ex. 3 Bought, a horse for £66, and sold it again for £57. What did I lose per cent.?

Here the loss on £66 is £9,

and the loss on 100 is $\dfrac{\overset{150}{\cancel{900}}}{\underset{11}{\cancel{66}}} = 13\frac{7}{11}$.

Ex. 4. If coffee is bought at 3s. 4d. per lb., what must it be sold at to gain $24\frac{1}{2}$ per cent.?

Here we perceive that 100 are sold for $124\frac{1}{2}$,

∴ 3s. 4d. are sold for $\dfrac{3\frac{1}{3}\text{s.} \times 124\frac{1}{2}}{100}$ or 4s. $1\frac{3}{4}$d. nearly.

Ex. 5. Bought, velvet at 8s. 8d. per yard; but it getting damaged, I am compelled to sell it at a loss of $39\frac{11}{26}$ per cent. What is the selling price per yard?

Here 100 are sold for $100—39\frac{11}{26}$, or $60\frac{15}{26}$,

∴ 8s. 8d. worth, or 1 yard of velvet is sold for $\dfrac{104\text{d.} \times 60\frac{15}{26}}{100}$ or 5s. 3d.

Ex. 6. Having sold a certain quantity of cloth, at 16s. 6d. per yard, I gained 12 per cent. What was the prime cost?

By the question, we see that for 112 the prime cost is 100,

∴ for 16s. 6d. the prime cost is $\dfrac{100 \times 16\text{s. } 6\text{d.}}{112} = 14\text{s. } 8\frac{4}{7}\text{d.}$

Ex. 7. By selling goods at 6s. 10d., I lost $18\frac{3}{4}$ per cent. What did I buy them for?

It is evident that for $81\frac{1}{4}$ the prime cost is 100,

$\therefore$ for 6s. 10d. the prime cost is $\dfrac{100\times 6\text{s. }10\text{d.}}{81\frac{1}{4}}=8\text{s. }4\frac{12}{13}\text{d.}$

Ex. 8. By selling cloth at 14s. per yard I gain 10 per cent. What did I gain per cent. by selling at 17s. per yard?

1st method: Here 110 cost 100,

$\therefore$ 14s. worth, or 1 yard cost $\dfrac{100\times 14\text{s.}}{110}=\frac{140}{11}\text{s.}=12\frac{8}{11}\text{s.}$;

$\therefore$ 17s.—$12\frac{8}{11}$s. $=4\frac{3}{11}$s. $=$ gain on $12\frac{8}{11}$s.

Now, $\because$ $12\frac{8}{11}$s. gain $4\frac{3}{11}$s.,

$\therefore$ 100 gain $\dfrac{100\times 4\frac{3}{11}}{12\frac{8}{11}}=\dfrac{\overset{235}{\cancel{4700}}}{\underset{7}{\cancel{140}}}=33\frac{4}{7}$.

2nd method: When sold for 14s., I get 110 out of 100,

$\therefore$ when sold for 17s. I get $\dfrac{110\times 17}{14}$, or $133\frac{4}{7}$ out of 100.

Hence $133\frac{4}{7}$—$100=33\frac{4}{7}=$ the gain per cent.

Ex. 9. By selling linen at 5s. per yard, I lost 16 per cent. What did I lose by selling it at 4s. 4d.?

1st method: Here 84 cost 100,

$\therefore$ 5s. worth, or 1 yard cost $\dfrac{\overset{125}{\cancel{500}}}{\underset{21}{\cancel{84}}}=5\frac{20}{21}$s.

Hence, $5\frac{20}{21}\text{s.}-4\frac{1}{3}\text{s.}=1\frac{13}{21}\text{s.}=$ loss on $5\frac{20}{21}$s.;

And $\therefore$ $5\frac{20}{21}$ lose $1\frac{13}{21}$;

$\therefore$ 100 lose $\dfrac{100\times 1\frac{13}{21}}{5\frac{20}{21}}=\dfrac{\overset{136}{\cancel{3400}}}{\underset{5}{\cancel{125}}}=27\frac{1}{5}$.

2nd method: When sold for 5s. I get 84 out of 100,

$\therefore$ when sold for $4\frac{1}{3}$s. I get $\dfrac{84\times 4\frac{1}{3}}{5}=72\frac{4}{5}$ out of 100.

Hence, loss per cent. is 100—$72\frac{4}{5}$, or $27\frac{1}{5}$,

Ex. 10. If $10\frac{1}{2}$ per cent. be gained by selling tea at 4s. 6d. per lb., what must it be sold at to gain 36 per cent.?

1st method: $\because$ $110\frac{1}{2}$ cost 100,

$\therefore$ 4s. 6d. worth, or 1 lb. of tea cost $\dfrac{100\times 4\text{s. }6\text{d.}}{110\frac{1}{2}}=4\frac{16}{221}$s.

By the second part of the question, we have:

100 sold for 136,

$\therefore 4\frac{16}{221}$s. worth, or 1 lb. of tea is sold for $\frac{136 \times 4\frac{16}{221}\text{s.}}{100}$ or $5\frac{119}{221}$s., or 5s. $6\frac{1}{4}$d. $\frac{187}{221}$

2nd method: $\because$ if 100 sold for $110\frac{1}{2}$, I get 4s. 6d. per lb,,

$\therefore$ if 100 sold for 136, I get $\frac{136 \times 4\text{s. } 6\text{d.}}{110\frac{1}{2}}$ or 5s. $6\frac{1}{4}$d. $\frac{187}{221}$.

Ex. 11. A person sold a horse for 70 guineas, and lost 15 per cent. What should he have sold it at to gain 20 per cent.?

Here, if 100 sold for 85, I obtain 70 guineas,

$\therefore$ if 100 sold for 120, I obtain $\frac{120 \times \overset{14}{\cancel{70}}}{\underset{17}{\cancel{85}}} = 98\frac{14}{17}$ guineas = £103. 15s. $3\frac{1}{2}$d.

Ex. 12. By selling 514 cwt. of sugar, at 3 per cent. profit, I gained £80. 11s. What was it bought and sold at per cwt.?

$\because$ 3 are gained by 100,

$\therefore$ £80. 11s. are gained by $\frac{100 \times \text{£}80.\ 11\text{s.}}{3}$ or £2685 = whole selling price.

And $\therefore$ 514 cwt. are sold for £2685,

$\because$ 1 cwt. is sold for $\frac{\text{£}2685}{514}$, or £5. 4s. $5\frac{1}{2}$d. $\frac{202}{257}$ = selling price per cwt.

Also 103 are bought for 100,

$\therefore \frac{\text{£}2685}{514}$ are bought for $\frac{\text{£}268500}{514 \times 103}$, or £5. 1s. 5d. $\frac{4793}{26471}$ = prime cost per cwt.

302. EXERCISES.

1. What must linen, which cost 9s. $4\frac{1}{2}$d. per yard, be sold for, to gain 16 per cent.?
2. If paper, which cost £1. 6s. 6d. per ream, be sold at £1. 16s. 4d. per ream, what is the gain per cent.?
3. Goods, bought at £171.25, were sold at £183.475. What is the whole gain, and how much per cent.?
4. A draper bought 78 yards of cloth, at £1. 8s. 9d. per yard. and 147 yards of linen, at 3s. 3d. per yard. The whole was sold for £141. 9s. $0\frac{3}{4}$d. Find the gain, and also the gain per cent.?
5. Bought, 87.9 yards of linen, at 6.4s. per yard; also, 698 yards, at 4.57s. per yard; and also, 78 yards, at 6.9s. per yard. The selling of the whole brought a profit of $3\frac{1}{4}$ per cent. The selling price of the whole is required.

6. Bought, goods at £1. 17s. 6d., and sold them at £1. 5s. 4d. Find the loss per cent.?

7. What must goods, which cost £6. 10s. 6d. be sold at to gain 18½ per cent.?

8. Find the selling price of goods, bought at £1. 6s. 7½d., and sold so as to lose 10 per cent.

9. What is the prime cost of a cricket ball, which, when sold at 8s. 6d., brings a gain of 35 per cent.?

10. If I gain 20 per cent. by selling tea at 6s. 8d., how much is gained per cent. by selling it at 7s. 6d.?

11. Bought, 56 pieces of stuff, at £6 per piece, and sold 20 pieces at £9, and 16 at £7. 10s. each. At what rate per piece must the remainder be sold, to gain 40 per cent. on the whole?

12. A grocer bought rice at £1. 15s. per cwt., and vermicelli at £1. 17s. 6d.; he sold the rice at 9½d. per lb., and the vermicelli at 11¼d. per lb. Find the difference of the gains, also the profit per cent. on each.

13. The produce of some land is £99. 10s., which is an increase of 8 per cent. on the preceding year. What was the value of the produce then?

14. By selling goods at 1s. 10d. per lb., I gain 12½ per cent. What is the cost price of 1 cwt.?

15. How much must an article, bought at £12. 16s. 6d. per cwt., be sold for to gain 24 per cent., and what quantity of it must be sold at that rate to make a profit of 100 guineas?

16. If 15 cwt. 1 qr. 2 lbs. of sugar cost £51. 19s. 8d., what must it be sold at per lb. to gain 15 per cent.?

17. If the brewing of 8741 gallons of ale cost £493. 10s., what must it be sold at per gallon to clear 125 per cent.?

18. If £27. 6s. be lost by the sale of 2 cwt. of raw silk, at £1. 18s. per lb., what was the prime cost, and what the loss per cent.?

19. I gained 25¾ per cent. by selling goods at 10d. What was the prime cost?

20. By selling some articles at 16s. 6d. I gained 25 per cent. What did I gain per cent. when selling the same articles at 18s. 4d.?

21. When tea is sold at 6s. 4d. I clear 12 per cent. What quantity did I sell when my gain was £42. 12s. 8d.?

22. Bought, sugar at 7¼d. per lb. What must I sell it at to gain 54⅝ per cent.?

23. How much per cent. is 4¼d. per shilling?

24. Cloth, which was bought at 14s. 9d. per yard is sold at 8d. per yard less. What is the loss per cent.?

25. A person bought 120 oranges at 2 a penny, and double that quantity at 3 a penny, and he sold them again at 5 for 2d. How much did he lose or gain, and how much per cent.?

26. Sold, 15 horses for £675, whereby I cleared as much as I sold 2 horses for. What is my gain per cent.?

27. A grocer had 2 cwt. of tea, of which he sold ¼ at 6s. 6d. per lb., and found that he was gaining 10½ per cent. At what rate must he sell the remainder to clear 16 per cent. on the whole?

28. The prime cost of 75 dozens of wine is £202. 10s., and 3 dozens are lost by leakage, &c. What must the remainder be sold at per dozen, so as to gain 12 per cent. on the prime cost?

29. Sold, 5 yards of stuff for £1. 3s. 9d., and gained at the rate of 15 per cent.; but had I sold it at 5s. 4d. per yard, what would be the gain per cent.?

30. A bought 648 yards of stuff, at 7s. 7d. per yard, 68 yards at 11s. 6d., 98 yards at 4½d., 487 yards at 10¼d., 98 yards at 9s. 4d. per yard. The whole was sold at 15 per cent. loss. Find the whole selling price, and also the selling price of each per yard.

EQUATION OF PAYMENTS.

303. It is sometimes required, both in commercial and banking business, to find the correct time at which two or more payments, or debts, payable at different times, should be discharged in one payment, so that neither debtor nor creditor will suffer loss. The operation is called *Equating the Time*, and questions relating to this subject are known under the appellation of *Equation of Payments*.

Ex. Suppose a banker has two bills, viz., one of £2500, payable in 9 months, and the other of £1600, payable in 16 months. What is the equated time?

Evidently, the time of payment lies between 9 months and 16 months, and that time must be such that the *interest* on £2500, which is paid after its time, should equal the *discount* on £1600, which is paid before its time. The rate being 5 per cent. For instance:

Suppose x to represent the equated time, then the interest of £2500 for $(x-9)$ months must be equal to the discount on £1600 for $(16-x)$ months.

$\therefore$ the interest of £2500 for $(x-9)$ months, at 5 per cent. $= \dfrac{£2500 \times 5(x-9)}{100 \times 12}$.

Also the discount of £1600 for $(16-x)$ months, at 5 per cent.

$$= \frac{£1600 \times \frac{5(16-x)}{12}}{100 + \frac{5(16-x)}{12}}.$$

Hence it follows that $\dfrac{£2500 \times 5\ (x-9)}{12} = \dfrac{£1600 \times \frac{5\ (16-x)}{12}}{100 + \frac{5\ (16-x)}{12}}$

From which quadratic equation x may be determined.

304. This method, though correct, is not the one adopted in practice, probably on account of its being complicated, and it becomes more so as the number of payments increase. In the method used, it is supposed that the sum of the interest of each debt for its respective time is equal to the interest of the sum of the debts for the equated time.

Thus, interest on £2500, for 9 months, at 5 per cent., is $\dfrac{5 \times 9 \times £2500}{12 \times 100}$.

Also, interest on £1600, for 16 months, at 5 per cent,, is $\dfrac{5 \times 16 \times £1600}{12 \times 100}$.

And interest on £4100, for equated time, at 5 per cent., is $\dfrac{5 \times 4100 \times \text{equated time}}{12 \times 100}$.

$$\therefore \frac{5 \times 9 \times 2500}{12 \times 100} + \frac{5 \times 16 \times 1600}{12 \times 100} = \frac{5 \times 4100 \times \text{equated time}}{12 \times 100}.$$

As every term contains the common factor $\frac{5}{12 \times 100}$, it may be omitted, and we have $9 \times 2500 + 16 \times 1600 = 4100 \times$ equated time.

Hence we infer that the sum of the two products, obtained by multiplying the value of each bill by the number of months that have to elapse before they are due, is equal to the product of the sum of the debts multiplied by the number of months of the equated time.

And since equated time $\times 4100 = 9 \times 2500 + 16 \times 1600 = 48100$,

$\therefore$ equated time $= \frac{48100}{4100} = \frac{481}{41} = 11\frac{30}{41}$ months.

The difference of the results, as found by this method and the true one is but small: the difference is in favour of the payer, because he reckons on his side the interest instead of the discount of those debts which he pays before they are due, whilst the other side of the operation is quite correct.

305. EXERCISES.

1. A owes B £1000, to be paid yearly, in five equal payments. What is the equated time to pay the whole?

By what has been said in the preceding article, we have:

$$\begin{array}{rr} & \text{yrs.} \quad \text{yrs.} \\ & 200 \times 1 = \ \ 200 \\ & 200 \times 2 = \ \ 400 \\ & 200 \times 3 = \ \ 600 \\ & 200 \times 4 = \ \ 800 \\ & 200 \times 5 = 1000 \\ \hline 1000) & 3000(3 \text{ years} \\ & 3000 \end{array}$$

2. A debt of £1600 is to be liquidated by three payments, as follows: £600 in 3 months, £700 in 7 months, and £300 in 10 months. When must the whole be paid together?

3. B owes £200, payable in 3 months; £200, payable in 4 months; and £200, payable in 5 months. In what time may the whole be paid, without loss to either party?

4. If a debt of £120 is due in 1 month, of £240 is due in 5 months, of £330 is due in 7 months, and of £480 in 8 months, the equated time for the payment of the four debts is required.

5. If $\frac{1}{4}$ of a debt is to be paid at present, $\frac{1}{3}$ in 4 months, $\frac{1}{5}$ in 9 months, and the remainder in 12 months, find the equated time for the payment of the whole.

6. Find the equated time of three debts, the first of £139. 10s., due at the end of 9 months; the second, of £235, due at 15 months' end; and the third, of £650, due in $1\frac{1}{2}$ years.

7. £200 are to be paid in monthly instalments of £20. Find the time when the whole may be paid in one sum.

8. Find the equated time of payment, when $\frac{1}{4}$ of a sum of money is due in 5 months, $\frac{1}{8}$ in 9 months, and the remainder in $1\frac{3}{4}$ years?

9. £541. 16s. are due now, and £473. 5s. are to be paid in 5 equal monthly instalments. Find the equated time for the whole.

10. £360 are to be paid as follows: £80 in 80 days, £100 in 100 days, £120 in 120 days, and the rest in 1 year. Find the equated time for paying the whole.

BARTER.

306. It frequently happens that one person buys goods of another, but instead of paying money for them, gives other goods in return. This kind of transaction is called *Barter.*

Ex. 1. Suppose tea, at 4s. 6d. per lb., is given for 90 yards of linen, at 3s. per yard, what quantity of tea must be given?

$\because$ 1 yard is bought for 3s.,

$\therefore$ 90 yards are bought for 90×3s.$=270$s.

Now, $\because$ 4s. 6d. buy 1 lb. of tea,

$\therefore$ 6d. will buy $\frac{1}{9}$ lb.;

$\therefore$ 270s., or 540 sixpences, buy $540 \times \frac{1}{9}$ lb., or 60 lbs. of tea.

Ex. 2. Bought, 12 quarters of wheat, at £2. 16s. per quarter, for which I paid in money £13. 12s., and the remainder in stuff, at 5s. per yard. How many yards had I to give?

$\because$ 1 quarter is bought for £2. 16s.,

$\therefore$ 12 quarters are bought for $12 \times$ £2. 16s., or £33. 12s.,

And the value paid in stuff is £33. 12s.—£13. 12s., or £20.

Now, 5s. buy 1 yard of stuff,

$\therefore$ £20 buy 80×1 yard, or 80 yards.

Ex. 3. A gave $96\frac{3}{4}$ yards of cloth, at 12s. 6d. per yard, for $336\frac{3}{8}$ yards of calico. The value of the calico per yard is required.

$\because$ 1 yard of cloth cost 12s. 6d.,

$\therefore$ 96¾ yards cost 96¾ × 12s. 6d., or 1209s. 4½d.

Also, 336⅝ yards of calico cost 1209s. 4½d.,

$\therefore$ 1 yard of calico cost $\frac{1209\text{s. } 4\frac{1}{2}\text{d.}}{336\frac{5}{8}}$, or $\frac{9675\text{s.}}{2693}$, or 3s. 7d. nearly.

Ex. 4. X has silk worth 16s. per yard, and Y has cloth worth 12s. per yard, which he barters at 14s. 6d. per yard. How much must the silk be raised to be equivalent to the rise of the cloth?

$\because$ 12s. are bartered for 14s. 6d.,

$\therefore$ 1s. is bartered for $\frac{14\text{s. } 6\text{d.}}{12}$.

And $\therefore$ 16s. are bartered for $\frac{16 \times 14\text{s. } 6\text{d.}}{12} = \frac{58}{3}\text{s.} = 19\text{s. } 4\text{d.}$

307. EXERCISES.

1. How many yards of cloth, at 15s. per yard, must be given in exchange for 450 yards of linen, at 4s. 6d. per yard?

2. Having bought 24¼ cwt. of sugar, at £3. 16s. 6d. per cwt., and paid in cash £22. 10s., how much coffee, at £4. 13s. 4d. per cwt., is wanted to make up the difference?

3. Bartered, 240 gallons of wine, at £2. 13s. per gallon, for 200 barrels of ale. What was the price of the ale per barrel?

4. How much must be charged for beef, which is sold for ready money at 10s. 6d. per stone, for mutton at 9s. 4d., which is bartered at 11s 3d. per stone?

5. Bartered 4½ cwt. of snuff, at £4. 7s. 6d. per cwt., and got for it eggs at 9d. per dozen, and butter at 1s. 4d. per lb.; and received 4 lbs. of butter to each dozen of eggs. How much did I get of each?

6. Cows, worth £14 each are exchanged for horses, worth £25 each, ready money; but in bartering, at £28. 15s., what ought to be charged for each cow?

7. Received 280 lbs. of tea, at 4s. 4d. per lb., for 360 yards of linen. What was it worth per yard?

8. D has 120 cwt of tallow at 18s. 2d. per cwt., E has 35 cwt. 2 qrs. 18 lbs. of sugar at 5¾d. per lb., and they barter their goods. What balance must D receive with the sugar?

9. How many lbs. of coffee, at 1s. per lb., must be given for 5 cwt. 2 qrs. 14 lbs. at 6d. per lb.?

10. Bartered, 82 cwt. of hops, at 30s. per cwt., for which I received £20 in money, and the rest in prunes at 5d. per lb. What quantity of prunes did I receive?

11. A grocer, with whom I bartered tea for sugar, raised his sugar from 1s. to 1s. $1\frac{1}{2}$d. per lb. What ought I to have charged for the tea, which I sold for ready money at 5s. 6d. per lb.?

12. A farmer bought of a draper 24 yards of cloth, at 9s. 4d. per yard; 30 yards of linen, at 3s. 9d. per yard; and 80 yards of printed calico, at 1s. 2d. per yard; for which he gave £13. 2s. 6d. in money and 32 cwt. of potatoes. Required, the value of the potatoes per cwt.

13. O has 1600 lbs. of coffee, at 1s. 5d. per lb., which he barters with P, for goods at 5d. and 8d. per lb., and to have $\frac{2}{5}$ money, and of each sort of goods an equal quantity. How much money, and how many lbs. of each kind of goods will he receive?

14. A merchant consigns 64 bales of cloth, at £86. 2s. 6d, per bale, to his agent, who, retaining $1\frac{1}{4}$ per cent. commission, sends him, in return, one-half of the value in sugar, at 44s. per cwt., and the other in rum, at 4s. 8d. per gallon. What quantity of each did he receive?

15. Bartered, 2400 dozen pairs of stockings, at £1. 10s. per dozen, for equal quantities of coal, at 8s., 10s. 6d. and 12s. 9d. per ton respectively, and received also £120 in money. How many tons of each kind of coal had I?

16. How much does cloth cost per yard, if I receive $55\frac{1}{4}$ yards for 10 cwt. of sugar, at £3. 9s. $0\frac{3}{4}$d. per cwt.?

17. A wants to exchange some land, at £22. 10s. per acre, for 16 acres of other land, worth £175 per acre. How many acres of the first land must be given?

TARE AND TRET.

308. When goods are weighed, together with the box, bag, barrel, &c., which contains them, the whole weight is called *Gross Weight*,

The allowance which the seller makes to the buyer for the weight of the box, bag, barrel, &c., which contains the goods bought, is called its *Tare*.

What remains after the tare is deducted from the gross weight is called *Tare Suttle*.

Tret is an allowance of 4lbs. on 104lbs. tare suttle, for waste, dust, &c.

After the deduction for tare and tret, what remains is called *tret suttle*.

Cloff is an allowance of 2lbs. in every 3 cwt. of tret suttle, given to retailers in order to turn the scales.

Net Weight is what remains after the allowances are deducted.

Questions treating of these allowances are classed under the head of *Tare and Tret*.

Ex. 1. What is the net weight of 16 barrels of rice, each containing 3 cwt. 2 qrs. 21lbs., tare 20lbs. per cwt., tret 4lbs. per 104 lbs., and cloff 2lbs. for every 3 cwt.

$\because$ The gross weight of 1 barrel is 3 cwt. 2 qrs. 21 lbs.,

$\therefore$ the gross weight of 16 barrels $= 16 \times 3$ cwt. 2 qrs. 21 lbs., or 59 cwt.

And $\because$ tare on 1 cwt. $= 20$ lbs.,

$\therefore$ tare on 59 cwt. $= 59 \times 20$ lbs., or 1180 lbs., or 10 cwt. 2 qrs. 4 lbs.

$\therefore$ tare suttle $= 59$ cwt.—10 cwt. 2 qrs. 4 lbs., or 48 cwt. 1 qr. 24 lbs.

Also, $\because$ the tret on 104 lbs. is 4 lbs.,

$\therefore$ the tret on 1 lb. is $\frac{4}{104}$, or $\frac{1}{26}$;

$\therefore$ the tret on 48 cwt. 1 qr. 24 lbs. is $\frac{48 \text{ cwt. } 1 \text{ qr. } 24 \text{ lbs.}}{26}$, or 1 cwt. 3 qrs. 12.77 lbs.

$\therefore$ tret suttle $= 48$ cwt. 1 qr. 24 lbs.—1 cwt. 3 qrs. 12.77 lbs. $= 46$ cwt. 2 qrs. 11.22 lbs.

And $\because$ the cloff on 3 cwt., or on 336 lbs. is 2 lbs..

$\therefore$ the cloff on 1 cwt. $= \frac{2}{336}$, or $\frac{1}{168}$;

$\therefore$ the cloff on 46 cwt. 2 qrs. 11.22 lbs. is $\frac{46 \text{ cwt. } 2 \text{ qrs. } 11.2\dot{2} \text{ lbs.}}{168}$, or 1 qr. 3.06 lbs.

$\therefore$ the net weight = 46 cwt. 2 qrs. 11.22 lbs.—1 qr. 3.06 lbs., or 46 cwt. 1 qr. 8.15 lbs.

The mechanical part of this work is exhibited in the following scheme;—

	cwt.	qrs.	lbs.	
	3	2	21	
			16	
16 lbs. = $\frac{1}{7}$ ...	59	0	0	gross weight
4 lbs. = $\frac{1}{4}$ ...	8	1	20	
	2	0	12	
	10	2	4	= tare
26)	48	1	24	= tare suttle
	1	3	12.77	= tret
168)	46	2	11.22	
		1	3.06	= cloff
	46	1	8.15	= net weight

Another method of obtaining the same result would be to find the net weight of 1 barrel, and multiply by the number of barrels.

Ex. 2, How much will 29 packs of wool cost, each weighing 3 cwt. 2 qrs. 25 lbs., tare 16 lbs. per cwt., tret and cloff as usual, at 3s. 9d. per lb. net?

$\because$ tare on 1 cwt. is 16 lbs., or $\frac{1}{7}$,

$\therefore$ tare on 1 pack, or 3 cwt. 2 qrs. 25 lbs. $= \dfrac{\text{3 cwt. 2 qrs. 25 lbs.}}{7}$ or 2 qrs. 3.571 lbs.

$\therefore$ tare suttle = 3 cwt. 2 qrs. 25 lbs.—2 qrs. 3.571 lbs. = 3 cwt. 0 qr. 21.428 lbs.

Tret is $\frac{1}{26}$ of the tare suttle, or $\dfrac{\text{3 cwt. 0 qr. 21.428 lbs.}}{26} =$ 13.747 lbs.

$\therefore$ Tret suttle = 3 cwt. 0 qr. 21.428 lbs.—13.747 lbs. = 3 cwt. 0 qr. 7.681 lbs.

Cloff is $\frac{1}{168}$ of 3 cwt. 0 qr. 7.681 lbs., or $\dfrac{\text{3 cwt. 0 qr. 7.681 lbs.}}{168}$ = 2.045 lbs.

$\therefore$ net weight per pack = 3 cwt. 0 qr. 7.681 lbs.—2.045 lbs. = 3 cwt. 0 qr. 5.635 lbs.

$\therefore$ net weight of 29 packs = 29 × 3 cwt. 0 qr. 5.635 lbs., or 88 cwt. 1 qr. 23.434 lbs.

The process may be thus exhibited :—

	cwt.	qrs.	lbs.	
7)	3	2	25	
		2	3.5714	= tare
26)	3	0	21.4285	= tare suttle
			13.7472	= tret
168)	3	0	7.6813	= tret suttle
			2.0457	= cloff
29 ×	3	0	5.6356	= 88 cwt. 1 qr. 23.434 lbs. = net weight.

Now, ∵ 1 lb. cost 3s. 9d.,

∴ 23.434 lbs. cost 23.434 × 3s. 9d. = 87.8775s.

Also, 1 qr. = 28 × 3s. 9d. .... = 105s.

And 88 cwt. = 88 × 112 × 3s. 9d. = 36960s.

37152.8775s.

Therefore, 29 packs of wool cost £1857. 12s. 10.530d.

Or ∵ 1 lb. cost 3s. 9d.,

∴ 88 cwt. 1 qr. 23.434 lbs., or 9907.434 lbs, cost 9907.434 × 3s. 9d. = £1857. 12s. 10.53d.

309. EXERCISES.

1. What is the net weight of 144 cwt. 2 qrs. 10 lbs. gross, tare 16 lbs. per cwt. ?
2. What is the net weight of 36 bags, each 4 cwt. 1 qr. 14 lbs. gross, tare 18 lbs. per cwt., and tret as usual ?
3. In 48 hhds. or sugar, each 10 cwt. 2 qrs. 20 lbs., how much net weight, allowing 16 lbs. per cwt., tare, tret, and cloff as usual ?
4. Bought, 3 packs of wool, No. 1 weighing 3 cwt. 3 qrs. 21 lbs.; No. 2, 4 cwt. 0 qr. 8 lbs.; No. 3, 3 cwt. 3 qrs. 14 lbs.; tare, 30 lbs. per pack; tret 8 lbs. for every 20 stones. What will the whole amount to, at 10s. 8d. per stone.
5. At the rate of 8 lbs. per cwt. the tare of some goods is 119 lbs. Required the net weight.
6. The net weight of some goods is 13 cwt. 3 qrs. 7 lbs., and the tare is 8 lbs. per cwt. Find the gross weight.
7. The tare on some goods comes to 119 lbs. at the rate of 8lbs. per cwt. What was the gross weight?
8. The tare of 14 cwt. 3 qrs. 14 lbs. is 119 lbs. What is it per cwt. ?

9. When the tare is at 8 lbs. per cwt., and subtracted from the gross weight, the net weight is 13 cwt. 3 qrs. 7 lbs. Find the tare.

10. The gross weight of 4 bags of pepper, bought at 1s. 6d. per lb. net, is 14 cwt. 3 qrs. 12 lbs., tare 14 lbs. per bag. What is the cost?

11. A buys 11 cwt. 1 qr. 20 lbs. gross, of sugar, at the rate of 8d. per lb., and pays £36. 8s. 2d. How much was allowed for the tare, and what was it per cent.?

12. What must be paid for six bags of rice, weighing 15 cwt. 1 qr. 4 lbs. gross, if 3 qrs. 16 lbs. net weight cost £2. 18s. 6d., tare $\frac{1}{8}$ of the gross?

EXCHANGE.

310. When it is required to find how much of the money of one city or country is equivalent to a given sum of money of another, the term applied to such a conversion is *Exchange.*

In every country, the mint fixes a precise weight and purity for its currency; for instance, in England and France the mint regulations, value £1 to be equal to 25 francs, 20 centimes, and this *is* said to be the *par of exchange* between the two countries. Therefore, when transactions between London and Paris are conducted on this footing, the exchange is at *par.* But it happens continually that £1 in London buys more or less than 25.20 francs in Paris; in the first case, the exchange is in favour of London, and in the second, against, Now, the sum of money of one country, given at any particular time, in exchange for a fixed sum of money of another, is called the *course of exchange.*

The circumstances which produce variation in the course of exchange depend on the state of trade, and the value of the coins of any country, which seldom correspond to their mint standard.

It happens that in cities or countries, which carry on considerable commercial transactions, the debts of the one are nearly equal to the debts of the other. In England there are always a considerable number of merchants indebted to America, for instance, and likewise about the same number in this country to whom America is indebted. Therefore, when A of England has a payment to make to B of America, he does not effect it in

coin, or as it is called, in *specie* or *bullion*, but he buys from C of England a *bill*, or an order upon his debtor, D of America, requesting him to pay the amount of the bill to A, who endorses this bill, and sends it to B, who is paid by B of the same city or country. This transaction might be worded in the following manner: A of England, who is debtor to B in America, buys from C in England, who is creditor to D in America, his claim upon D, which he sends to B, who easily obtains payment from D, of the same city or country. Thus all the parties concerned are benefitted by this mode of dealing, and the debts are liquidated without any specie leaving either country; thus avoiding all risk of loss by transmission, and at a very trifling expense.

The bills of exchange are either made payable at sight or at *usance*, which is a certain specified time after date.

It seldom occurs that debts reciprocally due by any two countries are of an equal amount. If the debts due by England to America, exceeded those due by America to England, the merchants of America have more bills, for which they want payment; and as the number of sellers of bills is greater than the number of buyers, they fall off in price; whilst in England, the bills being scarce, the buyers will exceed the sellers, and the bills will rise in value. Bills also fetch a higher price when the securities are of influence, and a lower price when a doubt exists as to the ability of the parties to meet them.

There is a limit to the rise of the value of bills of exchange; if in England, for instance, the expenses of insurance, and the loss of interest of conveying bullion, be less than the premium on bills, the merchants in England prefer remitting coins to America.

In some countries there is a difference between the current, or cash money, and the exchange, or bank money. The latter is of purer metal than the current money, and the difference is called the *Agio*.

311. TABLES OF FOREIGN MONEYS.

ALGIERS.

The French coins are in general use.

AUSTRALIA.

The same as in Great Britain.

AUSTRIA, BAVARIA, WURTEMBERG, BADEN, ETC.

Pfennige.	Kreuzer.	Groschen.	Sechser.	Half Florin.	Florin or Gulden.	Rix-dollar of Account.
4	1					
12	3	1				
24	6	2	1			
120	30	10	5	1		
240	60	20	10	2	1	
360	90	30	15	3	1½	1

The lowest piece of money is a copper coin, the pfennige, worth $\frac{1}{10}$d. sterling.

Exchange at ± 10 florins, or gulden, 3 kreuzer per £1 sterling.

BELGIUM.

Accounts are kept both according to the Dutch and French standards. See the tables under the heads of Holland and France.

BRAZIL AND PORTUGAL.

Rees.	Vintems.	Testoons.	Crusados (old)	Milrees.	Pataca, or Dollar.	Conto of Rees.
20	1					
100	5	1				
400	20	4	1			
1000	50	10	2½	1		
1200	60	12	3	1⅕	1	
1000000	50000	10000	2500	1000	823⅔	1

The lowest piece of money is a ree, worth $\frac{27}{400}$d. sterling.

Exchange at ± 5s. 2d. per 1 milree.

BIRMAH.

The same as in China.

BUENOS AYRES.

The same as in Spain.

CANADA (BRITISH AMERICA).

Accounts are kept both as in England and the United States.

CAPE OF GOOD HOPE.

The same as in Great Britain.

CEYLON.

The same as in Great Britain.

CHINA.

Cash.	Candarine.	Mace.	Tael.
10	1		
100	10	1	
1000	100	10	1

A cash = $\frac{2}{25}$d. sterling.
Exchange 1 dollar = $\pm$ 4s 7d.
Or 1 tael = $\pm$ 6s. 8d.

DENMARK.

Pfennings.	Schillings.	Marks.	Rix-dollars.
12	1		
192	16	1	
1152	96	6	1

The lowest piece of money is a pfenning, value $\frac{3}{64}$d. sterling.
Exchange at $\pm$ 4s. 9½d. per rix-dollar.
Or $\pm$ 34 schillings per £1 sterling.

EGYPT.

The same as in Turkey.

FRANCE.

Centimes.	Francs.	Napoleons.
100	1	
2000	20	1

The lowest coin is a centime, value $\frac{2}{21}$d. sterling.
Exchange at $\pm$ 25 francs 12 centimes per £1 sterling.

FRANKFORT ON THE MAIN, ETC.

Fennings.	Kreuzers.	Batzen.	Florins.	Rix-dollars.
4	1			
16	4	1		
240	60	15	1	
360	90	22½	1½	1

The lowest piece of money is a fenning, value $\frac{7}{60}$d. sterling.
Exchange at $\pm$ 10¾ florins per £1 sterling.

GREECE.

100 lepta = 1 drachma.
The lowest coin is the lepta, value $\frac{4}{47}$d. sterling.
Exchange at $\pm$ 28 drachma 15 lepta per £1 sterling.

GIBRALTAR.

Quartos.	Reals.	Dollars.
16	1	
192	12	1

The lowest coin is a quarto, value $\frac{5}{19}$d. sterling.
Exchange at $\pm$ 4s. 2d sterling per dollar.

GENOA.

100 centisimi = 1 lira nuova.
The lira is supposed to be of the same weight, fineness, and consequently, value as the franc.
The lowest coin is a centisimi, value $\frac{1}{10}$d sterling.
Exchange at $\pm$ 25 lira nuova 9 centisimi per £1 sterling.

HOLLAND.

Pennings.	Grotes, or Pence.	Stivers.	Schillings Flemish.	Florins, or Guilders.	Rix-dollars.	Pound Flemish.
8	1					
16	2	1				
96	12	6	1			
320	40	20	$3\frac{1}{3}$	1		
800	100	50	$8\frac{1}{3}$	$2\frac{1}{2}$	1	
1920	240	120	20	6	$2\frac{2}{5}$	1

Accounts are kept in florins, stivers, and pennings; and also in pounds, schillings, and pence Flemish.

The lowest piece of money is a penning, worth $\frac{21}{320}$d. sterling.

Exchange at $\pm$ 37 schillings 6d. Flemish per £1 sterling.

In Holland there are two kinds of money, called banco, or bank money, and currency, or current money. The agio is from 3 to 6 per cent: that is, 100 banco is valued at 103 or more currency.

HANOVER.

The same as Prussia.

HAMBURGH.

Pfennings.	Grotes, or Pence.	Schillings of Hambro'.	Schillings Flemish.	Marks.	Rix-dollars.	Pound Flemish.
6	1					
12	2	1				
72	12	6	1			
192	32	16	$2\frac{2}{3}$	1		
576	96	48	8	3	1	
1440	240	120	20	$7\frac{1}{2}$	$2\frac{1}{2}$	1

Accounts are kept in marks, schillings and pfennings; also in pounds, schillings, and pence, as in Holland.

The lowest coin is a pfenning, worth $\frac{3}{32}$d. sterling.

Exchange at $\pm$ 13 marks 6 schillings per £1 sterling.

In Hamburgh, as in Holland, there are two kinds of money, banco and currency. The agio varies from 18 to 25 per cent.: that is, 100 banco is valued at 118, or more, currency.

INDIA.—1ST. BOMBAY.

Reas.	Urdees.	Dooganeys, Sgle. Pice.	Dorias.	Fuddeas, Dble. Pice.	Quarters.	Rupees.	Mohur.
2	1						
4	2	1					
6	3	$1\frac{1}{2}$	1				
8	4	2	$1\frac{1}{3}$	1			
100	50	25	$16\frac{2}{3}$	$12\frac{1}{2}$	1		
400	200	100	$66\frac{2}{3}$	50	4	1	
1500	750	375	250	$187\frac{1}{2}$	60	15	1

Accounts are kept in rupees, quarters, and reas.

The rupee is also divided into 16 annas, or 50 pice.

The annas and reas are imaginary moneys.

The rea is equal to $\frac{7}{100}$d sterling.

Exchange at $\pm$ 2s. 3d. sterling per rupee.

2ND. CALCUTTA.

Cowries.	Gundas.	Pice.	Punns.	Annas.	Cahauns	Current Rupees.	Sicca Rupees	Gold Mohur
4	1							
$26\frac{2}{3}$	$6\frac{2}{3}$	1						
80	20	3	1					
320	80	12	4	1				
1280	320	48	16	4	1			
5120	1280	192	64	16	4	1		
$5939\frac{1}{5}$	$1484\frac{4}{5}$	$222\frac{16}{25}$	$74\frac{6}{25}$	$18\frac{14}{25}$	$4\frac{16}{25}$	$1\frac{4}{25}$	1	
$95027\frac{1}{5}$	$23756\frac{4}{5}$	$3563\frac{13}{25}$	$1187\frac{21}{25}$	$296\frac{24}{25}$	$74\frac{6}{25}$	$18\frac{14}{25}$	16	1

Accouuts are kept in imaginary money, called rupees, (either current or sicca) annas, and pice. Real specie is reduced to this currency before it is entered into books of accounts.

The cowrie is a small smooth shell, a species of cypræa, im ported from the Lacdive and Maldive islands. It is current as long as it continues entire.

A pice is equal to $\frac{1}{8}$d. sterling nearly.

A current rupee is reckoned at 2 shillings and a sicca rupee at $\pm$ 2s. 6d. A lac signifies 100000.

3RD. MADRAS.

Cash.	Fanams.	Rupees.	Star Pagoda.
80	1		
960	12	1	
3360	42	$3\frac{1}{2}$	1

The East India Company and European merchants keep accounts in star pagoda, rupees, fanams, and cash.

Copper coins, of 1, 5, 10, and 20 cash are struck in England, and sent to Madras for general circulation.

A cash is equal in value to $\frac{1}{35}$d. sterling, and a star pagoda is valued at 8s. sterling.

IONIAN ISLANDS.

Accounts are kept in sterling money. Spanish doubloons are valued at 3s. 6d. sterling; Spanish dollars, at 4s. 4d.; and Venetian dollars, at 4s. $4\frac{1}{2}$d. sterling.

LOMBARDO-VENETO.

Accounts are kept as in Genoa, in lire Italiane, divided into centisimi; but there is in circulation the lira Austriaca, valued at $4\frac{1}{2}$d. sterling.

MALTA.

Garni.	Tari.	Scudi.	Sicilian Dollar, or Pezza.
20	1		
240	12	1	
600	30	$2\frac{1}{2}$	1

British silver money is introduced into Malta. The Spanish dollar is a legal tender, worth 4s. 4d.; the Sicilian dollar, 4s. 2d.; The Scudo de Malta, 1s. 8d. sterling.

The lowest coin is a garni, valued at $\frac{1}{12}$d. sterling.

NAPLES.

Grani.	Carlini.	Tari.	Ducat de Regno.
10	1		
20	2	1	
100	10	5	1

The lowest piece of money is a grano, worth $\frac{2}{5}$d. sterling nearly.

Exchange at $\pm$ 3s. 5d. per ducat.

PRUSSIA, SAXONY, ETC.

Pfennings	Zwölftel Thaler Stücke.	Sechstel Thaler Stücke.	Drittel Thaler Stücke.	Silber Groschen.	Thaler.	Friedrich's D'or
$2\frac{1}{2}$	1					
5	2	1				
10	4	2	1			
12	$4\frac{4}{5}$	$2\frac{2}{5}$	$1\frac{1}{5}$	1		
360	144	72	36	30	1	
1800	720	360	180	150	5	1

The lowest coin is a pfenning, worth $\frac{10}{147}$d. sterling.

Exchange at $\pm$ 6 thaler 24 silber groschen per £1 sterling.

PORTUGAL.

Rees.	Vintems.	Testoons.	Old Crusados.	New Crusados.	Milrees.	Conto of Rees.
20	1					
100	5	1				
400	20	4	1			
480	24	$4\frac{4}{5}$	$1\frac{1}{5}$	1		
1000	50	10	$2\frac{1}{2}$	$2\frac{1}{12}$	1	
1000000	50000	10000	2500	$2083\frac{1}{3}$	1000	1

The lowest piece of money is a ree, valued at $\frac{27}{500}$d. sterling.
Exchange at $\pm$ 4s. 6d. per milree.

PALERMO.

The same money as in Naples.

PERSIA.

Dinars.	Mamoodies.	Abassi.	Lari.	Silver Rupees.	Double Rupees.	Toman.
100	1					
200	2	1				
$214\frac{2}{7}$	$2\frac{1}{7}$	$1\frac{1}{14}$	1			
$512\frac{1}{2}$	$5\frac{1}{8}$	$2\frac{17}{32}$	$2\frac{5}{13}$	1		
1025	$10\frac{1}{4}$	$5\frac{1}{32}$	$4\frac{10}{13}$	2	1	
10000	100	50	$46\frac{2}{3}$	$20\frac{30}{63}$	$40\frac{40}{63}$	1

A dinar is worth $\frac{87}{1000}$d. sterling.
A toman is worth £3. 12s. 6d. sterling.

PERU.

The same money as in Spain.

RUSSIA.

Copecks.	Bank Rubles.	Silver Rubles.	Half Imperial or Five Rubles Pieces
100	1		
360	$3\frac{3}{5}$	1	
1800	18	5	1

Throughout Russia accounts are kept in bank rubles of 100 copecks.

A copeck is valued at $\frac{11}{100}$d. sterling.
Exchange at $\pm$ 3s. $2\frac{3}{4}$d. sterling per silver ruble.

ROME.

Bajocchi.	Paoli.	Scudi Romani.	Scudo d'Oro.
10	1		
100	10	1	
150	15	$1\frac{1}{2}$	1

A bajoccho is worth $\frac{1}{2}\frac{2}{3}$d. sterling.
Exchange at $\pm$ 46 Paoli per £1 sterling

SARDINIA.

The same money as in Genoa

SPAIN.

Maravedis Vellon.	Maravedis of Plate.	Quartos.	Reals of Plate.	Dollars of Plate, Piastres, or Pezze.	Ducats of Plate.	Hard Dollars.	Pistole of Exchange
$1\frac{15}{17}$	1						
4	$2\frac{1}{8}$	1					
64	34	16	1				
512	272	128	8	1			
$705\frac{15}{17}$	375	$176\frac{8}{17}$	$11\frac{1}{34}$	$1\frac{103}{272}$	1		
1280	680	320	20	$2\frac{1}{2}$	$1\frac{61}{75}$	1	
2048	1088	512	32	4	$2\frac{338}{375}$	$1\frac{3}{5}$	1

There are two kinds of money in Spain, called plate and vellon. The former being to the latter as 32 to 17. Thus, 32 reals vellon are equivalent to 17 reals plate.

The lowest piece of money is the maravedi of plate, valued at $\frac{19}{136}$d. sterling.

Exchange at $\pm$ 3s. per dollar of plate.

SWITZERLAND.

The whole of that country has adopted the French system of moneys, and all accounts are now kept according to that system.

SWEDEN AND NORWAY.

Rundstycks.	Skillings.	Rix-dollars Banco.	Rix-dollars Species.	Dollar Species.
6	1			
108	18	1		
288	48	$2\frac{2}{3}$	1	
720	120	$6\frac{2}{3}$	$2\frac{1}{2}$	1

The rundstyck is worth $\frac{7}{36}$d, sterling.

Exchange at $\pm$ 12 rix-dollars banco 10 skillings per £1 sterling.

TUSCANY.

Centisimi.	Soldi.	Lira Toscani.
5	1	
100	20	1

The lowest piece of money, the centisimi, is worth $\frac{2}{25}$d. sterling.

Exchange at $\pm$ 30 lire 25 centisimi per £1 sterling.

TURKEY.

Mangars.	Aspers.	Paros.	Piastre or Dollar.
4	1		
12	3	1	
480	120	40	1

Exchange at $\pm$ 110 piastres per £1.

UNITED STATES.

Mills.	Cents.	Dimes.	Dollars.	Eagle.
10	1			
100	10	1		
1000	100	10	1	
10000	1000	100	10	1

Accounts are also kept, in some parts of the United States, and in the British possessions, in pounds, shillings, and pence, distinguished by the term currency. These pounds, in consequence of the scarcity of coins, and the use of paper money instead of them, are of much less value than the British pounds. £100 sterling $= \pm$ £166 American currency.

The lowest piece of money is a mill, valued at $\frac{27}{500}$d. sterling.

Exchange at $\pm$ 4s. 6d per dollar, or $\pm$ 4 dollars 60 centimes per £1 sterling.

312. All questions relating to exchange are easily solved, and it is merely necessary to consult the previous tables of foreign coinage. The following examples will show the manner of converting the currency of one country to that of another.

Ex. 1. Convert £640 into francs; the course of exchange is 25 francs 25 centimes.

Here $\because$ £1 = 25 francs 25 cents,

$\therefore$ £640 $= 640 \times 25.25$ francs $= 16160$ francs.

Ex. 2. Reduce 1840 francs 75 cents to British currency, exchange at 25 francs 35 cents.

Here we have: 25.35 francs = £1,

$$\therefore 1 \text{ franc} = \frac{£1}{25.35};$$

$$\text{And} \therefore 1840.75 \text{ francs} = \frac{£1840.75}{25.35} = \frac{£368.15}{5.07} = £72.\ 12s.\ 3.2d.$$

Ex. 3. A gentleman's bill amounts to 184 francs 75 cents. How much British money, will be required to settle it, at 25 francs 12 cents per £1?

$\because$ 25.12 francs are worth £1,

$\therefore$ 1 franc is worth $\frac{£1}{25.12}$;

And $\therefore$ 184.75 francs are worth $\frac{£184.75}{25.12}$, or £7. 7s. 1.12d. nearly,

Ex. 4. Reduce 14640 florins 40 kreuzers (Austrian) to British money, exchange at 10 florins 40 kreuzers per £1.

Here we have: 10 florins 40 kreuzers = £1,

$\therefore$ 14640 florins 40 kreuzers $= \frac{14640 \text{ fl. } 40 \text{ kr.} \times £1}{10 \text{ fl. } 40 \text{ kr.}} =$ £1372. 11s. 3d.

Ex. 5. Reduce £364. 18s. 9d. sterling to Austrian money, exchange at 10 florins 25 kreuzers per £1.

$\because$ £1 is worth 10 florins 25 kreuzers,

$\therefore$ £364. 18s. 9d. are worth £364. 18s. 9d. $\times$ 10 fl. 25 kr. = 3801 florins 25.9 kreuzers.

Ex. 6. If a quantity of port wine be bought for 3245 milrees 435 rees, what is the cost, in British money, at 61d. per milree, 6 per cent. being paid for insurance?

$\because$ 1 milree = 61d.,

$\therefore$ 3245 milrees 435 rees = 3245.435 $\times$ 61d. = 197971.535d.

And $\because$ 100 pays 6 insurance,

$\therefore$ 197971.535d. pay $\frac{197971.535 \times 6\text{d.}}{100}$, or 11878.292d.

$\therefore$ whole cost = 197971.535d. + 11878.292d. = 209849.827d., or £874. 7s. 5.827d.

Ex. 7. A merchant at Amsterdam is possessed of 5179 florins 16 stivers currency, which he wishes to turn into British money; exchange at 36s. 11d. (Flemish) per £1 sterling, and agio at $4\frac{1}{8}$ per cent.

By the question, $104\frac{1}{8}$ currency = 100 banco,

$\therefore$ 5179 fl. 16 stiv. $= \frac{100 \times 5179 \text{ fl. } 16 \text{ stiv.}}{104\frac{1}{8}} = \frac{4143840 \text{ fl.}}{833} =$ 4974.59784 fl., or 198983.9136 grotes.

Also, $\because$ 36s. 11d., or 443 grotes = £1 sterling,

$\therefore$ 198983.9136 grotes $= \frac{198983.9136 \times £1}{443} =$ £449.17362, or £449. 3s. $5\frac{3}{4}$d. nearly.

Ex. 8. Required the value, in Flemish money, of a bill for £290. 11s. 10d. sterling; exchange at 33s. 10d. Flemish per £1 sterling, and agio at $4\frac{1}{4}$ per cent.

$\because$ £1 = 33s. 10d., or 406d. Flemish banco,

$\therefore$ £290. 11s. 10d. = $290\frac{71}{120} \times 406$d. = $117980\frac{1}{6}\frac{3}{0}$d. banco.

Now, $\because$ 100 banco = $104\frac{1}{4}$ currency,

$\therefore$ $117980\frac{1}{6}\frac{3}{0}$d. = $\dfrac{117980\frac{1}{6}\frac{3}{0} \times 104\frac{1}{4}\text{d.}}{100}$, or 123289.32d., or £513. 14s. $1\frac{1}{4}$d. Flemish.

Ex 9. Reduce 4500 marks current of Hamburgh into pounds sterling: exchange at 13 marks 5 schillings per £1 sterling, and agio 20 per cent.

$\because$ 120 currency = 100 banco,

$\therefore$ 4500 marks = $\dfrac{45000}{12}$ = 3750 marks banco.

But 13 marks 5 schillings = £1,

$\therefore$ 3750 marks = $\dfrac{3750 \times £1}{13\text{m. } 5\text{sch.}} = \dfrac{£60000}{213}$ = £281. 13s. $9\frac{1}{4}$d.

Ex. 10. In £550 sterling, how many Prussian thalers, &c.; exchange at 6 thalers $25\frac{1}{2}$ silber groschen per £1?

$\because$ £1 = 6 thalers $25\frac{1}{2}$ silbg.,

$\therefore$ £550 = 550 × 6 thalers $25\frac{1}{2}$ silbg. = 3767 thalers 15 silbg.

Ex. 11. If 3000 milrees were paid at Lisbon for a bill upon London of £666. 13s. 4d., what was the course of exchange.

$\because$ 3000 milrees = £666. 13s. 4d,

$\therefore$ 1 milree = $\dfrac{£666.\ 13\text{s.}\ 4\text{d.}}{3000}$, or 4s. $5\frac{1}{3}$d.

Ex. 12. If port wine be bought for £236. 14s. $8\frac{1}{2}$d. British, what is the cost in Portuguese money, at 4s. 11d. per milree?

$\because$ 4s. 11d., or 59d. = 1 milree,

$\therefore$ 1d. = $\frac{1}{59}$ milree;

$\therefore$ £1 = $\frac{240}{59}$ milree;

And £236. 14s. $8\frac{1}{2}$d, = £236. 14s. $8\frac{1}{2}$d. × $\frac{240}{59}$ = 962 milrees $991\frac{1}{2}$ rees.

Ex. 13. Bought, Seville oranges, to the amount of 1927 piastres 3 reals vellon. For what sterling money must the bill be drawn, exchange at 3s. $3\frac{3}{4}$d. per piastre?

$\because$ 32 vellon = 17 plate,

$\therefore$ 1927 pi. 3 reals = $\dfrac{1927\text{ pi. } 3\text{ reals} \times 17}{32}$ = 1023.851 piastres.

Now, $\because$ 1 piastre = 3s. $3\frac{3}{4}$d.,

$\therefore$ 1023.851 piastres $= 1023.851 \times$ 3s. $3\frac{3}{4}$d., or £169. 11s. 6.077d.

Ex. 14. If a bill of £89. 2s. $11\frac{1}{4}$d. be drawn upon London, what is the value at Cadiz, in reals of plate, &c.; exchange at 40d. per piastre ?

$\because$ 40d. = 1 piastre,

$\therefore$ £1 $= \frac{240}{40}$ pi. = 6 pi.;

$\therefore$ £89. 2s. $11\frac{1}{4}$d. = £89. 2s. $11\frac{1}{4}$d. $\times$ 6 pi. = 534.8875 piastres., or 4279 reals $3\frac{4}{5}$ maravedis.

Ex. 15. Reduce £984. 6s. 8d. British to dollars (American), exchange at 4s. 7d. per dollar.

$\because$ 4s. 7d., or 55d. = 1 dollar,

$\therefore$ £1 $= \frac{240}{55}$ dollars $= \frac{48}{11}$ dollars ;

$\therefore$ £984. 6s. 8d. $= \dfrac{\text{£984. 6s. 8d.} \times 48}{11} =$ 4295 dols. 27 cents.

Ex. 16. A merchant in New York consigns to his factor, in London, goods amounting to £7150. 14s. currency. What sterling money must the factor remit, after deducting $2\frac{1}{2}$ per cent. for commission ; exchange at 65 per cent. ?

$\because$ £165 American currency = £100 sterling,

$\therefore$ £7150. 14s American currency $= \dfrac{\text{£7150. 14s.} \times \overset{20}{\cancel{100}}}{\underset{33}{\cancel{165}}} =$ £4333. 15s. $1\dot{8}\dot{1}$d.

On accounts of commission, the factor remits 100 out of $102\frac{1}{2}$,

$\because$ $102\frac{1}{2}$ are worth 100,

$\therefore$ £4333. 15s. $1.\dot{8}\dot{1}$d. are worth $\dfrac{\text{£4333. 15s. } 1.\dot{8}\dot{1}\text{d.} \times \overset{\overset{40}{\cancel{200}}}{\cancel{100}}}{\underset{\underset{41}{\cancel{205}}}{102\frac{1}{2}}} =$ £4228. 1s. 1.48d.

Ex. 17. The course of exchange between Paris and Hamburgh is at 185 francs 25 centimes for 100 marks banco. What will be the change for $545\frac{1}{2}$ marks banco?

$\because$ 100 marks = 185 francs 25 cents,

$\therefore$ $545\frac{1}{2}$ marks $= \dfrac{545\frac{1}{2} \times \text{185 fr. 25 c.}}{100}$, or 1010.53875 francs.

Ex. 18. A has 2000 Prussian sechstelsthaler, valued at $17\frac{1}{2}$ kreuzers each; but by exchange he obtains 569 florins 20 kreuzers. How much did he lose?

$\because$ 1 sechstelsthaler is worth $17\frac{1}{2}$ kreuzers,

$\therefore$ 2000 sechstelsthaler are worth $2000 \times 17\frac{1}{2}$ kzr., or 35000 kzr.

By exchange he obtains 569 florins 20 kzr., or 34160 kzr.

$\therefore$ the loss $=$ 840 kzr., or 48 sechstelsthalers.

Ex. 19. Convert 1200 Austrian florins into bank rubles of Russia; exchange at 33 kreuzers per ruble.

$\because$ 33 kreuzers are equivalent to 1 ruble,

$\therefore$ 1 kreuzer is equivalent to $\frac{1}{33}$ ruble;

$\therefore$ 1200 fl., or 72000 kzr. are equivalent to $\frac{72000}{33}$ rubles, or 2181 rubles 82 copecks.

Ex. 20. A gentleman invests £1200, value 25.15 francs, in the French rentes or stocks, at 5 per cent., at 80. What will his income be in ducats, &c., of Naples; exchange at 4 francs 40 centimes per ducat?

$\because$ £1 $=$ 25.15 francs,

$\therefore$ £1200 $= 1200 \times 25.15$ francs, or 30180 francs.

Now, 80 bring 5 income,

$\therefore$ 1 brings $\frac{5}{80}$, or $\frac{1}{16}$;

$\therefore$ 30180 francs bring $\frac{30180}{16}$ francs, or 1886.5 francs.

But 4.50 francs are equivalent to 1 ducat,

$\therefore$ 1886.5 francs are equivalent to $\frac{1886.5 \text{ duc.}}{4.50}$, or 428 ducats 3.75 tari.

313. EXERCISES.

1. Convert 843 guilders 9 stivers (bank) into current, agio $5\frac{1}{8}$ per cent.

2. In £840. 16s. sterling, how many rix-dollars current; agio $4\frac{7}{8}$, and exchange at 34s. $7\frac{1}{2}$d. Flemish per £1 sterling?

3. In £640. 15s. sterling, how many marks banco; exchange at 35s. 6d. Flemish per £1 sterling?

4. A bill of £454 15s. 6d. is remitted to Paris by a merchant in London. What is the value in francs and centimes, exchange at 24 francs 75 centimes per £1?

5. In 12450 piastres 2 reals 40 maravedis of plate, how much sterling; exchange at 3s. 4d per piastre?

6. Leghorn receives from London a bill of £748. 18s. sterling, to be paid in lira, &c.; exchange at 28 lira 60 centimes per £1. How much will be received?

7. London exchanges with Portugal, at 4s. 9½d. per milree, and afterwards at 4s. 5¾d. How much is lost per cent. by the last transaction, compared to the former?

8. Reduce £364. 18s. 6½d. sterling to Flemish, exchange at 36s. 9d. per £1 sterling.

9. In 4646 marks 14 schillings of Hamburgh currency, how many pounds sterling; exchange at 13 marks 8 schillings per £1 sterling, agio at 22 per cent.?

10. In 24645 dollars specie 60 skillings, how many pounds sterling; exchange at 11 rix-dollars banco 12 skillings per £1 sterling?

11. New York is indebted to London £2989. 10s. currency, What sterling money may London reckon to be remittəd, when the exchange is 60 per cent.?

12. Reduce £345. 10s. 6d. British to guilders, &c., currency; exchange at 37s. 6d. Flemish per £1 sterling, and agio at 4⅛ per cent..

13. Required, the value in Hamburgh banco of a bill for £832. 14s. 6½d. sterling; exchange at 36s. 8d. Flemish per £1 sterling.

14. Sold, goods for 36748 francs 35 centimes. Find the value, exchange at 24 francs 50 centimes per £1 sterling.

15. In 8464 dollars 40 cents, how many pounds sterling; exchange at 4s. 4d. per £1 British?

16. Reduce £364. 4d. sterling to rix-dollars, &c., of Vienna; exchange at 3s. 8d. sterling per rix-dollar.

17. Convert 2480 rubles of St. Petersburgh into francs; and conversely, convert 14580 francs into rubles; exchange at 1 franc 60 centimes per ruble.

18. Frankfort remits 7000 francs to Paris; the course of exchange at Frankfort is 200 francs per 94⅞ florins, and in Paris 100 florins per 212 francs. What sum, in florins, must be remitted, according to each course, and which is the one preferred?

19. What is the value of 3560 rubles, drawn on Holland, exchange at 48 stivers per ruble, commission there ⅛ per cent.; and from Holland to Londou, exchange at 35s. 4d. Flemish per £1 sterling?

20. How many rix-dollars are equal to £2420. 10s. 6d. sterling, drawn on Hamburgh; exchange at 250 marcs banco per 100 rix-dollars Danish?

ARBITRATION OF EXCHANGE.

314. Instead of carrying on the business of exchange directly between two given places, merchants find it often more advantageous to exchange with some intermediate place, we shall, therefore, determine the course of exchange between the first place and the last corresponding to these courses; and also the value of any sum of money of the first place in that of the last: this is called *Arbitration of Exchange.*

Ex. 1. Suppose the exchange between London and Amsterdam be $35\frac{1}{2}$ florins per £1 sterling, and the exchange between Amsterdam and Paris 4 florins per 3 francs; required the par of arbitration between London and Paris, and also the value of £720 in Paris.

Here 4 florins = 3 francs,

$\therefore$ 1 florin = $\frac{3}{4}$ franc;

And $\therefore$ £1, or $35\frac{1}{2}$ florins $= \dfrac{35\frac{1}{2} \times 3 \text{ frs.}}{4}$, or $26.62\frac{1}{2}$ francs = arbitrated value of £1 sterling.

$\therefore$ £720 $= 720 \times 26.62\frac{1}{2}$, or 19170 francs.

It follows that if a merchant in London wishes to remit a sum of money to Paris, and the course between London and Paris be below $26.62\frac{1}{2}$, the indirect exchange should be preferred; if the exchange between London and Paris were at $26.62\frac{1}{2}$, both courses offer the same advantages; but if the course between London and Paris were above, then a direct remittance should be adopted: the reasons are obvious.

Ex. 2. The exchange between Amsterdam and London is $56\frac{3}{4}$ florins for £5; between Amsterdam and Hamburgh $35\frac{1}{2}$ florins for 40 marks; and between Hamburgh and Petersburgh $9\frac{1}{2}$ schillings per bank ruble. Find how many pounds sterling are equal to 3400 rubles, and also how many rubles make £280.

The following proof is as simple as it is satisfactory:—

$\because 56\frac{3}{4}$ florins = £5,

$\therefore 1 \text{ florin} = \dfrac{5}{56\frac{3}{4}}$;

$\because 35\frac{1}{2}$ florins, or 640 schillings $= \dfrac{£5 \times 35\frac{1}{2}}{56\frac{3}{4}}$;

$\therefore 1$ schilling $= \dfrac{5 \times 35\frac{1}{2}}{640 \times 56\frac{3}{4}}$;

$\therefore 9\frac{1}{4}$ schillings, or 1 ruble $= \dfrac{9\frac{1}{4} \times £5 \times 35\frac{1}{2}}{640 \times 56\frac{3}{4}}$;

And $\because$ 3400 rubles $= \dfrac{\overset{85}{\cancel{3400}} \times 9\frac{1}{4} \times 5 \times 35\frac{1}{2}}{\underset{16}{\cancel{640}} \times 56\frac{3}{4}} = \dfrac{139559\frac{3}{8}}{908} = £153$. 14s. nearly.

To convert £280 into rubles we have the following solution:—

$\because 9\frac{1}{4}$ schillings $= 1$ ruble,

1 schilling $= \dfrac{1}{9\frac{1}{4}}$.

$35\frac{1}{2}$ florins, or 640 schillings $= \dfrac{640}{9\frac{1}{4}}$ rubles,

$\because 1$ florin $= \dfrac{640}{9\frac{1}{4} \times 35\frac{1}{2}}$;

$\therefore 56\frac{3}{4}$ florins, or £5 $= \dfrac{640 \times 56\frac{3}{4}}{9\frac{1}{4} \times 35\frac{1}{2}}$ rubles;

$\therefore £1 = \dfrac{640 \times 56\frac{3}{4}}{9\frac{1}{4} \times 35\frac{1}{2} \times 5}$ rubles;

$\therefore £280 = \dfrac{280 \times 640 \times 56\frac{3}{4}}{9\frac{1}{4} \times 35\frac{1}{2} \times 5} = \dfrac{2033920}{329\frac{3}{8}} = 6175$ rubles.

The last answer could have been found by dividing the value of 1 ruble, viz., $\dfrac{9\frac{1}{4} \times 35\frac{1}{2} \times 5}{640 \times 56\frac{3}{4}}$ into £280, as may be easily ascertained.

In practice, the following method is usually employed, called the *chain rule*:—

£5 $= 56\frac{3}{4}$ florins.
$35\frac{1}{2}$ florins $= 40$ marks, or 640 schillings.
$9\frac{1}{4}$ schillings $= 1$ ruble.
3400 rubles $= £x$.

$\therefore x = \dfrac{3400 \times 9\frac{1}{4} \times 35\frac{1}{2} \times £5}{640 \times 56\frac{3}{4}} = £153$. 14s. as before.

In the arrangement of the quantities we must take care to begin each line with the term which is of the same kind as the last term of the preceding line.

Ex. 3. A French merchant desires to remit £1200 to London, and he applies to his banker for that purpose, who charges 1 per

cent. How many francs must the banker receive, supposing that

$$\begin{aligned} \pounds 20 &= 125 \text{ rubles}, \\ 60\tfrac{1}{2} \text{ rubles} &= 144 \text{ marks of Hamburgh}, \\ 95 \text{ marks} &= 46 \text{ dollars of plate (Spanish)}, \\ 11 \text{ dollars} &= 41 \text{ francs?} \end{aligned}$$

Solution by fractions: $\because$ £20 = 125 rubles, it follows that £1 $= \frac{125}{20}$ rubles; similarly, $\because$ 62½ rubles = 144 marks, it follows that 1 ruble $= \frac{144}{62\frac{1}{2}}$ marks, and $\therefore$ £1 $= \frac{125}{20}$ of $\frac{144}{62\frac{1}{2}}$ marks; but $\because$ 95 marks = 46 dollars, it follows that 1 mark $= \frac{46}{95}$ dollar, and $\therefore$ £1 $= \frac{125}{20}$ of $\frac{144}{62\frac{1}{2}}$ of $\frac{46}{95}$ dollars; but $\because$ 11 dollars = 41 francs it follows that 1 dollar $= \frac{41}{11}$ francs; and therefore that £1 =

$$\frac{\overset{\overset{\overset{1}{\cancel{5}}}{\cancel{25}}}{\cancel{125}}}{\underset{\underset{1}{\cancel{4}}}{\cancel{20}}} \text{ of } \frac{\overset{36}{\cancel{144}}}{\underset{\underset{2\frac{1}{2}}{\cancel{12\frac{1}{2}}}}{\cancel{62\frac{1}{2}}}} \text{ of } \frac{46}{95} \text{ of } \frac{41}{11} \text{ francs} = \frac{36 \times 46 \times 41}{2\frac{1}{2} \times 95 \times 11}; \quad \therefore \pounds 1200 =$$

$$\frac{\overset{\overset{48}{\cancel{240}}}{\cancel{1200}} \times 36 \times 46 \times 41}{\underset{\frac{1}{2}}{\cancel{2\frac{1}{2}}} \times \underset{19}{\cancel{95}} \times 11} = 31186.68 \text{ francs}.$$

And commission at 1 per cent. = 312 francs.

$\therefore$ the banker receives 31186.68 + 312 = 31498.68 francs,

By the chain-rule method, we have:

$$\begin{aligned} \pounds 20 &= 125 \text{ rubles}, \\ 62\tfrac{1}{2} \text{ rubles} &= 144 \text{ marks}, \\ 95 \text{ marks} &= 46 \text{ dollars}, \\ 11 \text{ dollars} &= 41 \text{ francs}, \\ x \text{ francs} &= \pounds 1200. \end{aligned}$$

$$x=\frac{\overset{\overset{\overset{1}{\cancel{5}}}{\cancel{25}}}{\cancel{125}}\times\overset{36}{\cancel{144}}\times 46\times 41\times\overset{\overset{48}{\cancel{240}}}{\cancel{1200}}}{\underset{\underset{1}{\cancel{7}}}{\cancel{70}}\times\underset{\underset{\underset{\frac{1}{2}}{\cancel{2\frac{1}{2}}}}{\cancel{12\frac{1}{2}}}}{\cancel{62\frac{1}{2}}}\times\underset{19}{\cancel{95}}\times 11}=\frac{6518016\text{ francs}}{209}=31186.68\text{ frs.}$$

$$\begin{array}{rl} & 312\ \text{com.} \\ \hline & 31498.68\ \text{frs.}\end{array}$$

The principle of arbitration of exchange may be extended to a variety of questions, which an example will illustrate.

Ex. 4. 14 Welshmen perform a piece of work in 8 days.
6 Irishmen in 2 days do as much as 5 Englishmen in 3 days.
3 Englishmen in 4 days „ 2 Scotchmen in 6 days.
4 Scotchmen in 5 days „ 7 Welshmen in $2\frac{1}{2}$ days.
How long will 10 Irishmen, 12 Englishmen, and 16 Scotchmen be doing the same work respectively?

$\because$ 6 I. in 2 d., or 12 I. in 1 d. = 5 E. in 3 d., or 15 E. in 1 d.
3 E. in 4 d., or 12 E. in 1 d. = 2 S. in 6 d., or 12 S. in 1 d.
4 S. in 5 d., or 20 S. in 1 d. = 7 W. in $2\frac{1}{2}$ d., or $17\frac{1}{2}$ W. in 1 d.

$\therefore$ 1 I. in 1 d. $=\dfrac{\overset{\overset{1}{\cancel{3}}}{\cancel{15}}\times\cancel{12}\times 17\frac{1}{2}}{\underset{4}{\cancel{12}}\times\cancel{12}\times\underset{4}{\cancel{20}}}$ W. in 1 day, or $\frac{35}{32}$ W. in 1 day.

$\therefore$ 10 I. in 1 d. $=\frac{350}{32}$ W. in 1 day;
$\therefore$ 10 I. in 32 d. = 350 W. in 1 day;
$\therefore$ 10 I. in $\dfrac{32\text{ d.}}{25}$ = 14 W. in 1 day;
$\therefore$ 10 I. in $\dfrac{32\times 8}{25}$, or $10\frac{6}{25}$ d. = 14 W. in 8 days. 1st answer.

$\therefore$ 1 E. in 1 day $=\dfrac{12\times 17\frac{1}{2}}{12\times 20}$ W. in 1 d., or $\frac{7}{8}$ W. in 1 day;
$\therefore$ 12 E. in 1 day $=\frac{21}{2}$ W. in 1 day;
$\therefore$ 12 E. in 8 days $=\frac{21}{2}$ W. in 8 days;
$\therefore$ 12 E. in $\frac{8}{21}=\frac{1}{4}$ W. 8 days;
$\therefore$ 12 E. in $\dfrac{8\times 28}{21}$, or $10\frac{2}{3}$d. = 14 W. in 8 days. 2nd answer.

$\because 1$ S. in 1 d. $= \dfrac{17\frac{1}{2}}{20}$ W. in 1 day, or $\frac{7}{8}$ W. in 1 day.

$\therefore 16$ S. in d. $= \dfrac{16 \times 7}{8}$ W. in 1 day, or 14 W. in 1 day;

$\therefore 16$ S. in 8 d. $= 14$ W. in 8 days. 3rd answer.

315. EXERCISES.

1. If London exchanges with Russia, at £24 for 150 rubles; between Russia and Hamburgh, at 275 rubles for 600 marks; between Hamburgh and Venice, at 112 marks for 42 piastres; and between Venice and Paris, at 12 piastres for 65 francs. Find how many francs there are in £600.

2. Of four different kinds of cloth, 4 yards of the 1st kind are worth 7 yards of the 2nd kind, 10 yards of the 2nd are worth 8 yards of the 3rd, and 3 yards of the 3rd are worth 5 yards of the 4th. Required, how many yards of the 4th kind are worth 51 yards of the 1st.

3. If the course of exchange be £8 for $112\frac{3}{4}$ florins Amsterdam, $56\frac{1}{4}$ florins for 64 marks of Hamburgh, and $19\frac{1}{2}$ schillings of Hamburgh for 2 rubles of Russia. Determine how many pounds sterling are equivalent to 4000 rubles, and also how many rubles to £4000.

4. The rates of exchange are at £1 for every 25.15 francs, at 30 francs for $16\frac{2}{3}$ marks of Hamburgh; also at £$\frac{3}{20}$ for 1 thaler of Berlin, and at 51 thalers for 100 marks of Hamburgh; also at £$7\frac{3}{5}$ for 100 marks. Which is the most advantageous way to exchange 1500 marks in pounds sterling, through Paris, Berlin, or direct from Hamburgh to London?

5. If 12 lbs. at London = 10 lbs. at Amsterdam.
   100 lbs. at Amsterdam = 120 lbs. at Toulouse.
   18 lbs. at Toulouse = 24 lbs. at Leghorn.
   36 lbs. at Leghorn = 31 lbs. at Antwerp.
   49 lbs. at Antwerp = 58 lbs. at Dantzick.
   How many lbs. at Dantzick = 540 lbs. at London?

6. Admit that 48 Russians perform a piece of work in 17 days,
   and 27 Poles in $14\frac{1}{2}$ days = 38 Russians in 13 days,
   11 Prussians in 8 days = 16 Austrians in 6 days,
   9 Austrians in 15 days = 10 Italians in 7 days,
   18 Italians in 4 days = 17 Dutchmen in 5 days,
   13 Dutchmen in 19 days = 25 Russians in 9 days.

How long will 17 Poles, 19 Prussians, 7 Austrians, 26 Italians, and 11 Dutchmen be in doing the same work respectively?

7. The exchange between Amsterdam and Cadiz is at 90d. per 1 ducat.

1 florin is worth 40 pence.
1 ducat is worth 375 maravedis.
34 maravedis are worth 1 real.
8 reals are worth 1 piastre.
4 piastres are worth 1 pistole.

The exchange between England and Spain is 12 shillings for 1 pistole. It is required to find how much 1088 florins of Holland are worth in pounds sterling.

8. Convert 4.3 inches decimals into duodecimals, granting that 100 inches decimal $=$ 144 inches duodecimal.

9. How much will 26 square feet 2 square inches 8 square lines in decimals amount to in duodecimals, when 10000 square lines decimal $=$ 20736 square lines duodecimal?

10. Reduce 6 cubic feet 520 cubic inches 50 cubic lines decimals into duodecimals, when $100 \times 100 \times 100$ cubic lines decimal $=$ $144 \times 144 \times 144$ cubic lines duodecimal.

11. A merchant in England has to receive 1240 piastres from Venice, for which he can obtain directly 50d. per piastre; by the circular way, he remits first to Leghorn, at 48 piastres for 51 ducats; thence to Madrid, at 325 maravedis per ducat; thence to Oporto, at 626 rees per piastre of 272 maravedis; thence to Amsterdam, at 51 pence per crusado of 400 rees; thence to Paris, at 55 pence per 3 francs; and thence to London, at 30 pence per 3 francs. How much more profitable is the circular way than the direct, allowing commission at $\frac{1}{4}$ per cent.?

ALLIGATION.

316. It often happens that in business, goods of the same kind are mixed together, either to improve an inferior kind by mixing it with a superior one, or in order to sell goods of a superior quality, the price of which alone is too high for sale.

When several ingredients of different values are mixed together, a mixture of a certain rate is obtained. Thus, if 10 lbs.

of sugar at 4d. were mixed with 6 lbs. at 6d., and 8 lbs. at $6\frac{3}{4}$d., what is the price of the mixture per lb.?

Here 10 lbs. at 4d. per lb. = 40d.
6 lbs. at 6d. per lb. = 36d.
8 lbs. at $6\frac{3}{4}$d. per lb. = 54d.

∴ 24 lbs. are worth 130d.
∴ 1 lb. of mixture = $\frac{130}{24}$d., or $5\frac{1}{2}$d. nearly.

317. We may also have to determine the quantity of each of the ingredients which are to be mixed together.

Ex. How much wine, at 4s., 5s., and 7s. must be mixed together, so that the mixture may be worth 6s.?

Since the 4s. wine is sold at 6s., the gain is 2s; the 5s. wine being sold at 6s., there is another gain of 1s.; but the 7s. being sold at 6s., the loss is 1s. Thus, by mixing 1 measure of each sort, the gain is 3s., and the loss 1s.; therefore, the 7s. wine must be increased, so that the loss equal the gain; then taking 3 measures of the 7s. wine to 1 measure of each of the others, we have a composition in which there is neither gain nor loss. The proof is evident.

318. The inference from these considerations is, that when several ingredients are mixed together, we may have to find the rate of the mixture when the price and the quantity of each is given; and we may also have to find how much of each kind must be mixed, when the rates of each ingredient, and of the mixture are known. Questions treating of this subject belong to a section of arithmetic called *Alligation.*

319. Treated algebraically, alligation offers no difficulty whatever; but its principles are not so easily investigated by arithmetic: at least, most arithmeticians have failed in their way of handling it, We shall present the solutions of various cases, hoping to render the subject intelligible to the learner.

Ex. 1. A mixture being made of 8 lbs. of tea, at 6s. 6d. per lb.; 10 lbs., at 4s. 6d. per lb.; and 12 lbs., at 5s. 8d. per lb. What is 1 lb. of it worth?

8 lbs. of tea, at 6s. 6d. per lb. = 52s.
10 lbs. „ 4s. 6d. „ = 45s.
12 lbs. „ 5s. 8d. „ = 68s.

∴ 30 lbs. of tea cost .... = 165s.
∴ 1 lb. of tea cost $\frac{165}{30}$s. = 5s. 6d.

Ex. 2. How many carats fine in a mixture of 3 lbs. of gold bullion, 18 carats fine; and 5 lbs., 22 carats fine?

[Note. Pure gold contains 24 carats, but if a composition is said to be 20 carats fine, there is in it 4 parts of alloy and 20 parts of pure gold.]

In 3 lbs. there are 3×18, or 54 carats fine.
In 5 lbs. „ 5×22, or 110 carats fine.
$\therefore$ in 8 lbs. „ 164 carats fine.
$\therefore$ in 1 lb of mixture there is $\frac{164}{8}$, or 20.5 carats fine.

Ex. 3. A merchant has wines at 12s., 15s., 18s., and 20s. per gallon, which he mixes to make a composition worth 16s. per gallon. How much of each sort must be taken?

The 12s. wine being sold at 16s., the gain is 4s. per gallon.
„ 15s. „ „ „ „ „ 1s. „
„ 18s. „ „ „ „ the loss is 2s. „
„ 20s. „ „ „ „ „ 4s. „

$\therefore$ by mixing one gallon of each sort there is 5s. gain, and 6s. loss: but the gain must equal the loss, then we perceive that by mixing 2 gallons of the 15s. wine, we increase the gain by 1s. which was wanted. Therefore we obtain a proper composition by putting 1 gallon of the first, third, and fourth kind, and 2 gallons of the second.

Which is easily proved to be correct, for

1 gallon at 12s. = 12s.
2 gallons at 15s. = 30s.
1 gallon at 18s. = 18s.
1 gallon at 20s. = 20s.
$\therefore$ 5 gallons of the mixture = 80s.
and $\therefore$ 1 gallon of the mixture $= \frac{80}{5}$ or 16s.

The process is as follows:

$$16\begin{cases} 12+4 \times 1 \text{ gallon, or 4 gain.} \\ 15+1 \times 2 \text{ gallons, or 2 gain.} \\ 18-2 \times 1 \text{ gallon, or 2 loss.} \\ 20-4 \times 1 \text{ gallon, or 4 loss.} \end{cases}$$

It need scarcely be mentioned that in questions of this kind the number of answers is unlimited.

Ex. 4, How much alloy must be added to gold, 10 oz. fine, to bring it to $7\frac{1}{4}$ oz. fine?

There are 10 oz. fine in 1lb. or 12 oz.

$\therefore$ there is 1 oz. fine in 10×12 oz,, or 120 oz.

$\therefore$ there are $7\frac{1}{4}$ oz. fine in $\frac{120}{7\frac{1}{4}}$ or $16\frac{16}{29}$ oz.

and $\therefore 16\frac{16}{29} - 12 = 4\frac{16}{29}$ oz. alloy to be added.

Ex. 5. How much gold of 21 and 23 carats fine, must be mixed with 30 oz. of 20 carats fine, to bring it to 22 carats fine?

$$22\begin{cases} 21+1 \times \ \ 1 \text{ oz.} = \ \ 1 \text{ gain.} \\ 23-1 \times 61 \text{ oz.} = 61 \text{ loss.} \\ 20+2 \times 30 \text{ oz.} = 60 \text{ gain.} \end{cases}$$

We observe that by mixing 1 oz. of each, the gain in fineness is 3, and the loss 1, but as 30 oz. of 20 carats fine are to be mixed, the gain is 61, therefore taking 61 oz. of the 23 carats fine, we get a composition where there is neither gain nor loss.

It will be noticed that the number of answers is unlimited.

Ex. 6. My labourers consist of men at 1s. 6d., and women at 1s. per day, and the amount of the whole wages is the same as if each of them received 1s. 4d.; the number of women is 20, find the number of men.

$$1\text{s. } 4\text{d.}\begin{cases} 1\text{s. } 6\text{d.} - 2 \times 40 = 80 \text{ loss.} \\ 1\text{s. } 0\text{d.} + 4 \times 20 = 80 \text{ gain.} \end{cases}$$

The loss of each man is 2d., and the gain of 20 women 80d., it is evident the number of men must be 40, because $40 \times 2\text{d.} = 80\text{d.}$; the men's loss = the women's gain.

Ex. 7. What quantity of tea, worth 8s., 7s. 6d., and 6s. 6d, per lb., must be mixed together to form a parcel containing 60lbs. worth 7s. 4d. per lb.

$$7\text{s. } 4\text{d.}\begin{cases} 8\text{s. } 0\text{d.} - \ \ 8 \times 1 = \ \ 8 \text{ loss.} \\ 7\text{s. } 6\text{d.} - \ \ 2 \times 1 = \ \ 2 \text{ loss.} \\ 6\text{s. } 6\text{d.} + 10 \times 1 = 10 \text{ gain.} \end{cases}$$

When 1lb. of each kind is taken, we perceive that the gain = the loss, and as a parcel of 60lbs. is to be mixed, $\frac{60}{3}$ or 20lbs. Express how many lbs. of each sort are required.

Ex. 8. A dealer in spirits has 200 gallons, worth 12s. 8d. per gallon, which he mixes with three other kinds, worth 12s. 4d., 15s. 6d., and 16s. 8d. per gallon, in order to sell the whole at 16s. 2d. How much of each must be taken?

$$16\text{s. } 2\text{d.}\begin{cases} 12\text{s. } 8\text{d.} + 42 \times \ \ 200 = 8400 \text{ gain.} \\ 12\text{s. } 2\text{d.} + 48 \times \ \ \ \ \ \ 1 = \ \ \ \ 48 \text{ gain.} \\ 16\text{s. } 6\text{d.} - \ \ 4 \times \ \ \ \ 12 = \ \ \ \ 48 \text{ loss.} \\ 16\text{s. } 8\text{d.} - \ \ 6 \times 1400 = 8400 \text{ loss.} \end{cases}$$

The gain on the 200 gallons, is 8400d., and since 6d. is lost on 1 gallon of the last kind, 8400d. will be lost on $\frac{8400}{6}$ or 1400 gallons, also since 48d. are gained on 1 gallon of the second, and 4d. lost on 1 gallon of the third, therefore $\frac{48}{4}$ or 12 gallons, will give a loss of 48d. Here then the loss = the gain as required.

Questions of this kind admit of many answers.

320. EXERCISES.

1. A labourer performs 34.2 yards on Monday, 37.8 yards on Tuesday, 36.9 yards on Wednesday, 35.7 yards on Thursday, 36.6 yards on Friday, and 34.8 yards on Saturday. What is the average daily work?
2. Brass is made by casting 3 parts of zinc to 7 parts of copper. if 1 lb. of zinc cost 4d., and 1 lb. of copper cost 1s. 3d. What is the price of 1 lb. of brass?
3. Bronze for cannons is obtained by melting 11 parts of tin, to 100 parts of copper, the value of tin is 11d. per lb., and of copper, 1s. 4d. per lb. The price of 1 lb. of bronze is required?
4. A bell is cast by melting together 220 lbs. of tin, 780 lbs. of copper, 10 lbs. of zinc, and 8 lbs. of lead. Tin is 1s. per lb.; copper, 1s. 2d. per lb.; zinc, 5d. per lb.; and lead, 2d. per lb. Find the value of the bell, and also of 1 lb. of this bronze.
5. Printing types are composed of 20 parts of antimony, 80 parts of lead, and 5 parts of copper. What is the value of 1 lb. of this composition, if antimony is 4s. 6d. per lb., lead 2d. per lb., and copper 1s. 3d. per lb.?
6. What quantity of wines at 24s., 21s., 18s. and 15s. per gallon, must be mixed so as to make a composition of 1000 gallons at 20s. per gallon?
7. How many gallons of water must be added to 20 gallons of wine at 18s. per gallon, so that the mixture be worth 15s. per gallon?
8. Some sea water contains 1 lb. of salt in 32 lbs of water. How much spring water must be added to it so that in 32 lbs. there may be only 2oz. of salt?
9. A dealer has spirits at 32 degrees which he desires to be reduced to 21 degrees, by mixing water with it. How must he do that?

10. A cask containing 480 bottles of wine, has been filled with wines at 6s. and 4s. a bottle, and the whole cask is worth £108. how many bottles of each does it contain?

11. A wine merchant has wine at 4s., 5s. and 6s. a bottle, which he desires to mix in equal parts with 108 bottles of another wine at 5s. 8d., so that the mixture be worth 5s. 6d., what quantity of each sort of wine is he to mix?

12. A jeweller has gold at 15, 17, 18 and 22 carats fine, with which he wants to make a composition of 40oz. 20 carats fine, how much of each must he take?

13. How many gallons of wine at £·75, £·90, £1·20, and £·60 are required to make 150 gallons, at £1. per gallon?

14. A spirit merchant, who has 500 gallons of spirits, worth 13s. 4d. per gallon, wishes to mix it with three other kinds, worth 12s. 6d., 14s., and 16s. 6d. per gallon, in order to sell the whole at 15s. 4d. per gallon. How much of each must he take?

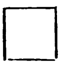
15. A druggist has two sorts of bark, one worth 5s. 9d., and the other 10s. per lb. What portion of each must he take to make a mixture of 2½ cwt. that will be worth 8s. 6d. per lb.?

16. It is required to mix British spirits at 9s., French wine at 17s., ginger wine at 3s., and water at 0 per gallon together, so that the mixture may be worth 6s. per gallon. How much of each must be taken?

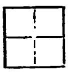
17. If I melt 8 lbs. 5¼ oz. of bullion, of gold 14 carats fine, with 12 lbs. 8 oz., of 18 carats fine, how many carats fine is the mixture?

INVOLUTION AND EVOLUTION OF NUMBERS.

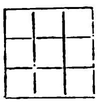
321. A square is a figure, the four sides of which are equal, and the angles right-angled. A square surface, the side of which is one foot or one inch is called a square foot or a square inch.



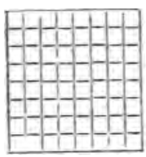
If the side were two feet, the surface would be four square feet.



If the surface were three feet long and three feet broad the square would, consequently, be nine feet square.



Then, to find the number of square feet contained in a square surface the dimension of one side must be known, since that dimension is to be multiplied by itself; for, if each of the sides be 8 inches, joining the points of division of the opposite sides in order, we have 64 squares, each one inch.

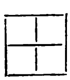


For this reason the name of *square numbers* has been given to the product of any number multiplied by itself, or to the product of two equal factors. 16 is a square number, because it is the product of 4×4. 25, 36, 49, 64, 81, 100, &c., are square numbers.

Therefore, to find the square of any number it must be multiplied by itself,

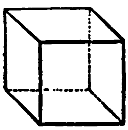
322. The number expressing the dimension of one side of a square, or one of the two equal factors, is called the *square root*. Thus, 4 is the square root of 16, 7 of 49, &c.

Let it be required to find the square of $\frac{1}{2}$. For that purpose, suppose a line divided into two equal parts, and a square described upon that line, it will contain 4 small equal squares, and the square formed upon the half of the line is $\frac{1}{4}$ of the whole square; therefore, the square of $\frac{1}{2}$ is $\frac{1}{4}$, viz., $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. It would appear that the square of a fraction is smaller than that fraction; but we must remember that the square root of $\frac{1}{2}$ is a line or length, whilst the square $\frac{1}{4}$ is a surface, which is one-fourth part of the whole square.



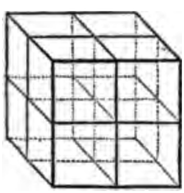
The square of $\frac{5}{7}$ is $\frac{25}{49}$. Let a base be divided into 7 equal parts, and a square described upon it, that square contains 49

small squares, but the square described upon $\frac{5}{6}$ of the base contains only 25 of them. Therefore, to square a fraction, multiply the numerator by itself and divide it by the denominator multiplied by itself. The square of $\frac{5}{6}$ is $\frac{5}{6}\times\frac{5}{6}$, or $\frac{25}{36}$. The square of $2\frac{1}{3}$, or of $\frac{7}{3}=\frac{7}{3}\times\frac{7}{3}=\frac{49}{9}$, or $5\frac{4}{9}$. The square of $3.4=3.4\times3.4$, or 11.56.



323. The cube is a geometrical solid in the form of a die, viz., having the same dimensions in every way. A solid of this shape, measuring one foot or one inch in every way, is called a cubic foot or a cubic inch.

On a line 2 feet long let a square be described, and with a height equal to the breadth or the length, complete the scheme as in the figure, we obtain a solid composed of 8 cubes, one foot every way. The same may be exemplified by placing 8 small cubes, supposed to be one foot every way, so that four stand on a square base of four square feet, and four on the top of these, thus a solid having 2 feet every way is formed. The same may be done on other square bases, and we arrive at the conclusion that a cube or cubic number is the product of a number multiplied twice by itself, or is the product of three equal factors.



324. The number expressing the length of one side of the cube, or one of the equal factors, is called the *cube root*, thus: 3 is the cube root of 27, because $3\times3\times3=27$.

The cube of $4=4\times4\times4=64$.
The cube of $10=10\times10\times10=1000$.
The cube of $\frac{1}{2}=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$. (See preceding page.)
The cube of $\frac{3}{4}=\frac{3}{4}\times\frac{3}{4}\times\frac{3}{4}=\frac{27}{64}$.
The cube of $2\frac{2}{3}=\frac{8}{3}\times\frac{8}{3}\times\frac{8}{3}=\frac{512}{27}=18\frac{26}{27}$.
The cube of $4.24=4.24\times4.24\times4.24=76.225024$.

325. A number which is multiplied once, twice, three times, four times, &c., by itself, is said to be raised or involved to the second, third, fourth, fifth power, &c. The number itself is in the first power. The second power of a number is the same as its square, and the third power the same as its cube.

As an exercise, the pupil may construct a table similar to the following, which can easily be extended

TABLE OF POWERS.

NUMBERS.	1	2	3	4	5	6	7	8	9	10
Square or 2nd Power	1	4	9	16	25	36	49	64	81	100
Cube or 3rd Power	1	8	27	64	125	216	343	512	729	1000
Biquadrate or 4th Power	1	16	81	256	625	1296	2401	4096	6561	10000
5th Power	1	32	243	1024	3125	7776	16807	32768	59049	100000
6th Power	1	64	729	4096	15625	46656	117649	262144	531441	1000000
7th Power	1	128	2187	16384	78125	279936	823543	2097152	4782969	10000000
8th Power	1	256	6561	65536	390625	1679616	5764801	16777216	43046721	100000000
9th Power	1	512	19683	262144	1953125	10077696	40353607	134217728	387420489	1000000000
10th Power	1	1024	59049	1048576	9765625	60466176	282475249	1073741824	3486784411	10000000000

This process is termed *Involution*, and the reverse process, viz., that of finding the original number, called the *root*, is termed *Evolution*.

326. If the same number be repeated or multiplied by itself, it is expressed by placing rather above the number a figure called *index* or *exponent*, denoting how often it is repeated. Thus, if 7 were to be repeated three times, or raised to the third power, it is expressed in this manner:—7^3 and 20^8 is the eighth power of 20.

327, A sign is also used to express the root of a number, $\sqrt{}$ or $\sqrt[2]{}$, $\sqrt[3]{}$, $\sqrt[4]{}$, &c, denote the square root, the third or cube root, the fourth root, &c. The same is also expressed by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. placed a little above the number.

Thus $\sqrt{36}$ or $36^{\frac{1}{2}}$ express the square root of 36.

„ $\sqrt[4]{48}$ or $48^{\frac{1}{4}}$ „ fourth 48.

„ $\sqrt[3]{20^4}$ or $20^{\frac{4}{3}}$ represent the third root of the fourth power of 20.

„ $\sqrt[8]{12^5}$ or $12^{\frac{5}{8}}$ „ eighth „ fifth „ 12

SQUARE ROOT.

328. We shall consider, in the first place, the squares of the natural series of numbers, from which we shall make some observations which will be found useful for explaining the extraction of the square root.

The squares of the numbers of one digit

	1,	2,	3,	4,	5,	6,	7,	8,	9,
are	1,	4,	9,	16,	25,	36,	49,	64,	81.

The square of 10 is 100, of 99 is 9801, of 100 is 10000, of 999 is 998001, &c., &c., &c

From this we infer that the figures of a square are twice those of the root, except when the first figure of the root is 1, 2, or 3, then the figures are twice as many, minus one.

Therefore—1st, every square of one or two figures has only one figure in its root; 2nd, every square of three or four figures has only two figures in its root; 3rd, every square of five or six figures has only three figures in its root; 4th, &c.

By this law the square roots of 567 and 4236 have two figures, 53641 and 168478 have three figures. For this reason before

extracting the square root of a number it is divided into periods of two figures, beginning at the units' place, above which a point is placed, as also on every alternate figure. The number of points show the number of figures in the root. Thus $\dot{2}4\dot{3}$, because it has three figures, has two in the root, and the points show it also.

It follows also that 1 is the integer part of the square root of all numbers from 1 to 4. The square root of 2 is 1 with a fraction, the same applies to 3; a fraction is found in the root of every number which is not a *perfect square*, and the square root of such a number is the root of the preceding perfect square plus a fractional part. Thus the square of 20 is 4 and a fraction, that of 90 is 9 plus a fraction. It shall be shown that the roots of such quantities can only be *approximately* found, and these quantities are called surds or *irrational quantities*.

329. If we analyse what takes place when squaring a number we shall derive a formula or law which will enable us to extract the square root of numbers.

Every number of two or more figures allows of its being divided into two quantities. For example $27=20+7$; $424=420+4$; $84625=84620+5$. Let us express generally the first part by a, and the second by b, so that $a+b$ represent all numbers of two or more figures; then squaring $a+b$ we have

$$(a+b)^2=(a+b)\ (a+b)=a\times a+a\times b+b\times a+b\times b=a^2+2\times a \times b+b^2.$$

330. Hence it follows that the square of a number of two or more figures contains the square of the first part $(a^2)+$twice the product of the first part by the second $(2\times a\times b)+$the square of the second part (b^2).

According to this law the multiplication is thus carried on—

$$\begin{aligned}36^2&=(30+6)^2=(30+6)\ (30+6.)\\&=30\times 30+30\times 6+6\times 30+6\times 6.\\&=30^2+2\times 30\times 6+6^2.\\&=1296.\end{aligned}$$

Also,
$$\begin{aligned}524^2&=(520+4)^2=(520+4)(520+4).\\&=520\times 520+520\times 4+4\times 520+4\times 4.\\&=520^2+2\times 520\times 4+4^2.\\&=274576.\end{aligned}$$

331. Ex. 1. Let it now be required to extract the square *root of 576.*

$$\begin{array}{r}
\dot{}\ \dot{}\quad a\quad b \\
\sqrt{576}=20+4 \\
\underline{400=a^2} \\
2\times a=40)176=2\times a\times b+b^2. \\
160=2\times a\times b \\
\underline{16=b^2}
\end{array}$$

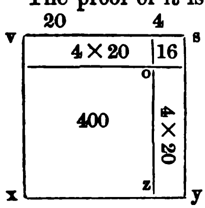
The process in practice is this:

$$\begin{array}{r}
\dot{5}7\dot{6}(24 \\
\underline{4=a^2} \\
44)176 \\
\underline{176=2\times a\times b+b^2}
\end{array}$$

Since 576 is a number of three figures, and there are two in the root, as the points show. Therefore extracting the square root of the first period, or of 500, the root is found to be between 20 and 30, take 20 and subtract 20^2 (a^2), or 400 from 500, the remainder is 100, to which the second period is added, and 176 contains twice the first part by the second + the square of the second part, then dividing the whole remainder by $2\times 20=40$, the quotient is 4, which is the second part of the root; lastly we subtract 40×4 $(2\times a\times b)+4^2(b^2)=176$, from the remainder 176 and 0 is left.

$\therefore \sqrt{576}=20+4=24$.

The proof of it is $24^2=576$.



The annexed diagram shows plainly what we have just explained, here $xs=576$, $xz=20$, $\therefore xo=400$, and $vo+os+oy=176$, zy is found to be 4, then $oy+ov=2\times 20\times 4$, and os is the square of 4 or 16, which completes the square xs.

We must observe that the first figure of the root (2) expresses tenths, twice its product is 40, or 4 tenths; so we might have divided 17 tenths by 4 tenths, instead of 176 by 40, and the quotient would have been the same.

Ex. 2. 7225 is the square of what number?

$$\begin{array}{r}
\dot{}\ \dot{}\quad ab \\
\sqrt{7225}=85 \\
\underline{64}\quad =a^2 \\
2a+b=165)825 \\
\underline{825=2\times a\times b+b^2}
\end{array}$$

This number of four figures has two periods. $\sqrt{72}$ is between 8 and 9, therefore 8 is the first part of the root, and 8^2 or 64 being subtracted from 72, the remainder is 8, the next period 25 being brought down, we divide 82 by 2×8 or 16, and the quotient is 5, the second part of the root, we set 5 after 16 tenths to

fill up the units' place, for (16 tenths + 5 units) $\times$ 5 = $(2\times a+b)b=2a\times b+b^2$, and subtracting 825 from 825 the remainder is 0, $\therefore\ \sqrt{7225}=85$. The proof of the operation is $85^2=7225$.

332. Having thus explained the extraction of the square root of two figures, it will not be difficult to find a root of three figures, for the two first are determined as before, and to find the third we consider the two first figures, as composing the first part of the square root.

Were we to extract the square root of a number of 7 or 8 figures, the root would consist of four, we should find the three first, as just mentioned, and then considering those three figures as the first part, the fourth would be the second part of the square root.

Ex. 3. Find the square root of 294849.

$$\begin{array}{rl}
 & \dot{2}9\dot{4}8\dot{4}\dot{9}(543 \\
 & \underline{25=a^2} \\
2\times50+4=104) & 448 \\
 & \underline{416=2a\times b+b^2} \\
2\times540+3=1083) & 3249 \\
 & \underline{3249=2a\times b+b^2}
\end{array}$$

Here we find the number consisting of three periods, from the first period the square root is 5; the square of which being subtracted leaves 4. Bring down the next period 48, then 448 is the second part, divide 44 by 2×5 or 10, the double of the first figure in the root, we set the quotient 4, as the second figure in the root, then we subtract $(2\times50+4)\times4$, or 416, from 448, the remainder is 32. We then bring down the last period 49. Now we are to divide 3249 by 2×54, or 108, twice the first figures of the square root, which we regard as 1080, the quotient is 3, which is the third figure of the square root, and we subtract $1080\times3+3^2=3249$ from the dividend 3249, the remainder is 0. $\therefore\ \sqrt{294849}=543$.

The proof is $543^2=294849$.

If after having brought down a period by the remainder, the double of the root was not contained in this dividend, the last figure being cut off, a cipher is set at the root, and also after the double of the root, and a new period is set down by the side of the other one, and proceed as before.

333. EXERCISES.

1. Extract the square roots of 676, 1689, 9801.

2. Find the square roots of 16129, 258064, 61340224.

3. What is the square root of 258064? 826281? 31360000?

334. The square roots of irrational quantities or surds cannot be exactly expressed, but we can get an approximate root, correct to as many decimals as we please, by affixing to the right hand of the last remainder as many periods of ciphers, as we require decimal figures in the root.

Thus, if it were required to extract the square root of 345, true to three places of decimals, or within .001 the operation is carried on as follows:—

$$\begin{array}{r r}
 & \sqrt{\dot{3}4\dot{5}}\,(=18.574 \\
 & 1 \\
28) & \overline{245} \\
 & 224 \\
365) & \overline{2100} \\
 & 1825 \\
3707) & \overline{27500} \\
 & 25949 \\
37144) & \overline{155100} \\
 & 148576 \\
 & \overline{6524}
\end{array}$$

The square root of 345 is found as before; to the remainder 21 a period of ciphers is appended, and the quotient is found to be 5, the remainder is 275, to which a period of ciphers is added, and so on. Since three periods of ciphers are appended to the integral part, there are three decimal figures in the root, viz., as many decimal figures as periods of ciphers, which are pointed from the right hand.

In this and like cases where ciphers are appended, the root can never terminate, because no figure multiplied by itself can produce a cipher.

335. If the number from which the square root is to be extracted were a mixed number, the fractional part is converted into decimals, and periods of decimals are brought down as mentioned with regard to the ciphers; or if we had to extract the

square root of a decimal, the pointing is made from the units' place to the right hand.

Ex. 1. Required the square root of $28\frac{5}{8}$.

$\sqrt{28\frac{5}{8}} = \sqrt{28.625}(=5.35023$

25

103) 362

309

1065) 5350

5325

107002) 250000

214004

1070043) 3599600

3210129

389471

After what has been said the operation is easily understood:— The square root has been extracted to five places of decimals.

Ex. 2. What is the square of .2, true to .00001 or to 5 places?

$\sqrt{.20}$ (=.44721

16

84) 400

336

887) 6400

6209

8942) 19100

17884

89441) 121600

The periods are perfectly similar to the cases we have examined; the first period is .20, and not .2, because $.4 \times .4 + .04 = .20 = .2$.

336. By what has been said of the squaring of vulgar fractions it is evident that the square root of such quantities is found by extracting the square root of both numerator and denominator. Thus the square root of $\frac{9}{25}$ is $\frac{3}{5}$, since $3 \times 3 = 9$ and $5 \times 5 = 25$. But it generally happens that the numerator, or denominator, or both, are not square numbers, then divide the square root of their product by the denominator.

For $\sqrt{\frac{3}{5}} = \sqrt{\frac{3 \times 5}{5 \times 5}} = \frac{\sqrt{15}}{5} = \frac{3.873}{5} = .774$

The fraction may be reduced to decimals, and the square root of the decimals extracted. Thus

$$
\begin{array}{r@{}l}
\sqrt{\tfrac{3}{5}} = \sqrt{.\overline{60}}(& = .774 \\
49 & \\
\hline
147)1100 & \\
1029 & \\
\hline
1544)\ \ 7100 & \\
6176 & \\
\hline
924 &
\end{array}
$$

337. EXERCISES.

1. Find the squares of 23, 549, 467825.

2. Required the squares of $\frac{2}{3}$, $\frac{49}{112}$, $\frac{4}{7}$, $\frac{485}{913}$.

3. What are the squares of $3\frac{12}{17}$, $18\frac{3}{4}$, $27\frac{16}{49}$, $462\frac{153}{289}$?

4. Raise to the squares 2.5, .3, .08, 3.216, .0000092, 9.0006.

5. What number is that, the square root of which being increased by 13, the sum is 29?

6. The treble of the square root of a number is $5\frac{3}{7}$, required the number.

7 Take the fifth part of the square of 3.26.

8. Extract the square root of 81, 1444, 32041, 2939483089.

9. Required the square root of $\frac{9}{64}$, $\frac{25}{169}$, $1\frac{156}{1444}$, $\frac{2209}{6241}$.

10. Find the value of $\sqrt{33.64}$, $\sqrt{.49}$, $\sqrt{29.16}$, $\sqrt{.011664}$.

11. 729 men are to be formed into a solid square, how many will be on one side.

12. Several persons gained £841, each received as many pounds as there were persons, how many were there?

13. A piece of land, 108 yards long and 12 broad, is to be exchanged for an equivalent square plot, what is its length?

14. The two sides of the right angle of a right angled triangle are 6 and 8 yards long, how many yards is the hypothenuse?

15. The square root of a number multiplied by $7 = 161$. Find the number.

16. A farm of 363 acres is three times as long as broad, find both its length and breadth in yards.

17. A ladder, 17 feet long, stands against a wall, its base is 8 feet from the bottom of the wall, how high does it reach?

18. A rectangular garden contains 180 square yards, if it were as broad as it is long it would contain 225 square yards, required its length and breadth? Also what is the length of the diagonal of the said garden?

19. In constructing a railway, a tunnel is to be made through a hill, whose top is 180 feet above the level of the line, and the slant sides are 560 and 960 feet respectively: What is the length of the tunnel?

20. A creditor received the same sum from each of two debtors, the product of the two sums is £177241. What was the amount of each debt?

21. The diagonal of a square is 33.64 chains, find the area of the field and the cost of fencing it round at 2s. 6d. per yard.

22. Two ships set sail from the same port, one of them has sailed due east 75 knots, the other due north 112 knots. How far are they asunder?

CUBE ROOT.

338. It is necessary to have a perfect idea of the formation of a cube, by the multiplication of its factors, in order to understand the extraction of the cube root.

The cubes of the first nine numbers must be remembered, they are:—

Digits,	1,	2,	3,	4,	5.	6,	7,	8,	9.
Cubes,	1,	8,	27,	64,	125,	216,	343,	512,	729.

The cube of 10 is 1000, 99 is 97029, 100 is 1000000, 999 is 9970029999, 1000 is 100000000, 9999 is 999700029999.

We observe by the above:—

1st, that every	number	of one	figure	contains	1 to 3	in its cube.
2nd,	,,	,,	two	,,	,,	4 to 6	,,
3rd,	,,	,,	three	,,	,,	7 to 9	,,
4th,	,,	,,	four	,,	,,	10 to 12	,,

From this it follows that for every figure in the root those in the cube are increased by three, therefore the units of the root can only be contained in the three right-hand figures of the cube,

the tens of the root in the three following, and so on. For this reason, before extracting the cube root of a number, its figures are divided by points into periods of three, beginning at the units' place

Hence $\dot{3}37\dot{5}$, $2\dot{8}78\dot{4}$, and $46\dot{2}56\dot{9}$ have two figures in the cube root, and $\dot{5}64\dot{3}02\dot{6}$, $3\dot{4}69\dot{7}32\dot{4}$, and $53\dot{4}60\dot{1}20\dot{2}$ have three figures in the cube root.

339. Every number of two or more figures can be divided into two quantities. Let a represent the first and b the second part of a number of several figures ; $\therefore$ $a+b$ denote any number whatever.

By cubing $(a+b)$ we have:

$(a+b)^3=(a+b)\times(a+b)\times(a+b)=(a+b)^2\times(a+b)=(a^2+2ab+b^2)(a+b)=a^3+2\times a^2\times b+a\times b^2+a^2\times b+2\times a\times b^2+b^3=a^3+3\times a^2\times b+3\times a\times b^2+b^3$.

Hence we say, the cube of a number of many figures contains the cube of the first part (a^3)+three times the product of the square of the first part by the second $(3\times a^2\times b)$+three times the product of the first part by the square of the second $(3\times a\times b^2)$+the cube of the second part (b^3).

To illustrate this law, we shall give two examples.

Ex. 1. $36^3=(30+6)^3=(30+6)^2\times(30+6)$
$=(30^2+2\times30\times6+6^2)\,(30+6)$
$=30^3+2\times30^2\times6+30\times6^2+30^2\times6+2\times30\times6^2+6^3$
$=30^3+3\times30^2\times6+3\times30\times6^2+6^3$
$=27000+16200+3240+216$
$=46656$.

Ex. 2. $524^3=(520+4)^3=(520+4)^2\times(520+4)$
$=(520^2+2\times520\times4+4^2)\times(520+4)$
$=520^3+2\times520^2\times4+520\times4^2+520^2\times4+2\times520\times4^2+4^3$
$=520^3+3\times520^2\times4+3\times520+4^2$
$=140608000+3244800+24960+64$
$=143877824$.

From what has been said with regard to the cube of numbers, we can detect at once, by the inspection of the numbers, two particulars connected with the extraction of the cube root, viz., the number of figures in the root, as well as the first figure of it; for instance, $\sqrt[3]{1\dot{3}82\dot{4}}$ has two figures, and the first of it is 2.

$\sqrt[3]{246894757}$ has three figures, and the first of it is 6.

Now, let us endeavour to explain the operation of the extraction of the cube root.

Ex. 1. Let it be required to find $\sqrt[3]{405224}$.

$$\begin{array}{rl}
 & \quad\quad\quad\quad\quad\; a \quad\; b \\
 & 405224(70+4=74 \\
 & \underline{343000=a^3} \\
3\times a^2=14700) & 62224 \\
 & \left\{\begin{array}{r} 58800=3\times a^2\times b \\ 3360=3\times a\times b^2 \\ 64=b^3 \end{array}\right. \\
 & \overline{62224} \\
 & \cdot\cdot\cdot\cdot\cdot
\end{array}$$

This cube consists of two periods, therefore the root has two figures. We find the first part of the cube root to be 70; subtracting 343000 from 405000, the remainder is 62000, and with the second period it becomes 62224, which contains three times the product of the square of the first part by the second + three times the product of the first part by the square of the second + the cube of the second part. Then dividing 62224 by $3\times 70^2=14700$, the quotient is 4, which is the second figure, or the second part of the cube root. Lastly, subtract 14700×4 $(3\times a^2\times b)+3\times 70\times 4^2$ $(3\times a\times b^2)+4^3$ $(b^3)=58800+3360+64=62224$, and the remainder is 0.

$\therefore\ \sqrt[3]{405224}=74$. The proof is $74^3=405224$.

In practice, it is usual to omit the ciphers of the periods following the one from which the root is extracted, so as to simplify the work. Thus:

$$\begin{array}{rl}
 & \sqrt[3]{405224}(=74 \\
 & \underline{343} \\
147) & 62224 \\
 & \left\{\begin{array}{l} 588 \\ \;\;336 \\ \;\;\;\;\;64 \end{array}\right. \\
 & \overline{62224} \\
 & \cdot\cdot\cdot\cdot\cdot
\end{array}$$

Here $3\times a^2=147$ is divided into 622, because $3\times a^2=147$ hundreds; and further, $3\times a\times b^2=336$ is set one place more towards the right, or under the tens' place, because $3\times a\times b^2=336$ tens; and, lastly, the remaining term $b^3=64$ is set one place more towards the right, or under the units' place.

When the second part of a cube root is to be found, it will sometimes occur that the quotient is too large; this arises from not taking into account all the parts which are to be subtracted when the quotient is determined.

After what has been said of the extraction of the cube root of two figures, it becomes easy to extract the cube root of more than two figures.

If it were required to extract a cube root of three figures the two first are found as mentioned, then these are considered as the first part, and the third, which is unknown, as the second part of the cube root.

Ex. 2. It is required to find $\sqrt[3]{17173512}$.

$$
\begin{array}{rl}
 & \sqrt[3]{1\dot{7}17\dot{3}51\dot{2}}(=258 \\
 & 8 \quad = 2^3 \\
\hline
3\times 2^2 = 12) & 9173 \\
 & \left\{\begin{array}{l} 60 \quad = 12\times 5 \\ 150 \quad = 3\times 2\times 5^2 \\ \ 125 \quad 5^3 \end{array}\right. \\
\hline
 & 7625 \\
\hline
3\times 25^2 = 1875) & 1548512 \\
 & \left\{\begin{array}{r} 15000 \quad = 1875\times 8 \\ 4800 \quad = 3\times 25\times 8^2 \\ 512 = 8^3 \end{array}\right. \\
\hline
 & 1548512 \\
\hline
 & \cdot\cdot\cdot\cdot\cdot\cdot\cdot
\end{array}
$$

Then $\sqrt[3]{17173512} = 258$. The proof is $258^3 = 17173512$.

Ex. 3. The product of three equal factors is 6372785864, what are they?

To say that a quantity is the product of three equal factors is the same as saying that a quantity is the cube of another. Therefore a factor will = the cube root of the given quantity.

Ex. 3. The product of three equal factors is 6372783864, which are they?

$$
\begin{array}{rl}
 & \sqrt[3]{\dot{6}37\dot{2}78\dot{3}86\dot{4}}\,(=1854 \\
 & 1 \\
\cline{2-2}
3\times 1^2=3) & 5372 \\
 & 24 \quad =3\times 8 \\
 & 192 \quad =3\times 1\times 8^2 \\
 & 512=8^3 \\
\cline{2-2}
 & 4832 \\
\cline{2-2}
3\times 18^2=972) & 540783 \\
 & \left\{\begin{array}{l} 4860 \quad =972\times 5 \\ \;1350 \quad =3\times 18\times 5^2 \\ \quad 125= \;5^3 \end{array}\right. \\
\cline{2-2}
 & 499625 \\
\cline{2-2}
3\times 185^2=102675) & 41158864 \\
 & \left\{\begin{array}{l} 410700 \quad =102675\times 4 \\ \quad 8800 \quad =3\times 185\times 4^2 \\ \qquad 64= \;4^3 \end{array}\right. \\
\cline{2-2}
 & 41158864 \\
\cline{2-2}
 & \cdots\cdots
\end{array}
$$

Therefore each of the three factors $=1854$. The proof of which is $1854^3=6372783864$.

340. We shall now show another method of extracting the cube root, which is preferable on account of its brevity.

Taking Example 2 over again, we have :—

I	II.	
		17173512(258
		$8 \quad = a^3$
	$12 = 3a^2$	9173
$65 = 3a + b$	$325 = (3a+b) \times b$	
	$1525 = 3a^2 + 3ab + b^2$	$7625 = 3a^2b + 3ab^2 + b^3$
	$25 = b^2$	
	$1875 = 3a^2 + 6ab + 3b^2 = 3a_{,}^2$	1548512
$758 = 3a_{,} + 6_{,}$	$6064 = (3a_{,} + b_{,}) \times b_{,}$	
	$193564 = 3a_{,}^2 + 3a_{,}b_{,} + b_{,}^2$	$1548512 = 3a_{,}^2b_{,} + 3a_{,}b_{,}^2 + b_{,}^3$

$= a_{,}^3$

Here the number is divided into periods as before, and the cube of 2 is subtracted. To the remainder, 9, the next period, 173, is subjoined. The triple of the first figure of the root, 2, is set in column I; then multiplying this triple by the figure in the root, the product 12 is placed in column II.

With 12, as trial divisor, divide 91 (the two last figures of the remainder being omitted), the quotient is 5, set it as the second figure of the root, and also in column I, after 6, making 65 ; now multiply 65 by 5, and the product is set in column II, under 12, extending two places of figures more to the right, and 1525, the sum of these two numbers, is the corrected divisor; this sum is multiplied by the second figure in the root, and the product 7625 is subtracted; the last period is brought down to the remainder, and the number we have is 1548512.

Now we must repeat the same process to obtain the third figure of the root as we did for the second, viz., triple the root, which gives 75, and set the product in column I.; now set the square of 5, the second figure of the root, in column II., and add the three last numbers, those enclosed by the brackets, the sum 1875 is the trial divisor, which is found to be contained in 15485 (the two last figures being omitted) 8 times; then proceed as before.

We have appended to each line the quantities we have made use of before, in order to render the process intelligible, and after what has been previously said this needs no explanation.

Our third example worked after this method is here represented :—

I.	II.	6372783864(1854
		1
	3	5372
38	304	
	604	4832
	64	
	972	540783
545	2725	
	99925	499625
	25	
	102675	41158864
5554	22216	
	10289716	41158864
		....

If a number be not a perfect cube, its root is extracted approximately, by adding as many periods of three ciphers as there are decimal figures required in the root.

Ex. 4. Find $\sqrt[3]{12}$.

		12(2.28
		8
	12	4000
62	124	
	1324	2648
	4	
	1452	1352000
668	5344	
	150544	1204352
		147648

341. After what has been said about the extraction of the square root of fractions and the extraction of the cube root of numbers, as well as the observations made upon the formation of the cube of fractions, no explanation is required to elucidate the following cases of the extraction of the cube root of both decimal and vulgar fractions.

Ex. 1. Extract $\sqrt[3]{280.001}$.

```
                                 280.001(6.54214
                                 216
               108 ..........    64001
      185 ...... 925 \
               11725  } ........ 58625
                  25 /
               12675 ..........    5376000
     1954 ....... 7816 \
             1275316    } ........ 5101264
                  16   /
             1283148 ..........     274736000
    19622 ......... 39244 \
           128354044      } ...... 256708088
                   4     /
           128393292 ..........       18027912000
   196261 ........... 196261 \
           12839525461       } ...... 12839525461
                     1      /
           12839721723 ..........       5188386529000
  1962634 ........... 7850536
           1283980022836 .........      5135920091344
                                          52466437656
```

Ex. 2. What is the cube root of $\frac{2}{3}$?

```
                                $\sqrt[3]{2/3}$ = ∛.666(.8735
                                      512
               192 ................. .. 154666
    247 ..............1729 \
                     20929  } ................. 146503
                        49 /
                     22707 .................        8163666
   2613................. 7839 \
                     2278539   } .................. 6835617
                           9  /
                     2285377                       1328049666
  26195................. 130975
                     228668075 ...............     1143343375
                                                    184706291.
```

Instead of reducing $\frac{2}{3}$ to a decimal, we might have multiplied both numerator and denominator by the square of the denominator, and the denominator becomes thus a perfect cube; for

$$\sqrt[3]{\frac{2}{3}}=\sqrt[3]{\frac{2\times 9}{3\times 9}}=\sqrt[3]{\frac{18}{3^3}}=\frac{\sqrt[3]{18}}{3}=\frac{2.6205}{3}=.8735.$$

342. EXERCISES.

1. Find the cubes of 548, 74302, 129458.
2. Express the cubes of $\frac{2}{7}$, $\frac{12}{25}$, $12\frac{2}{3}$, $3\frac{152}{241}$.
3. What are the cubes of .3, 2.006, .004, 34.005?
4. Extract the cube roots of 512, 140608, 11089567.
5. Determine the cube roots of $\frac{8}{125}$, $3\frac{3}{8}$, $1\frac{2402}{6859}$, $162\frac{378}{1331}$.
6. What are the approximate cube roots of 50, .653, 4258, .28, .00459, $\dot{0}3\dot{7}$, $1587.\dot{9}6\dot{2}$, 00000943.
7. What number is that whose cube root decreased by 3 is equal to 14?
8. In a train of 25 carriages there are 25 persons in each carriage, and each person carries £25, what sum of money do these people possess?
9. What number is that whose half, third, and fourth, multiplied together, the product is 9?
10. Find a number whose third multiplied by its square the product is 1944.
11. The third part of the cube of a number is 171307467, find that number.
12. When 137 is added to the cube of a number, the sum is 2334, what is the number?
13. What is the contents of a reservoir, 30.44 yards long, 20.21 yards broad, and 3.4 yards deep?
14. The product of a number by its square is 4.096, find the number.
15. What is the side of a cubical mound that will be equivalent to one which is 288 feet long, 216 feet broad, and 48 feet high?
16. What is the cube root of the square root of 346, correct to two decimal places?

17. What is the square root of the cube root of 3261, correct to .1?

18. What is the cube root of the cube root of 421634, correct to .1?

PROPORTIONS.

343. Two quantities are either equal or unequal to one another. In the latter case, when one is greater than the other, we may consider their inequality in two different points of view; we may either inquire how much one of the quantities is greater than the other, or how many times the one is contained in the other. Thus, when I say that between 12 and 4 the difference is 8, these numbers are considered with regard to their difference; and when I say that 4 is contained three times in 12, they are considered with regard to their quotient.

344. We shall now make a few observations upon the first connexion. When between any two numbers, such as 15 and 10, the difference is the same as between any two other numbers, 11 and 6; we say that 15 stands to 10 in the same respect as 11 stands to 6. These four quantities form an *arithmetical proportion*, or an *equidifference*, which is written thus: $15-10=11-6$; the first and last terms are called the *extremes*, and the second and third the *means*. Now, were we to add to each difference the sum $10+6$, or the subtracted parts, the results are $15+6$ and $11+10$. Therefore, *in every arithmetical proportion, the sum of the extremes is equal to the sum of the means.* And conversely, if $15+6=11+10$, we have an arithmetical proportion: $15-10=11-6$.

If three members or terms of an equidifference were given, the fourth could easily be found, for let the three first terms, 15, 10, 11 be given find the fourth x.

$\because$ x increased by $15=10+11$, $\therefore$ $x=10+11-15=6$, and thus we have the arithmetical proportion $15-10=11-6$.

Hence, if three terms of an arithmetical proportion be given, the fourth (if an extreme) will be found by subtracting from the sum of the means the extreme given; and if it be a mean, by subtracting from the sum of the extremes, the mean given.

To find x in the following arithmetical proportion: $36-21=49-x$.

$\because$ $x+36=21+49$, it follows that $x=21+49-36=34$; $\therefore$ $36-21=49-34$.

Likewise, in the arithmetical proportion $35-17=x-8$, we have $x+17=35+8$; $\therefore$ $x=35+8-17=26$, and $\therefore$ $35-17=26-8$.

If three quantities, as 25, 18, and 11, be in arithmetical proportion, then $25-18=18-11$; hence it follows, from the equality between the sum of the means and that of the extremes, that $2\times18=25+11$, or twice the *arithmetical mean*, is equal to the sum of the extremes; and $\therefore$ the arithmetical mean is equal to half their sum, $\frac{25+11}{2}=18$.

345. When we consider how many times one quantity is contained in another, or what part or parts one is of the other, such a relation is called a *ratio;* thus the ratio of 6 to 9 is written 6:9, or $\frac{6}{9}$. These numbers, thus compared, are called terms of the ratio, the former being the *antecedent* and the latter the *consequent.*

When one antecedent is the same multiple, or part of its consequent, as another antecedent is of its consequent. The ratios are equal, thus $\frac{6}{9}$ and $\frac{8}{12}$ are two equal ratios, since $\frac{6}{9}=\frac{8}{12}=\frac{2}{3}$. The four quantities which form two equal ratios, are said to be *proportionals*, or to determine a *geometrical proportion*, or an *equiquotient*, which is usually read, 6 is to 9 as 8 is to 12, and written $6:9=8:12$, or $\frac{6}{9}=\frac{8}{12}$. Here 6 and 8 are the antecedents, and 9 and 12 the consequents; also, 6 and 12 are the extremes, and 9 and 8 the means.

If we were to multiply two equal ratios, $\frac{6}{9}$ and $\frac{8}{12}$, by 9×12, the product of the denominators or consequents, we have on one part 6×12, and on the other 8×9. Therefore, *when four quantities are proportionals, the product of the extremes is equal to the product of the means.*

Conversely, if four quantities, 6, 9, 8, 12, are such that 6×12 and 8×9 are equal, they are proportionals, $6:9=8:12$, or $\frac{6}{9}=\frac{8}{12}$. Then a proportion may be formed with the factors of two equal products, the factors of one product being the extremes and those of the other the means.

If the first be to the second as the second to the third, the product of the extremes is equal to the square of the means; therefore, the *mean proportional*, or the *geometrical mean*, between two numbers is the square root of their product. The

mean proportional between 3 and 12 is $\sqrt{3\times 12}=6$, $\therefore \frac{3}{6}=\frac{6}{12}$. Conversely, if we have $6^2=3\times 12$, we infer a proportion $3:6=6:12$.

If a proportion contains one unknown term, as $6:9=8:x$, then $8\times 9=6\times x$, and we find $x=\frac{8\times 9}{6}=12$. $\therefore 6:9=8:12$. Then one of the extremes is found by dividing the product of the means by the other extreme, and one of the means is found by dividing the product of the extremes by the other mean.

346 Now, the four terms of a geometrical proportion may be transposed in several ways, without altering the proportion. The test of all these transformations will be that the product of the extremes is equal to that of the means.

$$\begin{array}{rll}
\text{Thus, since } 6:9=8:12, \text{ or} & \ldots\ldots\ldots\ldots & \frac{6}{9}=\frac{8}{12}, \\
\text{it follows that } 6:8=9:12 & \left.\begin{array}{l}\\ \\ \\ \end{array}\right\}\text{alternando} & \left\{\begin{array}{l}\frac{6}{8}=\frac{9}{12},\\ \frac{12}{9}=\frac{8}{6},\\ \frac{12}{8}=\frac{9}{6},\end{array}\right. \\
\text{or } 12:9=8:6 & & \\
\text{or } 12:8=9:6 & & \\
\text{or } 9:6=12:8 & \text{invertendo} & \frac{9}{6}=\frac{12}{8}, \\
\text{or } 6-9:9=8-12:12 & \text{dividendo} & \frac{6-9}{9}=\frac{8-12}{12}, \\
\text{or } 6+9:9=8+12:12 & \text{componendo} & \frac{6+9}{9}=\frac{8+12}{12}, \\
\text{or } 6:6\mp 9=8:8\mp 12 & \text{convertendo} & \frac{6}{6\mp 9}=\frac{8}{8\mp 12}, \\
\text{or } 6+9:6-9=8+12:8-12 & \text{miscendo} & \frac{6+9}{6-9}=\frac{8+12}{8-12}.
\end{array}$$

347. The corresponding terms of two or more proportions can be multiplied together, and the products will also be in proportion.

$$\begin{array}{r}
\text{Thus, if } 30:15=6:3, \text{ or } \frac{30}{15}=\frac{6}{3}; \\
\text{and } 2:3=4:6, \text{ or } \frac{2}{3}=\frac{4}{6}; \\
\therefore 30\times 2:15\times 3=6\times 4:3\times 6, \text{ or } \frac{30\times 2}{15\times 3}=\frac{6\times 4}{3\times 6}.
\end{array}$$

Therefore, the terms of a proportion may be raised to the square, cube, &c.; and also, the square root, cube root, &c., may be extracted, and the results will also be in proportion.

These properties of proportions are true for all numbers whatever.

348. In what has been said about the ratios of proportions, we have considered the quantities as abstract, and the ratios were likewise abstract; in the same manner do we find that the ratios of concrete quantities are abstract.

$$\text{Thus } 5 \text{ lbs.} : 7 \text{ lbs.} = 5 : 7, \text{ or } \frac{5 \text{ lbs.}}{7 \text{ lbs.}} = \tfrac{5}{7}.$$

$$12 \text{ yds.} : 8 \text{ yds.} = 12 : 8, \text{ or } \frac{12 \text{ yds.}}{8 \text{ yds.}} = \tfrac{12}{8} = \tfrac{3}{2}.$$

$$6 \text{ days} : 15 \text{ days} = 6 : 15, \text{ or } \frac{6 \text{ days}}{15 \text{ days}} = \tfrac{6}{15} = \tfrac{2}{5}.$$

It is scarcely necessary to mention that the quantities forming a ratio must be of the *same kind*, for it would be too absurd to attempt to compare £6 to 12 lbs. of cheese, or 12 gallons of beer to 20 days, &c.

349. EXERCISES.

To determine x in the following:—

1. $72 - 7 = 81 - x$.
2. $9\frac{3}{4} - 7\frac{1}{2} = 8\frac{3}{5} - x$.
3. $34\frac{3}{8} - x = x - 46\frac{2}{3}$.
4. $x - 35.5 = 16\frac{5}{6} - x$.
5. $\dot{3}41 - x = x - 12$.
6. $\dfrac{79\frac{1}{2}}{3\frac{2}{3}} - \dfrac{18\frac{5}{8}}{24\frac{1}{5}} = \dfrac{114\frac{1}{12}}{5\frac{7}{8}} - x$.
7. $8 : 9 = 46 ; x$.
8. $36 : 75 = x : 45$.
9. $\dfrac{245}{x} = \frac{124}{28}$.
10. $\dfrac{x}{56} = \frac{72}{96}$.
11. $7.8 : 31.3 = 4.84 : x$.
12. $.75 : x = 4.5 : .9$.
13. $\dfrac{9\frac{3}{4}}{16\frac{3}{8}} = \dfrac{24\frac{7}{9}}{x}$.
14. $\frac{7}{9} : x = \frac{7}{8} : \frac{4}{5}$.
15. $28\frac{4}{7} : 9\frac{1}{21} = 8.7 : x$.

16. $45.50 : .07 = \frac{4}{9} : x$.

17. $.704 : 3.52 = 6.5 : x$.

18. $\frac{34.56}{63} : 7\frac{1}{9} = \frac{3\frac{1}{7}}{.64} : x$.

19. $\frac{5\frac{1}{3} \times 12}{\frac{8}{9}} : 55.75 = \frac{3\frac{2}{11}}{.36} : x$.

20. $\frac{x \times 3.5}{49} : \frac{.54 \times 3\frac{1}{3}}{35} = 2\frac{1}{3} : \frac{12.4}{5}$.

21. $47 : \frac{3}{4} \times x = \frac{2}{3} : 49$.

PROGRESSIONS.

350. It has been shown that numbers connected thus:

$$3—8 = 8—13, \text{ or } 3—8—13;$$
$$\text{and } 5:10 = 10 : 20, \text{ or } 5:10 : 20,$$

are in proportion, viz, that the third term bears the same relation to the second as the second to the first. We can easily conceive a fourth term having the same relation to the third as the third has to the second; and also a fifth term having the same relation to the fourth as the fourth has to the third, &c; and the given proportions extended become:—

3—8—13—18—23—28—33—38—43—48—53—58
5 : 10 : 20 : 40 : 80 : 160 : 320 : 640 : 1280 : 2560 : 5120 : 10240

Such continued proportions are termed *progressions;* the first row constitutes an *arithmetical progression*, or an *equidifferent series*, and the second a *geometrical progression*, or an *equiquotient series*.

351. Since the terms of a progression may either increase or diminish by a common difference or by a common ratio, there are *increasing* and *decreasing* arithmetical progressions, and *increasing* and *decreasing* geometrical progressions.

Every increasing series may be converted into a decreasing one by inverting the terms, viz., by making the last term the first, &c.

It is evident that every progression can be supposed continued *sine fine*, for to every term a succeeding one may always be found.

ARITHMETICAL PROGRESSIONS, OR EQUIDIFFERENT SERIES.

352. We have shown that when a series of numbers increase or decrease by the same quantity, those numbers are said to be in arithmetical progression.

Thus 3, 6, 9 12, 15, 18, &c., or 21, 17, 13, 9, 5. 1,—3,—7, &c.

It must be observed that any term is composed of the preceding term, plus or minus the common difference. For instance, 6 consists of the first term 3 and of the common difference 3; 9 consists of the second term 6 and of the common difference 3; similarly, 17 consists of the first term 21, minus the common difference 4; 13 consists of 17 minus 4, &c.

353. The natural series of numbers, 1, 2, 3, 4, 5, 6, &c., determine an arithmetical progression, the first term of which is 1, and the common difference 1.

The even numbers 2, 4, 6, 8, 10, &c., as well as the uneven numbers 1, 3, 5, 7, 9, &c., constitute an arithmetical progression, the common difference being 2.

The series of numbers, 10, 20, 30, 40, &c., form an arithmetical progression, having 10 for its common difference.

354. From what has been mentioned on the formation of the terms of an arithmetical progression, we are enabled to determine any term of such a series. For an increasing arithmetical progression we have:

The 1st term $=3$.
,, 2nd ,, $=3+3$, or $3+1\times3$, or 6.
,, 3rd ,, $=3+3+3$, or $3+2\times3$, or 9.
,, 4th ,, $=3+3+3+3$, or $3+3\times3$, or 12.
,, 5th ,, $=3+3+3+3+3$, or $3+4\times3$, or 15.
,, 6th ,, $=3+3+3+3+3+3$, or $3+5\times3$, or 18.
,, 7th ,, $=3+3+3+3+3+3+3$, or $3+6\times3$, or 21.

Likewise, for a decreasing series, we have:

The 1st term $=21$.
,, 2nd ,, $=21-4$, or $21-1\times4$, or 17.
,, 3rd ,, $=21-4-4$, or $21-2\times4$, or 13.
,, 4th ,, $=21-4-4-4$, or $21-3\times4$, or 9.
,, 5th ,, $=21-4-4-4-4$, or $21-4\times4$, or 5.
,, 6th ,, $=21-4-4-4-4-4$, or $21-5\times4$, or 1.
,, 7th ,, $=21-4-4-4-4-4-4$, or $21-6\times4$, or -3.

It follows, then, that any term consists of the first plus or minus (according as the equidifference is increasing or decreasing) the common difference, multiplied by the number of terms preceding the one required. The seventh term is composed of the first plus, or minus six times the common difference.

355. Let us represent the first term of an A.P. by a, and the common difference by d, then the increasing series is:

1st, 2nd, 3rd, 4th. 5th, 6th, 7th nth term.
$a,\ a+d,\ a+2d,\ a+3d,\ a+4d,\ a+5d,\ a+6d \ldots a+(n-1)d.$

And the decreasing series is:

1st, 2nd, 3rd, 4th, 5th, 6th, 7th, nth term.
$a,\ a-d,\ a-2d,\ a-3d,\ a-4d,\ a-5d,\ a-6d \ldots a-(n-1)d.$

Which increasing and decreasing series may be thus written:

1st, 2nd, 3rd, 4th, 5th, 6th, 7th, nth term.
$a,\ a\pm d,\ a\pm 2d,\ a\pm 3d,\ a\pm 4d,\ a\pm 5d,\ a\pm 6d \ldots a\pm(n-1)d.$

Where it is seen that any term is found by adding to or subtracting from the first term, the common difference, multiplied by the number of terms which denote the place in the series, minus 1.

356 If l stands for the last or nth term of a series, then

$$l=a\pm(n-1)d.$$

Ex. Let the first term of an increasing series be 4, and the common difference 5, then the 24th term of the A.P. $=4+23\times5=119$. The 50th term $=4+49\times5=249$.

Or, if in a decreasing A.P. the first term be 248, and the common difference 2, then the 13th term $=248-12\times2=224$. The hundredth term $=248-99\times2=50$.

357. In the expression $l=a\pm(n-1)d$, which represents the nth term of an A.P., let us subtract a from each side of the sign of equality of the increasing series, then:

$$\begin{array}{l} l=a+(n-1)d. \\ \underline{a=a\qquad\qquad} \\ l-a=(n-1)\,d \end{array}$$, and then dividing these equal remainders by $(n-1)$ we have $\dfrac{l-a}{n-1}=d.$

Hence the common difference is equal to the last term, minus the first, divided by the number of terms, minus 1.

This is evident, since the last term consists of the sum of the first term, and a certain number of times the common difference; $l—a$, expresses that certain number of times the common difference, and this remainder being divided by the number the common difference is multiplied by, the quotient is necessarily the common difference.

Again, if in a decreasing progression we have $l=a—(n—1)\ d$, adding to each side of the sign of equality $(n—i)\ d$, then $(n—1)\ d+l=a$, from which l being subtracted from these equal sums, we have $(n—1)\ d=a—l$, and dividing these equal remainders by $(n—1)$, it is found that $d=\frac{a—l}{n—1}$

Here the common difference is equal to the first term minus the last, divided by the number of terms minus 1.

Example 1. Suppose an A.P. of 14 terms, in which the first term $a=3$, and the last $l=55$, it is required to find the common difference d.

$$d=\frac{55—3}{14—1}=\frac{52}{13}=4,$$

and the A.P. is 3, 7, 11, 15, 19.........55.

Ex. 2. What is the common difference of a series consisting of 10 terms, the first term is 11 and the last 4?

$$d=\frac{11—4}{10—1}=\frac{7}{9}$$

and the equi-difference is 11, $10\frac{2}{9}$, $4\frac{4}{9}$, $8\frac{6}{9}$, $7\frac{8}{9}$.........4.

358. In a similar manner do we determine n, the number of terms, by means of a, the first term, l the last, and d the common difference.

As before, $l=a+(n—1)\ d$.

Let a be subtracted from each side of the sign of equality, then $l—a=(n—1)\ d$, dividing both equal remainders by d, it follows that $\frac{l—a}{d}=n—1$, and adding 1 to both equal quantities, we have $1+\frac{l—a}{d}=n$.

Hence the number of terms is found by dividing the last term, minus the first, by the common difference and adding 1 to the quotient. For taking away the first term from the last, the remainder is the common difference multiplied by a certain number, which being divided by the common difference, the quotient is the

number by which the common difference was multiplied; this number is 1 less than the number of terms, therefore 1 must be added to the quotient.

For a decreasing A.P. we have: $l=a-(n-1)\,d$, adding to both sides of the sign of equality $(n-1)\,d$.

$\therefore\ l+(n-1)\,d=a$, substracting from both sides of the sign of equality l, we have $(n-1)\,d=a-l$, which being divided by d, gives $n-1=\frac{a-l}{d}$, and adding to both sides 1,

$$\therefore\ n=\frac{a-l}{d}+1.$$

Hence the number of terms is found, &c.

359. If we were to subtract $(n-1)\,d$ from both sides of $l=a+(n-1)\,d$, we have $l-(n-1)\,d=a$.

Therefore the first term of an A.P. = the last term, minus the product, of the common difference and the number of terms less 1.

This truth is rendered evident when we consider that the last term consists of two parts, the first term and the product of the common difference and the number of terms minus 1; then subtracting the second part, the remainder is the first term.

When the A.P. is decreasing, we have:
$l=a-(n-1)\,d$, and adding $(n-1)\,d$ to each side, $l+(n-1)\,d=a$.

Hence the first term is found, &c.

Example 1. The last term is 53, the common difference 3, and the number of terms 18 Required the first term.

$$a=53-(18-1)3=2.$$

Ex. 2. The last term of an A.P. is 5, the common difference 2, and the number of terms 52. What is the first term?

$$a=5+(52-1)2=107.$$

360. An important case in A.P. is to find the sum of the terms; the operation of adding all the successive terms together would be very tedious; we shall now determine a method by means of which the sum of the terms of any A.P. can at once be found.

Let it be required to ascertain the sum of the terms of the following A.P.:—

$$3,\ 7,\ 11,\ 15,\ 19,\ 23,\ 27,\ 31,\ 35,\ 39.$$

In order to determine the sum of the terms, let us write the

progression, both as an increasing and decreasing series, setting the terms in order, and adding together the corresponding ones, as follows :—

Increasing series...	3, 7, 11, 15, 19, 23, 27, 31, 35, 39.
Decreasing series...	39, 35, 31, 27, 23, 19, 15, 11, 7, 3.
	42, 42, 42, 42, 42, 42, 42, 42, 42, 42.

We obtain every term 42, as the sum of two terms, and this sum is repeated as many times as there are terms in the progression; therefore $10\times 42=420$ is the sum of both series, or twice the sum of the terms of the proposed progression. Whence the sum required is $\frac{10\times 42}{2}=210$.

Let us proceed in a similar manner with any A.P. :—

INCREASING SERIES.

$a,\quad a+d,\quad a+2d,\quad a+3d,\quad a+4d,\quad a+5d,\quad a+6d,\quad a+7d.$

DECREASING SERIES.

$a+7d,\quad a+6d,\quad a+5d,\quad a+4d,\quad a+3d,\quad a+2d,\quad a+d,\quad a.$

$2a+7d, 2a+7d, 2a+7d, 2a+7d, 2a+7d, 2a+7d, 2a+7d, 2a+7d.$

In the first series every following term increases by d, whilst in the second it diminishes by d, The sum of the terms of both series is $8\times(2a+7d)$, or the number of terms in one progression multiplied by the sum of any two corresponding terms; therefore the sum of the proposed series $=\frac{8\times(2a+7d)}{2}$.

Were we to take the number of terms to be n, we should have :

INCREASING SERIES.

$a,\quad a+d,\quad a+2d,\quad a+3d \ldots a+(n-2)d,\quad a+(n-1)d.$

DECREASING SERIES.

$a+(n-1)d,\quad a+(n-2)d,\quad a+(n-3)d,\quad a+(n-4)d, \ldots\ldots a+d.\ a.$

$2a+(n-1)d,\quad 2a+(n-1)d,$ &c. .... $2a+(n-1)d.$

The sum of every two corresponding terms is $2a+(n-1)d$, which quantity being repeated n times, we have, supposing s to represent the sum of the terms of one series, $2s=n\left\{\frac{2a+(n-1)d}{2}\right\}$ for the sum of both series; therefore the sum of the terms of any series is $s=n\left\{\frac{2a+(n-1)d}{2}\right\}$. From which is derived the following rule:

The sum of the terms of any A.P. is found by multiplying half the sum of the first and last terms by the number of terms in the series.

361. In this progression: 3, 7, 11, 15, 19, 23, 27, 31, 35, 39, we may observe that the sum of the first and last term is 42; the sum of the second and last but one is likewise 42; of the third and eighth; of the fourth and seventh, and so on of any two terms equally distant from the first and last.

Also for any arithmetical series;

$$a,\ a+d,\ a+2d,\ a+3d,\ a+4d \ldots\ldots a+(n-4)d,\ a+(n-3)d,\ a+(n-2)d,\ a+(n-1)d.$$

The sum of the first and last term is equal to the sum of the second and last but one, &c., viz., $2a+(n-1)d$.

The reason of this is manifest from what has been said, for, we find the sum of two terms, as just mentioned, to be the same as adding together the two corresponding terms of an increasing and decreasing series.

Therefore, if the sum of the first and last terms of a series, $a+l$, be multiplied by $\frac{n}{2}$, half the number of terms, we obtain the sum of the terms of the series; hence $s=(a+l)\frac{n}{2}=\frac{(a+l)n}{2}=\frac{\{2a+(n-1)d\}n}{2}$.

Ex. 1. In an arithmetical series, the first term $a=4$, the common difference $d=5$, and the number of terms $n=36$, find the sum s.

$$s=\frac{\{2\times 4+(36-1)5\}36}{2}=\frac{(8+175)36}{2}=263\times 18=4734.$$

Ex. 2. Given the first term $a=3$, the last term $l=36$, and the number of terms $n=18$, to find the sum of the A.P.:

$$s=\frac{(3+36)18}{2}=39\times 9=351.$$

362. In the following propositions will be found the different cases which present themselves in A.P., they are deductions from what has been explained:

1. Given a, d, n, to find l. *Ans.* $l=a+(n-1)d$.
2. Given a, d, s, to find l. *Ans.* $l=-\frac{d}{2}\pm\sqrt{2ds+\left(a-\frac{d}{2}\right)^2}$

3. Given a, n, s, to find l. *Ans.* $l=\frac{2s}{n}-a.$

4. Given d, n, s, to find l. *Ans.* $l=\frac{s}{n}+\frac{(n-1)d.}{2}$

5. Given a, d, n, to find s. *Ans.* $s=\left\{2a+(n-1)d\right\}\frac{n}{2}$

6. Given a, d, l, to find s. *Ans.* $s=\frac{a+l}{2}+\frac{a+l(l-a).}{2d.}$

7. Given a, n, l, to find s. *Ans.* $s.=(a+l)\frac{n}{2}.$

8. Given d, n, l, to find s. *Ans.* $s.=\left\{2l-(n-1)d\right\}\frac{n}{2}.$

9. Given a, n, l, to find d. *Ans.* $d=\frac{l-a}{n-1}.$

10. Given a, n, s, to find d. *Ans.* $d=\frac{2(s-an)}{n(n-1)}.$

11. Given a, l, s, to find d. *Ans.* $d=\frac{(l+a)\ (l-a)}{2s-l-a}.$

12. Given n, l, s, to find d. *Ans.* $d=\frac{2(nl-s)}{n(n-1}.$

13. Given a, d, l, to find n. *Ans.* $n=1+\frac{l-a}{d}.$

14. Given a, d, s, to find n. *A.* $n=\frac{d-2a}{2d}\pm\sqrt{\frac{2s}{d}+\left(\frac{2a-d}{2d}\right)^2}$

15. Given a, l, s, to find n. *Ans.* $n=\frac{2s}{a+l}.$

16. Given d, l, s, to find n. *A.* $n=\frac{2l+d}{2d}\pm\sqrt{\left(\frac{2l+d}{2d}\right)^2-\frac{2s}{d}}$

17. Given d, n, l, to find a. *Ans.* $a=l-(n-1)d.$

18. Given d, n, s, to find a. *Ans.* $a=\frac{s}{n}-\frac{n-1}{2}d.$

19. Given d, l, s, to find a. *Ans.* $a=\frac{d}{2}\pm\sqrt{\left(l+\frac{d}{2}\right)^2-2ds.}$

20. Given n, l, s, to find a. *Ans.* $a=\frac{2s}{n}-l.$

363. EXERCISES.

1. A debt was discharged by forty weekly payments in A.P., the first payment was £3 and the last £10. What is the common difference and the amount of the debt?

2. A gentleman bought a horse upon this condition, that for the first nail in its shoes he should pay 6d., for the second 10d., for the third 14d., and so on for every succeeding nail. Now the number of nails being 32, find the price of the horse.

3. A person travelled from Worksop to London, the distance being 148 miles. The first day he went 12 miles, and the last he went 25 miles, increasing his speed regularly every day. How long was he on his journey?

4. Suppose 100 stones were placed in a right line, a yard asunder and the first a yard from a basket, what distance will a man have travelled after he has brought them one by one to the basket?

5. Two persons, A and B, travel to meet each other. A goes at the uniform rate of 10 miles per hour. B goes 5 miles the first hour, and increases his speed 1 mile every hour. When they meet it is found that both have travelled the same distance. How many hours were they travelling, and how far were the two starting places from each other?

6. A workman served his master for 18 years. For the first year he received £20, and every successive year his salary was raised £3. What was the amount of his wages, and what did he receive for the last year?

7. What is the sum of all the numbers from 1 to 1000, both included?

8. A body falling in vacuo descends in the first second of time through a space of $16\frac{1}{12}$ feet, and in every successive second through $32\frac{1}{6}$ feet more. Through what space would it fall in 40 seconds, and how much would it descend the last second?

9. It is required to find eight arithmetical means between 1 and $1\frac{1}{2}$?

10. Twelve points being taken on the circumference of a circle, it is required to determine the greatest number of straight lines by which these points can be connected.

11. A carter takes 56 loads of stones, which are unloaded in as many heaps, equally distant from each other, on a road 1120

yards in length, the first heap is 250 yards from the quarry. What is the distance gone over by the carter in the performance of this undertaking?

GEOMETRICAL PROGRESSION, OR EQUIQUOTIENT SERIES.

364. A series of numbers, the terms of which are continually increased or diminished by a multiplication or a division, is said to be in geometrical progression. Thus:

1 5 25 125 625 3125 15625 78125, &c.

4096 1024 256 64 16 4 1 $\frac{1}{4}$ $\frac{1}{16}$ $\frac{1}{64}$, &c.

The first series being an increasing and the second a decreasing G.P. The terms of the former series are found by multiplying successively by 5, and those of the latter by dividing successively by 4. This constant multiplier or divisor is termed the ratio.

If it be noticed that every term is deduced from the preceding one by multiplying it by the ratio, these progressions might be put under the form of:

$$1,\ 1\times 5,\ 1\times 5^2,\ 1\times 5^3,\ 1\times 5^4,\ 1\times 5^5,\ 1\times 5^6,\ 1\times 5^7, \text{\&c.}$$

$$1\times 4^6,\ 1\times 4^5,\ 1\times 4^4,\ 1\times 4^3,\ 1\times 4^2,\ 1\times 4^1,\ 1,\ \frac{1}{4},\ \frac{1}{4^2},\ \frac{1}{4^3}, \text{\&c.}$$

Or if the first term be expressed by a, the ratio by r, the number of terms by n, the geometrical series is represented thus:

1st term	2nd	3rd	4th	5th	6th	7th	8th	nth
a,	ar,	ar^2,	ar^3,	ar^4,	ar^5,	ar^6,	ar^7 ..	ar^{n-1}.

Here is plainly exhibited how any term of a G.P. is found by means of the first term and the ratio, viz., every term is the product of the first term by the ratio raised to the power of the number of terms less one. Thus the sixth term is the product of the first and of the ratio raised to the fifth power; the twelfth term is the product of the first by the ratio raised to the eleventh power; the last term, when the number of terms of the series is n, is the product of the first term by the ratio raised to the $(n-1)$ power.

Suppose l to express the last term, then: $l = ar^{n-1}$.

365. An important case in G P. is to determine the sum of the terms of the series. For this purpose let the terms of the G.P. be:

1, 5. 25, 125, 625, 3125, 15625, &c.

Multiplying by the ratio 5, we obtain :

5, 25, 125, 625, 3125, 15625, 78125, &c.

Subtracting the first series from the second, there remains 78125—1, which is equal to four times the sum of the terms of the proposed series, or to $4s$ (s representing the sum of the terms of the given G.P.)

$$\therefore s = \frac{78125-1}{4} = \frac{78124}{4} = 19531.$$

The same reasoning will apply to any other series, thus:

$a,\ ar,\ ar^2,\ ar^3,\ ar^4,\ ar^5 \ldots\ldots\ldots ar^{n-2},\ ar^{n-1}$; multiplying by r,

$ar,\ ar^2,\ ar^3,\ ar^4,\ ar^5 \ldots\ldots ar^{n-2},\ ar^{n-1},\ ar^n.$

And the first series being taken from the second, leaves:

$$sr-r=s(r-1)=ar^n-a.$$

Now, dividing each side of this equality by $(r-1)$, there remains :

$$s=\frac{ar^n-a}{r-1}=\frac{a(r^n-1)}{r-1}.$$

Hence the sum of the terms of any geometrical series may be found by multiplying the first term by the ratio raised to the power of the number of terms, and dividing the difference between this product, and the first term by the ratio, less 1.

366. The several cases which may offer themselves in G.P. are presented in the following table; they are deductions from what has been investigated :

1. Given a, r, n, to find l. Ans. $l=ar^{n-1}$.

2. Given a, r, s, to find l. Ans. $l=\dfrac{a+(r-1)s}{r}$.

3. Given a, n, s, to find l. Ans. $l(s-l)^{n-1}=a(s-a)^{n-1}$.

4. Given r, n, s, to find l. Ans. $l=\dfrac{(r-1)sr^{n-1}}{r^n-1}$.

5. Given a, r, n, to find s. Ans. $s=\dfrac{a(r^n-1)}{r-1}$.

6. Given a, r, l, to find s. Ans. $s=\dfrac{rl-a}{r-1}$.

7. Given a, n, l, to find s. Ans. $s=\dfrac{l^{\frac{n}{n-1}}-a^{\frac{n}{n-1}}}{l^{\frac{1}{n-1}}-a^{\frac{1}{n-1}}}$.

8. Given r, n, l, to find s. Ans. $s=\dfrac{l(r^n-1)}{(r-1)r^{n-1}}$.

9. Given r, n, l, to find a. Ans. $a=\dfrac{l}{r^{n-1}}$.

10. Given r, n, s, to find a. Ans. $a=\dfrac{(r-1)s}{r^n-1}$.

11. Given r, l, s, to find a. Ans. $a=rl-(r-1)s$.

12. Given n, l, s, to find a. Ans. $a(s-a)^{n-1}=l(s-l)^{n-1}$.

13. Given a, n, l, to find r. Ans. $r=\left(\dfrac{l}{a}\right)^{\frac{1}{n-1}}$.

14. Given a, n, s, to find r. Ans. $r^n-\dfrac{rs}{a}=\dfrac{a-s}{a}$.

15. Given a, l, s, to find r. Ans. $r=\dfrac{s-a}{s-l}$.

16. Given n, l, s, to find r. Ans. $r^n+\dfrac{sr^{n-1}}{l-s}=\dfrac{l}{l-s}$.

17. Given a, r, l, to find n. Ans. $n=\dfrac{\log l-\log a}{\log r}+1$.

18. Given a, r, s, to find n. Ans. $n=\dfrac{\log\{s(r-1)+a\}-\log a}{\log r}$.

19. Given a, l, s, to find n. Ans. $n=1+\dfrac{\log l-\log a}{\log(s-a)-\log(s-l)}$.

20. Given r, l, s, to find n. Ans. $n=\dfrac{\log l-\log\{rl-(r-1)s\}}{\log r}+1$.

367. Ex. 1. The first term of a G.P. is 1, the last 128, and the sum 255. Find the ratio.

It follows that $r=\dfrac{s-a}{s-l}=\dfrac{255-1}{255-128}=\dfrac{254}{127}=2$.

Ex. 2. What is the twelfth term of a G.P., if the first be 4, and the ratio 2,

We have: $l = ar^{n-1} = 4 \times 2^{11} = 8192$.

Ex. 3. What is the sum of this series: $1\frac{1}{2}$, $2\frac{1}{4}$, $3\frac{3}{8}$, &c., to 10 terms?

Here $r = \dfrac{3\frac{3}{8}}{2\frac{1}{4}} = \dfrac{3}{2}$, $\therefore s = \dfrac{a(r^n-1)}{r-1} = \dfrac{1\frac{1}{2}((\frac{3}{2})^{10}-1)}{\frac{3}{2}-1} = \dfrac{3\{(\frac{3}{2})^{10}-1\}}{1}$ 169.995.

Ex. 4. The first term of a G.P. is 1, the last 65536, and the ratio 4. Find the sum of the series.

We have: $s = \dfrac{rl-a}{r-1} = \dfrac{4 \times 65536 - 1}{4-1} = 87380$.

Ex. 5. The sum of 12 terms of the series 64, 16, 4, &c., is required.

Here $r = \frac{1}{4}$, $\therefore\ s = \dfrac{a(r^n-1)}{r-1} = \dfrac{64\{(\frac{1}{4})^{12}-1\}}{\frac{1}{4}-1} = \dfrac{4^3(1-4^{12})}{4^{12}\left(\frac{1-4}{4}\right)} =$

$\dfrac{4^3(4^{12}-1)}{4^{11}(4-1)} = \dfrac{4^{12}-1}{4^8 \times 3} = 85.333328$.

Ex. 6. Insert five geometrical means, between 7 and 448.

By the question, $a = 7$, $l = 448$, $n = 2+5 = 7$.

$\therefore\ r = \left(\dfrac{l}{a}\right)^{\frac{1}{n-1}} = \left(\dfrac{448}{7}\right)^{\frac{1}{6}} = 64^{\frac{1}{6}} = 2$.

Then the five means are: 14, 28, 56, 112, and 224.

Ex. 7. Find the sum of the series $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$, &c., *ad infinitum.*

Here we have to use the expression $s = \dfrac{ar^n - a}{r-1}$; but it must be observed that the number of terms n being without limit, and r a proper fraction, the value r^n or ar^n, the last term of the series becomes less than any quantity that can be assigned, and, therefore, may be considered as nothing; and the value $\dfrac{ar^n-a}{r-1}$ or $\dfrac{ar^n}{r-1} - \dfrac{a}{r-1}$, becomes $0 - \dfrac{a}{r-1}$ or $\dfrac{a}{1-r}$. This quantity is the limit to which the sum of the terms converges, and is the true expression of the sum of the terms of a series, continued *sine fine.* Hence $s = \dfrac{a}{1-r} = \dfrac{\frac{3}{10}}{1-\frac{1}{10}} = \frac{3}{9}$ or $\frac{1}{3}$.

A question, comprising this example, might be worded thus: if a body were propelled by a force which moved it $\frac{3}{10}$ of a mile

in the first second of time, $\frac{3}{100}$ of a mile in the second, $\frac{3}{1000}$ of a mile in the third, &c., for ever, it would only move $\frac{1}{3}$ of a mile during all eternity.

368. These infinite series are applied with advantage to the reduction of circulating decimals.

Ex. 1. Required the vulgar fraction which is equivalent to $.\dot{3}$.

This decimal may be represented by the geometrical series .3, .03, .003, &c.; hence $.\dot{3}=\frac{.3}{1-.1}=\frac{3}{9}=\frac{1}{3}$.

Ex. 2. Find the value of $.\dot{3}\dot{2}$.

Here the series is $\frac{32}{10^2}, \frac{32}{10^4}, \frac{32}{10^6}$, &c : then $a=\frac{32}{10^2}, r=\frac{1}{10^2}$.

Therefore, $.\dot{3}\dot{2}=\dfrac{\frac{32}{10^2}}{1-\frac{1}{10^2}}=\frac{32}{99}$.

Ex. 3. Find the value of the infinite decimal $.51\dot{8}\dot{5}$.

The series of fractions representing the value of the decimal are $\frac{51}{100}$ + the G.P. $\frac{85}{10000}, \frac{85}{1000000}$, &c.; and the sum of the G.P. is

$$s=\frac{\frac{85}{10000}}{1-\frac{1}{100}}=\frac{85}{10000-100}=\frac{85}{9900}$$

$$\therefore\ .51\dot{8}\dot{5}=\frac{51}{100}+\frac{85}{9900}=\frac{5049+85}{9900}=\frac{14}{27}\text{nearly.}$$

369. EXERCISES.

1. What would the price of a horse be, which is sold at 1 farthing for the first nail, 2 for the second, 4 for the third, &c., allowing 8 nails in each shoe?

2. Sessa, an Indian, having first discovered the game of chess, showed it to his Prince, Sheram, who was so delighted with the invention that he bid him ask what he would require as a reward for his ingenuity; upon which Sessa requested that he might be allowed one grain of wheat for the first square, two for the second, four for the third, &c., doubling continually to 64, the whole number of squares. Now, supposing a pint to contain 7680 of these grains, it is required to find what number of ships, each carrying 1000 tons burden,

might be freighted with the produce, allowing 40 bushels to a ton; also what would be the value of the corn, at £1. 7s. 6d. per quarter.

3. A puts 6d. into a lottery, which being lost, he risks now 1s. 6d., this being likewise lost, he risks 4s. 6d.; now this process he repeated 11 times. How much must he win to recover all that he risked?

4. A charitable person gave alms to 10 poor people, each received twice as much as the one preceding; the tenth got £2. 4s. What did the first receive, and how much was distributed altogether?

5. A ball is discharged by a force which carries it 10 miles in the first minute, 9 miles in the second, and so on, in the ratio of $\frac{9}{10}$ for ever. What distance would it go?

6. Required the value of the circulating decimal $.\dot{7}$.

7. Find the sum of the series $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, &c., *ad infinitum.*

8. Find the value of $.95\dot{6}$.

9. What debt can be discharged in a year by monthly payments, giving 4s. 2d. the first month, and three times as much every succeeding month?

370. An important application of G.P. is seen in the computation of compound interest (See Art. 283.). Let p represent any principal, R the rate per cent.; then the interest at the end of first year is $p \times \frac{R}{100}$, and the amount at the end of that time is $p+\frac{pR}{100}=p\left(\frac{100+R}{100}\right)$ let this quantity $=p'$; the interest at the end of second year is $p' \times \frac{R}{100}$, and the amount at that time $= p' + \frac{p'R}{100} = p'\left(\frac{100+R}{100}\right) = p\left(\frac{100+R}{100}\right)\left(\frac{100\times R}{100}\right) = p\left(\frac{100+R}{100}\right)^2$, which quantity let p'' represent; the interest at the end of the third year is $p'' \times \frac{R}{100}$, and the amount at that time $= p'' + \frac{p''R}{100} = p''\left(\frac{100+R}{100}\right) = p\left(\frac{100+R}{100}\right)^2 \times \left(\frac{100+R}{100}\right) =$

$p\left(\frac{100+R}{100}\right)^3$, let this quantity $=p'''$; the interest at the end of the fourth year is $p'''\frac{R}{100}$ and the amount then $=p'''+p'''\frac{R}{100}=$ $p'''\left(\frac{100+R}{100}\right)=p\left(\frac{100+R}{100}\right)\times\left(\frac{100+R}{100}\right)^3=p\left(\frac{100+R}{100}\right)^4$.

Suppose the quantity $\frac{100+R}{100}=r$, the principal $=a$, the amount of the given principal $=l$, and the number of years n; therefore $l=ar^n$, and it follows that $a=\frac{l}{r^n}$, $r^n=\frac{l}{a}$ or $r=\left(\frac{l}{a}\right)^{\frac{1}{n}}$ $n=\frac{\log l-\log a}{\log r}$.

371. EXERCISES.

1. What is the amount of £3,600, at 3 per cent., for 80 years, compound interest?
2. How many years must £6000 be lent, at 6 per cent. compound interest to amount to £34461. 4s.?
3. At what rate must £1000 be lent so as to amount to £1675 in 6 years?

PERMUTATIONS AND COMBINATIONS.

372. If it were required to ascertain how many positions two persons, a and b, can take, with regard to their order, we should find that a may be on the right of b or on his left; thus they can take two different positions. Also taking two persons out of three, a, b, c, we should find that they may be placed, with regard to their order, in the following different ways:

$$ab,\quad ba,\quad ac,\quad ca,\quad bc,\quad cb.$$

And if taken all three together, they may be arranged thus:

$$abc,\quad acb,\quad bac,\quad bca,\quad cab,\quad cba.$$

These different orders in which any number of persons or things can be arranged, are called *permutations*.

373. The permutations of two things, a and b, are ab, ba, viz., 1×2 or 2.

To find the number of permutations of three things a, b, c, it must be observed that a may remain first, while $b\,c$ change; thus we have $a\ b\ c$, $a\ c\ b$; likewise b and c may remain first while the others change, so we have $1\times2\times3=6$ permutations.

In the permutations of four things, we notice likewise that every one thing may be first, while the other three change; and as three things give $1\times2\times3$ permutations, we shall have $1\times2\times3\times4$, or 24 permutations of four things.

Similarly with five things; every one may be first, while the four others change. Thus we obtain as the number of permutations of five things:

$$1\times2\times3\times4\times5 \text{ or } 120.$$

The permutations of six things, taken all together, are:

$$1\times2\times3\times4\times5\times6=720.$$

The law which is here perceived may be generalised thus:

To find the number of permutations of n things, taken all together, we have $1\times2\times3\times4\times\ldots\ldots(n-3)\,(n-2)\,(n-1)n$ for the required number.

Ex. 1. Required the number of different ways in which 10 persons can be placed round a table.

Ex. 2. How many changes can be rung on 12 bells? and how long would they take in ringing once over, supposing 10 changes might be rung in a minute, and the year to consist of $365\frac{1}{4}$ days.

Ex. 3. How many different ways can the 7 notes in music be varied?

374. Let us now determine how many permutations can be found out of a certain number of things, by taking two and two, three and three together, &c. We shall find that the premutations of six things, a, b, c, d, e, f, taken two together, are with

a	as first letter	ab,	ac,	ad,	ae,	af,
b	"	ba,	bc,	bd,	be,	bf,
c	"	ca,	cb,	cd,	ce,	cf,

and so on; and the whole number is $6\times5=30$.

When three are taken together:

ab,	as first, gives	abc,	abd,	abe,	abf,
ac,	"	acb,	acd,	ace,	acf,
ad,	"	adb,	adc,	ade,	adf.

Thus every permutation of two letters gives 4 of three letters, and as there are 6×5 changes of two letters, we have $6\times5\times4$ or 120 permutations of three letters.

If four be taken together:

a b c	as first, gives	*abcd*,	*abce*,	*abcf*,
a b d	,,	*abdc*,	*abde*,	*abdf*,

Then each permutation of three gives 3 changes, and $6\times5\times4\times3$ or 360 expresses the number of permutations of six things, four together.

Similarly the number of permutations of five things out of six is $6\times5\times4\times3\times2$ or 720.

It follows that if 12 different things be given, out of which 6 are taken, the number of changes they form is $12\times11\times10\times9$ 8×7, or 6652880.

Then generally the number of permutations of n things, taken

Two together, are		$n\,(n-1)$
Three	"	$n\,(n-1)\,(n-2)$
Fonr	"	$n\,(n-1)\,(n-2)\,(n-3)$
Five	"	$n\,(n-1)\,(n-2)\,(n-3)\,(n-4)$
$\therefore p$	"	$n\,(n-1)\,(n-2)\,(n-3)\,(n-4)\ldots\ldots(n-p+1)$.

The pupil will do well to express this law in correct language.

Ex. 1. How many changes can be made by taking 4 digits out of the nine?

Ex. 2. How many changes can be rung on 6 bells, out of 12?

Ex. 3. How many permutations can be made with 4 letters out of the 26 which compose the alphabet?

375. We shall now inquire how the permutations decrease, when some of the things have the same letter. Suppose two letters are given, and both alike, then the two changes are reduced to one; therefore, $\frac{1\times2}{1\times2}$, or 1, is equal to the expression when both letters are the same. If three letters be alike, the six permutations are reduced to one; therefore, $\frac{1\times2\times3}{1\times2\times3}$, or 1, is the expression of the changes when the three letters are the same. For four letters alike, we have: $\frac{1\times2\times3\times4}{1\times2\times3\times4}$, or 1, and so on.

Now, to ascertain the permutations of *aaabbc*, we have to observe that six letters, if they are all different, would give $1\times2\times3\times4\times5\times6$ changes; but since a occurs three times, these changes must be divided by $1\times2\times3$; also, because b

occurs twice, we must divide by 1×2. Hence the required number of permutations is $\frac{1\times 2\times 3\times 4\times 5\times 6}{1\times 2\times 3\times 1\times 2}$, or 60.

Let a be repeated p times, b repeated q times, c repeated r times, &c., and we shall then have the general expression: $\frac{1\times 2\times 3\times\ldots\ldots p\times 1\times 2\times 3\times\ldots q\times 1\times 2\times 3\times\ldots r}{1\times 2\times 3\times 4\times\ldots\ldots\ldots(n-3)\ (n-2)\ (n-1)n}$ for the permutations of any number of letters, when there are p of any sort q of another, r of another, &c.

Ex. 1· How many permutations can be made of the letters in the word commemorate?

The whole number of letters is 11; o is repeated twice, m three times, and e twice. $\therefore$ The permutations are:

$$\frac{1\times 2\times 3\times 4\times 5\times 6\times 7\times 8\times 9\times 10\times 11}{1\times 2\times 1\times 2\times 3\times 1\times 2}=1663200.$$

Ex. 2. How many permutations can be made of 4 oranges, 4 apples, and 4 pears?

Ex. 3. Required the number of permutations that can be formed out of the letters of the word "Disinterestedness."

Ex. 4. How many different numbers can be made out of the following digits: 1223334444?

376. Now, if instead of finding how many changes and quantities a, b, c, d, e may undergo, we find the different groups that can be formed out of them, so that no two groups consist of the same things, we shall determine the *combinations* of those quantities. Thus: *ab*, *ac*, *ad*, *ae*, *bc*, *bd*, *be*, *cd*, *ce*, *de*, are the combinations of five things, taken two and two

abc, *abd*, *abe*, *acd*, *ace*, *ade*, *bcd*, *bce*, *bde*, *cde*, are the combinations of five things, taken three and three.

abcd, *abce*, *abde*, *acde*, *bcde*, are the combinations of five things, taken four and four.

abcde is the only combination of five things taken altogether.

We have seen that the number of permutations of five things taken together is 5×4, and each combination admits of two permutations; therefore, $\frac{5\times 4}{2}$ expresses the number of combinations of five things, taken three together.

We have already found that $n(n-1)(n-2)(n-3)\ldots(n-p+1)$ express the permutations of n things, taken p and p together; hence the number of combinations of n things taken, also p and p together, is:

$$\frac{n(n-1)(n-2)(n-3)\ldots\ldots\ldots(n-p+1)}{1\times2\times3\times4\times\ldots\ldots\ldots p}$$

Ex. 1. In how many ways can four soldiers be taken to mount guard from a company of 50 men?

The number of different guards is evidently the same as the number of combinations of 50 things, taken four at a time.

$$\therefore \frac{50\times49\times48\times47}{1\times\ 2\times\ 3\times\ 4}=230300$$

Ex. 2 A groom is ordered to bring 4 horses from a stable containing twelve horses, but having forgotten which were wanted, he brings 4 taken at random. What chance has he of being right?

Ex. 3. How many different three-coloured flags can be made with the primitive colours, viz., red, orange, yellow, green, blue, indigo, and violet?

Ex. 4. How many signals may be made by a telegraph consisting of six boards, each making one motion?

Ex. 5. There are four companies, in one of which there are 18 men, in another 20, and in each of the other 24 men. What are the combinations that can be made with 4 men, one out of each company?

In this problem let us establish the principle on which we should proceed. If there were two companies containing 18 and 20 men respectively, and every man of one company is combined with every man of the other, the number of combinations is evidently the product of the two numbers of men, or 18×20. Again if there be another company of 24 men introduced, each of these being combined with 18×20 combinations, will make $18\times20\times24$ combinations of three companies, one man being taken out of each company. The same method of reasoning will be true for any number of companies. Therefore the number of combinations required is the product of the numbers which express the number of men in each company. If there were the same number of men in each company, that number raised to the power expressed by the number of companies would be the answer.

Hence, generally, if n represent the number of companies containing respectively, p, q, r, &c., men, and one man being taken out of each company, the number of combinations is $p \times q \times r \times$ &c., viz. the continued product of the numbers expressing the number of men in each company. If there be the same number of men in each company, then p^n is the number of combinations.

Thus in our question the answer is $18 \times 20 \times 24 \times 24$ or 207360.

Ex. 6. In a school of 106 boys, it is found that 12 of them are in room No. 1; 16 in room No. 2; 18 in room No 3; and the others are equally divided in rooms Nos. 4, 5, and 6. How many ways may 6 pupils be taken, one out of each room?

Ex. 7. Required the number of ways in which 8 men, 6 women, and 5 boys can be taken, so as always to have one out of each set?

Ex. 8. How many combinations are there in throwing 4 dice?

Ex. 9. Find the number of different triangles into which a polygon of 12 sides may be divided by joining the angular points.

Ex. 10. How many different sums may be formed with a sovereign, a half-sovereign, a crown, a half-crown, a shilling, a sixpence, a fourpence, and a penny?

Ex. 11. How many words can be found, consisting of 3 consonants and a vowel, from an alphabet which consists of 21 consonants and 5 vowels?

Ex. 12. In a box of figures of animals and riders every figure is divided into four parts. Now taking four parts together, viz., any head and neck, stomach and legs, hind quarters, and rider, they form a complete figure. There are 12 sets. Find the number of combinations.

LOGARITHMS.

377. If there be written the natural series of numbers which form an arithmetical progression, the difference of which is 1; and also different geometrical progressions, the ratio being 2, 3, 4. 5, &c.......10, &c.; and placing these series under one another, as follows,

RATIO 2.

0,	1,	2,	3,	4,	5,	6,	7,	8,	9,	10, &c.
1,	2,	4,	8,	16,	32,	64,	128,	256,	512,	1024, &c.
2^0,	2^1,	2^2,	2^3,	2^4,	2^5,	2^6,	2^7,	2^8,	2^9,	2^{10}, &c.

RATIO 3.

1,	3,	9,	27,	81,	243,	829,	2187,	6561,	19683,	59067, &c.
3^0,	3^1,	3^2,	3^3,	3^4,	3^5	3^6,	3^7,	3^8,	3^9,	3^{10}, &c.

RATIO 4.

1,	4,	16,	64,	256,	1024,	4096,	16384,	65536,	262144,	1048576 ,,
4^0,	4^1,	4^2,	4^3,	4^4,	4^5,	4^6,	4^7,	4^8,	4^9,	4^{10}, &c.

&c. &c. &c.

RATIO 10.

1,	10,	100,	1000,	10000, &c.		&c.		&c.		&c.
10^0,	10^1,	10^2,	10^3,	10^4,	10^5,	10^6,	10^7,	10^8,	10^9,	10^{10}, &c.

&c. &c. &c.

the terms of the arithmetical series, or the indices of the geometrical progressions, are called the *Logarithms* of the corresponding terms of the geometrical series. The terms of the arithmetical series, with the corresponding terms of each geometrical series, form a *System of Logarithms*, and the ratio 2, 3, 4, &c....10, &c., is called the *base* of the system of logarithms.

In the preceding series, it is observed that the numbers 1, 2, 3, 4, &c., which are the terms of the arithmetical series, or the indices of the geometrical series, correspond to different numbers, according as the base is 2, 3, 4, &c,; for instance, 5 corresponds to 32 when the base is 2, to 243 when the base is 3, to 1024 when the base is 4; then, the same number may be the logarithms of several quantities. Moreover, we notice the same number in different series; for instance, 64 corresponds to 6 when the base is two, to 3 when the base is 4; likewise, 1024 corresponds to 10 when the base is 2, to 5 when the base is 4; then, the same number may correspond to different logarithms. Hence, in order to ascertain, on one side, which logarithm corresponds to a given number; and on the other side, which number corresponds to a given logarithm, it is necessary to know the base of the system of logarithms.

378. If any two terms of the arithmetical series, or any two indices, be added up, as $2+5=7$, and the corresponding terms of the geometrical series be multiplied by each other, viz., $4 \times 32 = 128$ (when the ratio is 2), this product corresponds to the

sum; also $3+7=10$, the corresponding terms (when the ratio is 4), are $64 \times 16384 = 1048576$. Hence, the product of two terms of the geometrical series is equal to the sum of the logarithms of the corresponding terms.

379. Let us divide two terms of the G.S. by one another; for instance, 19683 by 81 (when the ratio is 3), the quotient is 243, and subtract 4, logarithm of 81 from 9, logarithm of 19683, the difference is 5, which corresponds to 243, the quotient; also (when the ratio is 4) we have $65536 \div 1024 = 64$, and $8-5 = 3$, which corresponds to 64. Hence, the quotient of two terms of the G.S. is equal to the difference of the logarithms of these terms.

380. If any term of the A.S., or if any index, as 4, be doubled, the result, 8, is the logarithm of 256 (when the ratio is 2), which is the square of 16 corresponding to 4, the given term; the result 8 is also the logarithm of 6561 (when the ratio is 3), which is the square of 81, corresponding to 4, the given term. If a term of the A.S., or if an index, as 3, be trebled, the result, 9, is the logarithm of 19683 (when the ratio is 3), which is the cube of 27, corresponding to 3. Hence, the power of any number of the G.S. is formed by multiplying its logarithms by its index.

From this last observation it follows, that if any term of the A.S., or if any index, be divided by 2, 3, 4, &c., the 2nd, 3rd, 4th, &c., root of the corresponding term of the G.S. will be determined; for example, $8 \div 2 = 4$, the logarithm of 81 (when the ratio is 3), which is the square root of 6561, corresponding to 8; likewise, $8 \div 4 = 2$, the logarithm of 16 (when the ratio is 4), which is the fourth root of 65536, corresponding to 8.

381. These properties enable us to form an idea of the importance of logarithms in facilitating certain arithmetical operations. As the base of the system of logarithms in use the number 10 has been adopted, as presenting great advantages over every other; then:

$0 = \log 10^0$ or 1, $\quad 1 = \log 10^1$ or 10, $\quad 2 = \log 10^2$ or 100
$3 = \log 10^3$ or 1000, $4 = \log 10^4$ or 10000, $5 = \log 10^5$ or 100000
$6 = \log 10^6$ or 1000000, &c.

It follows from this that the logarithms of the numbers 10, 100, 1000, 10000, &c., are alone whole numbers; the logarithm

of a quantity between 1 and 10 will be between 0 and 1, or a fraction; that of a quantity between 10 and 100 will be between 1 and 2, or 1 + a fraction; that of a quantity between 100 and 1000 will be between 2 and 3, or 2 + a fraction, and so on.

382. Suppose, now, it were required to find the logarithm of 5. Since 5 is between 1 and 10, find a geometric mean x of 1 and 10 (§345), then $x=\sqrt{10}=3.1622776$; also an arithmetical mean X of 0 and 1 (§344); then $X=\frac{1}{2}$ or .5, which is evidently the log of $\sqrt{10}$, since $\log \sqrt{10}=\frac{\log 10}{2}=\frac{1}{2}$.

Because 5 is between 3.162...and 10, find the geometric mean of 3.162...and 10, and the arithmetic mean of .5 and 1. Then $x=\sqrt{31.622766}$, or 5.623, and $X=\frac{1.5}{2}=.75$. Therefore, .75 is the logarithm of 5.623.

Because 5 is between 3.162 and 5.623...continuing the same process upon these and other numbers, we shall at last find two quantities differing as little as possible from 5; and there will not be any sensible error in taking 5 for one geometric mean, and the corresponding arithmetic mean is the logarithm of 5. Thus, by similar operations, we shall determine the logarithm of every quantity.

We may remark that it would only be necessary to calculate the logarithms of *prime numbers*, for the logarithms of *multiples* can be found by their factors.

Having shown a method for finding the logarithms of all numbers, the results form a *Table of Logarithms*.

383. A logarithm consists of two parts, one on the left of the decimal point, called the *characteristic*, and the other, on the right, called the *mantissa*. It must be observed, that the characteristic contains as many units, but one, as there are figures in the corresponding number.

384. We owe the invention of logarithms to John Napier, a Scotch nobleman, born in 1550; but to Briggs, professor of geometry at Oxford, we are indebted for many improvements: he published the first table of logarithms, in 1624.

With regard to the directions for using the tables, every necessary information will be found in the introduction accompanying the Tables of Logarithms. We shall, therefore, suppose every

student familiarized with the manner of finding the logarithms of any given number; and conversely, of finding the number corresponding to any given logarithms.

385. Here are some applications :

Ex. 1. What is the product of 564 and 792 ?

$$\begin{aligned} \text{We have: } \log 564 &= 2.7512791 \\ \log 792 &= \underline{2.8987252} \\ \text{log of product} &= 5.6500043 \\ \therefore\ \text{product} &= 446688. \end{aligned}$$

Ex. 2. Find the quotient of 37812 and 454.

$$\begin{aligned} \log 37812 &= 4.5776296 \\ \log 454 &= \underline{2.6570559} \\ \therefore \text{log of quotient} &= 1.9205737 \\ \therefore \text{quotient} &= 83.287. \end{aligned}$$

Ex. 3. Determine by logarithms the value of the expression $\dfrac{54.6 \times 1.764}{6.72}$.

$$\begin{aligned} \log 54.6 &= 1.7371926 \\ \log 1.764 &= 0.2464986 \\ -\log 6.72 &= \underline{0.8273693} \\ \therefore \text{log of answer} &= 1.1563219 \\ \therefore\ \text{answer} &= 14.3325. \end{aligned}$$

Ex. 4. Find $2^{\frac{1}{3}}$.

$$\log 2^{\frac{1}{3}} = \frac{\log 2}{3} = \frac{0.3010300}{3} = 1.259921.$$

Ex. 5. What is the value of $\left(\frac{6}{5}\right)^{15}$?

$$\log \left(\frac{6}{5}\right)^{15} = 15(\log 6 - \log 5) = 15 \times 0.07918125 = \log 1.1877187$$
$$= 15.407.$$

Ex. 6. £24. 12s. 6d. $\times (3.56)^4$.

$$\begin{aligned} \log \text{£}24.\ 12\text{s}.\ 6\text{d}. = \log 24.625 &= 1.3913762 \\ \log (3.56)^4 = 4 \log 3.56 &= \underline{2.2058000} \\ \text{log of answer} &= 3.5971762 \end{aligned}$$

$\therefore$ answer = £3955.271 = £3955. 5s. $5\frac{1}{2}$d. nearly.

Ex. 7. What is the amount of £1210, left unpaid for 54 years 6 months, at $3\frac{1}{2}$ per cent. per annum, C.I. ?

The amount $= 1210 \left(\frac{103.5}{100}\right)^{45.5}$

$\therefore \log a = \log 1210 + 54.5 \times \log 1.035 = \log 3.8970318.$

$\therefore$ amount $=$ £7889.18 $=$ £7889. 3s. 7d.

Ex. 8. In how many years will a principal double itself, at 3 per cent. per annum, C.I.?

Here $2\,P = P(1.03)^n$.

$\therefore 2 = (1.03)^n$.

$\therefore n \log 1.03 = \log 2.$

$\therefore \; n = \frac{\log 2}{\log 1.03} = \frac{0.3010300}{0.0128372} = 23.53$ years nearly.

386. EXERCISES.

1. Determine a mean proportional between 64.5 and .73.

What will £1000 amount to in 64 years, at 4 per cent. per annum, C.I.?

3. In what time, at C.I., reckoning $4\frac{1}{2}$ per cent. per annum, will £100 amount to £1000?

4. At what rate per cent. per annum will £400 amount to £1600. 12s. 6d., in 12 years, C.I.?

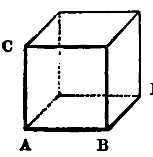
5. Find the value of the expression $\sqrt[12]{(564)^7}$; also of $\left(\frac{21}{64}\right)^{\frac{3}{5}}$.

PART VII.

APPLICATION OF ARITHMETIC TO GEOMETRY, OR MENSURATION.

387. Generally speaking a *Body* is anything that can be seen, touched, or weighed.

Bodies are either *solid*, *liquid*, or *gaseous;* metals, stones, wood are solid bodies; water, wine, &c., are liquid bodies, and lastly, the atmosphere, lighting gas, &c., are gaseous bodies.



A body cannot exist without occupying a part of space; or a body has three dimensions, length, breadth, and thickness or depth. Thus, in the accompanying diagram, A B is the length, B E the breadth, and A C the thickness.

388. The *content*, *volume* or *solidity* of a body is the part of space which it occupies; the boundaries or limits of a body are its *faces*, or its *surface.*

Surfaces may be *plane* like the front of a house, the top of a table, a mirror, &c., and *curved* as in a ball, a turnip, &c., there are also *mixed surfaces*, composed of plane and curved surfaces. It must be observed that no account is taken of thickness when speaking of surfaces.

The boundaries or limits of surfaces are *lines*, the *right* or *straight line* is represented by the edge of a well made ruler, the direction of a thread at the end of which is hung a heavy body; the curved line, of which we have representations in the circumference of a circle, the edge of a basin, &c., when a line is partly straight and partly curved it is said to be *mixed;* then we may consider lines without having regard to the surface. The place of intersection or of meeting of two or more lines, is called a *point*, thus we may speak of a point independently of the lines.

389. The distance between two points is measured by the straight line which connects them, for it is the shortest path between them.

390. An *angle* is the inclination of two lines, which meet, the point of meeting is called the *vextex* of the angle.

When one straight line meeting another straight line, makes with it two angles, which may be equal or unequal, in the first case, the lines are *vertical* to one another, and the equal angles are said to be *right*, in the second, when the lines are oblique to one another, the angle which is less than a right angle is called *acute*, and that which is greater *obtuse*.

391. The distance from a point to a line is measured by the vertical line drawn from the point to the line.

Two lines are parallel when they are everywhere equidistant. The vertical line which joins the parallels is their true or shortest distance.

392. By *polygon* is meant a plain surface, enclosed by right lines, which are called the *sides* of the polygon.

The simplest of all polygons is the *triangle*, or *trigon*, having three angles and three sides. A triangle is right-angled, obtuse-angled, or acute-angled, as one of its angles is right, obtuse, or all three acute. It is also *equilateral*, when the three sides are equal to one another; *isosceles*, when only two sides are equal; and *scalene*, when all three are unequal.

A quadrilateral, or tetragon, is a polygon of four sides and four angles.

A pentagon, a figure of five sides and five angles.

A hexagon, a figure of six sides and six angles.

A heptagon, a figure of seven sides and seven angles.

An octagon, a figure of eight sides and eight angles, &c.

If the sides of a polygon be equal, as also the angles, or if a polygon be equilateral and equiangular, it is called a *regular polygon*.

A quadrilateral, with the opposite sides parallel, is called *parallelogram*. A parallelogram, which has four equal sides, and its angles right, is called a *square*.

A parallelogram, which has four equal sides, and its angles oblique, is called a *rhombus*.

A parallelogram, with two pairs of equal sides, and its angles right, is called a *rectangle*.

A parallelogram with two pairs of equal sides and its angles oblique is called a rhomboid.

A four-sided figure with only one pair of parallel sides is called a trapezoid.

A four-sided figure with no parallel sides is called a trapezium.

393. The simplest of curves is the *circumference*, of which every point is equally distant from the *centre*. The circle is the surface contained within the circumference.

Any line drawn from the centre to the circumference is called *radius*, and a line drawn through the centre and terminated at the circumference is called the *diameter*, therefore a diameter is composed of two radii. All diameters of the same circle are equal to one another.

Every circumference of the circle is supposed divided into 360 parts, called degrees, the degree into 60 minutes, the minute into 60 seconds, &c.

An *arc* is a part of the circumference, and the right line joining its extremities is the *chord.*

394. To measure the length of a line, we fix upon some unit of measure, as an inch, a foot, &c., and this unit is repeated till it makes up the line, and the number of times it is contained gives the units of length of that line.

The perimeter of a figure is the sum of its sides.

395 Every circumference is 3.14159 times its diameter, or the ratio of the circumference to the diameter is $\frac{22}{7}$ nearly, which ratio is generally expressed by π.

Therefore to find the circumference multiply the diameter by 3.14159, or by $\frac{22}{7}$ in ordinary cases.

And to find the diameter, divide the circumference by 3.14159 or by $\frac{22}{7}$.

396. The unit used to measure angles is the angle of one degree. From the vertex of an angle, with any radius, suppose an arc described, the number of degrees contained in the arc between its sides, indicates the measure of the angle.

The instrument employed to find this number of degrees is called a *protractor*.

An angle of 36 degrees, 40 minutes, and 30 seconds, is expressed thus: $36° \; 40' \; 30''$.

The sum of the angle in every triangle is the same as that of two right angles, or of 180°.

397. EXERCISES.

1. What is the perimeter of a triangle, the sides of which are 35 feet, $28\frac{1}{4}$ feet, and 44 feet 7 inches respectively?

2. A rectangular piece of ground, the adjacent sides of which are 116 feet and 104 feet, is to be surrounded by a ditch; its length is required?

3. At what rate, per hour, does a horse go, that runs three times round a field of a rectangular shape, the adjacent sides of which are 960 and 1118 yards, in 8 minutes and 20 seconds?

4. What is the circumference of a circular well the diameter of which is 5 yards, 2 feet, 8 inches?

5. What will be the expense of planting with box a circular garden, whose diameter is 7 yards 10 inches, at 2d. per yard?

6. What is the diameter of a piece of water, the circumference of which is $124\frac{1}{2}$ yards?

7. The radius of the equator is 3962.824 miles, find at what rate per hour is any object on the equator carried round during the earth's rotation in 24 hours?

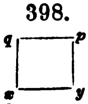
8. The mean distance of the earth from the sun is 95 millions of miles, at what rate per hour is the earth carried round the sun during its revolution in $365\frac{1}{4}$ days?

9. The radius of the equator is 3962:824 miles. What is the distance on the equator of two places which are $36° 24' 16''$ apart?

10. The diameter of a circle is 7.4 feet. Required the number of degrees in an arc, whose length is 5.6 feet.

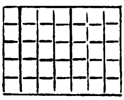
MENSURATION OF SURFACES.

398. The *unit* of superficial measure is a square surface,



the length of the side of which is the lineal unit; thus if $x\,y$ be the lineal unit, the square $x\,y\,p\,q$ is the superficial unit.

399. The surface or area of a rectangle is found by multiplying



together two adjacent sides; if one side is 7 feet, and the other 5 feet, then 7×5, or 35, expresses that the area of the rectangle contains 35 times the surface of the square foot, or is 35 square feet. This is easily shown by dividing one side of a rectangle into 7 equal parts, and the other into 5; and by drawing from the points of division, lines parallel to the sides, the area contains 7×5 equal squares.

400. The two contiguous sides of a rectangle are its length and breadth, and are called its dimensions. The area of a square is found by multiplying one side by itself; thus if the side is 12 yards, the square contains 12×12, or 144 square yards. This is evident, for a square is a rectangle whose contiguous sides are equal.

401. To find the area of a parallelogram multiply its length by its vertical breadth, or its base by its height. For a parallelogram has the same area as a rectangle of the same base and the same height.

402. To find the area of a triangle, multiply the base by the vertical height, and half the product, will be the area; any side may be taken as the base of the triangle, and its altitude or height is the vertical line drawn from the vertex of the opposite angle to the side taken as a base, This is evident, since the area of a triangle is the half of that of a parallelogram, having the same base and the same altitude.

The area of a triangle is also found as follows:

From half the sum of the three sides, subtract each side separately, multiply this half sum and the three remainders continually together, and extract the square root of the product.

Ex. 1. The base of a triangle is 32.5 feet, and its vertical height 18.8 feet. Required the area.

$$\text{Here area} = \frac{32{\cdot}5 \times 18.8}{2} = 305.5 \text{ square feet.}$$

Ex. 2. If the sides of a triangle are 15.4, 20, and 24 yards, find its area.

$$\text{Half sum of sides} = \frac{15.4 + 20 + 24}{2} = \frac{59.4}{2} = 29.7.$$

1st remainder $=29.7-15.4=14.3$
2nd remainder $=29.7-20=9.7$
3rd remainder $=29.7-24=5.7$

$\therefore$ Area $=\sqrt{29.7\times 14.3\times 9.7\times 5.7}=145.84$ square yards.

403. To find the area of a trapezium:

Multiply either of the diagonals by the sum of the verticals drawn to it from the opposite angles, and halve the product.

Or divide the trapezium into two triangles, and proceed as for triangles.

Ex. 1. Let a diagonal be 65 yards, and the verticals on it drawn from the opposite angles 28 and $32\frac{1}{2}$ yards; what is the area of the trapezium?

$$\text{Area}=\frac{65\times(28+32\frac{1}{2})}{2}=1966\tfrac{1}{4}\text{ square yards.}$$

404. To find the area of a trapezoid, multiply the sum of the parallel sides by the vertical distance between them, and take half the product.

Ex 1. Given the parallel sides of a trapezoid, equal $24\frac{1}{2}$ and $16\frac{1}{4}$ feet, and its breadth, or vertical distance equals 12 feet. Find the area.

$$\text{Area}=\frac{(24\frac{1}{2}+16\frac{1}{4})\times 12}{2}=244\tfrac{1}{2}\text{ square feet.}$$

405. The area of a polygon, or of any irregular figure, is found by dividing it into triangles, or trapeziums, or into both; the areas of these are determined and their sum is the area of the polygon.

406. To find the area of a circle, multiply the circumference by the diameter, and take one-fourth of the product.

Or, multiply the square of the radius by 3.14159.

Or, multiply the square of the diameter by .785398.

407. To find the area of a sector, multiply the subtending arc by half the radius.

Or, find the area of the whole circle, multiply it by the number of degrees in the arc, and this product, divided by 360°, gives the area.

408. To find the area of an ellipse, multiply the product of the two diameters by .785398.

Ex. 1. What is the area of a circle, the circumference of which is 15 feet, and diameter 4.77 feet?

$$\text{Area} = \frac{15 \times 4.77}{4} = 17.9049 \text{ square feet.}$$

Ex. 2. Find the area of a sector, the radius of which is 50 and the arc 56° 30′.

$$\text{Area} = \frac{50^2 \times 56° \ 30' \times 3.14159}{360°} = 1232.6377.$$

$$\text{Or, area} = \frac{2 \times 50 \times 3.14159 \times 56\frac{1}{2} \times 25}{360} = 1232.6377.$$

Ex. 3. The diameters of an ellipse are 18 feet and 8 feet; the area is required.

$$\text{Area} = 18 \times 8 \times .785398 = 113.0973 \text{ square feet.}$$

409. EXERCISES.

1. Find the area of a rectangular garden, the length of which is 100 yards, and breadth 84 yards.

2. If the side of a square table be 4 feet 8 inches, what is its area?

3. How many acres does a triangular field contain, the base of which is 1440 yards, and its altitude 960 yards?

4. How many paving stones, the surface of each of which is 9 inches by 5, will be required to pave a street 450 yards long and 8 yards broad?

5. What is the side of a square, the area of which is the same as that of a triangle, and the sides of which are 72.60 and 56 feet?

6. Find the area of a circle, the diameter of which is 5.6 yards; and the side of a square of equivalent area.

7. The circumference of a circular pond is 720 yards; find its area.

8. It is required to find the radius of a circle, of the same area as two other circles, the radii of which are, respectively, $5\frac{1}{4}$ and 7.6 yards.

9. A square is covered with half-crowns, the diameter of which is 1.3 inches, and there are 12 on each side. Find the vacant space between the coins.

10. The interior diameter of a building is 56 feet, and the thickness of the wall 1 foot 4 inches. Find the surface of the ground upon which the wall stands.

11. The areas of an elliptic ceiling are 32 feet 8 inches, and 20 feet 9 inches. What is its area?

12. Required, the area of the sector of a circle, the arc and radius of which are each 7.4 feet.

MENSURATION OF SOLIDS.

410. Geometrical bodies are either terminated by plane faces or by curved faces; solid bodies alone maintain their forms, liquid and gaseous bodies take the form of the vessels which contain them.

Solid bodies bounded by plane surfaces are called *polyhedron*, they are; the *prism*, whose bases are equal and parellel polygons, and the lateral faces parallelograms. A prism is said to be *trigonal*, *tetragonal*, *pentagonal*, &c., if its bases be triangles, tetragons, pentagons, &c, The expressions *three-sided, four-sided, five-sided prism*, &c., are sometimes employed

The altitude of a prism is the distance between its parallel bases.

A prism the bases of which are parallelograms is called *parallelepiped;* when the lateral faces are vertical to the bases, we have instances of right-parallelepipeds, such as a closed box, a brick, &c.

The *cube* is an example of a right parallelepiped, its six-faces, are equal squares.

A *pyramid* has for its base a polygon, and for its lateral faces triangles. the vertices of which meet in a point called the *vertex* of the pyramid.

Pyramids are trigonal, tetragonal, pentagonal, &c., if their base be a triangle, a tetragon, a pentagon, &c.

The altitude of a pyramid is the vertical drawn from the vertex to the base.

The *cylinder* is a prism having circles for its bases, such as a garden roller, a round pillar, &c.

The *cone* is a pyramid having a circular base, such as a sugar loaf. In the *right cone* the axis is vertical to the centre of the base, in all other cases the cone is *oblique*.

A *frustum* of a pyramid or a cone, is what remains when the top has been taken away, the solid is then said to have been *truncated*

A *sphere* is a solid, bounded in every direction by a curved surface, which is everywhere at the same distance from a certain point within it called the centre.

SUPERFICIAL MEASURE OF SOLIDS.

411. The *lateral surface of a right prism*, is equal to the product of the perimeter of its base by the altitude.

The *lateral surface of a regular pyramid*, is formed by multiplying the perimeter of its base by the altitude of the lateral faces and taking half of the product.

The *convex surface of a cylinder*, is obtained by multiplying the circumference of the base by the altitude.

The *convex surface of a cone*, is equal to the product of the circumference of the base and half the slant height.

The *convex surface of a truncated cone*, is equal to the sum of the circumferences of both bases, mutliplied by half the slant altitude.

The *surface of a sphere* is equal to the convex surface of the circumscribing cylinder, or to the product of the square of the diameter by 3.1415926.

The *surface of a spherical segment*, is equal to the convex surface of the corresponding portion of the circumscribing cylinder.

MEASURE OF CONTENTS OF SOLIDS.

412. The *dimensions* of a solid are the lines representing its length, breadth, and thickness.

The *unit* of bodies is a cube the length, breadth, and thickness of which are each equal to the lineal unit.

The *solidity of a prism*, or *cylinder*, is equal to the product of the base by the altitude.

The *contents of a right parallelepiped* is found by multiplying its three dimensions, or the three edges terminating at the same point.

The *solidity of a pyramid*, or *cone*, is equal to the third part of the area of the base multiplied by the altitude.

The *solidity of a frustum of a pyramid*, or *of a cone*, is found by adding the area of both bases to four times the area of the mean section parallel to the bases; multiply this sum by the altitude, and take one-sixth of the result.

The *content of a sphere* is equal to the cube of its diameter multiplied by .5236, or is equal to two-thirds of the solidity of the circumscribing cylinder.

413. EXAMPLES OF SUPERFICIAL MEASURE.

Ex. 1. How many square feet of wood are required to make a box, the length of which is 4 feet, depth 3 feet 6 inches, and breadth 1 foot 6 inches?

Here lateral faces $=(8+3)\ 3\frac{1}{2}=38\frac{1}{2}$ square feet.

Bases $=2\times4\times1\frac{1}{2}=12$ square feet.

$\therefore$ wood required $=38\frac{1}{2}+12=50\frac{1}{2}$ square feet.

Ex. 2. What quantity of canvass is required for an octagonal tent, each side being 8 feet, and the slant height 10 feet?

Here surface $=\dfrac{8\times8\times10}{2}=320$ square feet, or $35\frac{5}{9}$ square yards.

Ex. 3. A cylindrical iron chimney, 5 feet in diameter and 20 feet high, is made with sheet-iron, 8 feet 6 inches long and 1 yard broad. How many sheets were required?

Surface of chimney $=5\times3.1416\times20=314.16$ square feet.

Surface of each sheet $=8\frac{1}{2}\times3=25.5$ square feet.

$\therefore\dfrac{314.16}{25.5}=12.32$ sheets.

Ex. 4. If the diameter of the base of a right cone be .58 yards, and the distance of the vertex to any point of the circumference of the base .92 yard. Find the entire surface of the cone.

Area of convex surface $=\dfrac{.58\times3.1416\times.92}{2}=.8382$ square yard nearly.

Area of base $=.58^2\times.785398=.2642$ square yard.

$\therefore$ whole surface $=.8382+.2642=1.1$ square yards nearly.

Ex. 5. A pail has the shape of a frustum of a cone; the radii of its bases are 1.25 feet and .75 foot, the height of its side is .5 yard. Find its convex surface.

Here slant surface $=\frac{(1.25+.75)\,3.14159}{1.5}=4.1888$ square ft.

Ex. 6. If the earth were a sphere 7912 miles in diameter, what would be its surface ?

Surface $=7912^2\times 3.1415926=196662892.5123$ square miles.

Ex. 7. The diameter of a sphere is 3 feet 6 inches. What is the convex surface of a segment, the height of which is 9 inches ?

Surface $=3.1416\times 42\times 9=1187.5248$ square inches.

414. EXAMPLES OF SOLID MEASURE.

Ex. 1. How many gallons of distilled water will a cistern contain, the length being 8 feet, the breadth 3 feet 6 inches, and the depth 4 feet 2 inches ?

Contents of the cistern is $96\times 42\times 50=201600$ cubic inches.

And $\because$ 277.274 cubic inches $=1$ gallon,

$\therefore$ 201600 cubic inches $=\frac{201600\times 1}{277.274}=727$ gallons nearly.

Ex. 2. A cylindrical vessel, the base of which is 3 yards in circumference, and height 5 yards, is three-quarters full of distilled water. The weight of the water is required.

The diameter of the base $=\frac{36}{3.1416}=11.4$ inches.

$\therefore$ Surface of base $=\frac{36}{2}\times\frac{11.4}{2}=102.6$ square inches.

$\because$ The vessel is $\frac{3}{4}$ full, the water in it $=\frac{3\times 60}{4}\times 102.6=$ 4617 cubic inches, and $\therefore$ 27.7274 cubic inches of distilled water weigh 1lb.

$\therefore$ 4617 cubic inches weigh $\frac{4617\times 1}{27.7274}=166.5$lbs.

Ex. 3. Required the number of cubic feet of air in a room, the length of which is 32 feet, breadth 20 feet 6 inches, and height 9 feet 8 inches.

Content $=32\times 20.5\times 9.\dot{6}=6341.\dot{3}$ cubic feet.

Ex. 4. The sides of the base of a triangular pyramid are 4.5, 4, 3.5 feet, its altitude is 8.5 feet; find its solid content.

Area of base $=\sqrt{6\times 1.5\times 2\times 2.5}=6.708$ square feet.

Content $=\frac{6.708\times 8.5}{3}=19.006$ cubic feet.

Ex. 5. What is the solidity of the frustum of a square pyramid, the sides of bases being 8 inches and 6 inches, and the altitude 15 inches.

Here side of middle section$=\frac{8+6}{2}=7$ inches.

$\therefore$ Content$=\frac{1}{6}\times15\times(8\times8+6\times6+4\times7\times7)=740$ inches.

Ex. 6. The diameter of the moon is 2180 miles, find her solid content.

Solidity$=2180^3\times.5236=5424617475$ cubic inches.

415. EXERCISES.

1. The height of the largest pyramid is 477 feet, and a side of the base, which is square, is 720 yards ; find its solidity.
2. Required the content of a bale, the length being 5 feet 6 inches, breadth 4 feet 4 inches, and thickness 3 feet 8 inches.
3. How often may a conical glass, $2\frac{1}{2}$ inches deep, and $1\frac{1}{2}$ inches in diameter at the top, be filled out of an imperial gallon?
4. A cistern is 6 feet 4 inches long, 5 feet 6 inches broad, and 4 feet 8 inches deep, how many gallons of water does it contain?
5. The interior diameter of a metal pipe is 8 inches, the thickness of the metal is $.\dot{6}$ inch, and the length of the pipe is 35 yards, how much metal in the pipe?
6. How many times would a roller 3 feet 8 inches long and 1 foot 10 inches in diameter, turn on a walk 84 yards long, and 6 feet wide, supposing it never to pass twice over the same ground?
7. What length must be cut off from a board 9 inches wide, to contain 2 square feet?
8. The wheel of a carriage turns round 1640 times a distance of $4\frac{1}{2}$ miles, what is the diameter?
9. What length of a wire, $\frac{1}{12}$ of an inch in diameter, may be drawn out of a cubic inch of metal?
10. A roof, which is 36 feet 9 inches by 16 feet 8 inches, is to be covered with lead, at $8\frac{1}{2}$lbs. per square foot, find the price at 14s. per cwt.
11. What will the diameter of a sphere be, when the solidity and superficial area are expressed by the same quantity?

12. The ball on the top of St. Paul's Cathedral is 6 feet in diameter, what will the gilding of it cost at 2½d. per square inch?

13, How many gallons of water will be required to fill a ditch, ¾ mile long, 10 feet deep, 25 feet broad at the top and 18 feet at the bottom?

14. What is the surface of a sphere, the volume of which is 480 square yards?

15. Express the side of the cube equivalent in solidity to a sphere 2.5 feet in diameter.

16. Knowing that globes are to each other as the cubes of their diameters; if the diameter of the Earth be 1, that of the Sun is 109.93, of Mercury .39, of Venus .97, of Mars .56, of Jupiter 11.56, of Saturn 9.61, of Uranus 4.26, of Neptune 3.91, of the Moon, .27; it is required to find the solidity of these bodies, the earth's solidity being supposed unity.

17. The Sun weighs 354936 times as much as the Earth, Jupiter 332, Saturn 101, Neptune 29, Uranus 20, Venus .87, Mercury .18, Mars .14, and the Moon $\frac{1}{70}$; the solidity of these bodies being determined by the previous example, find their density or their weight under the same bulk.

18. Supposing the weight of a cubic inch of cast iron weighs 4.3 oz. avoirdupois, what is the weight of an iron ball of 10.5 inches diameter.

TO FIND THE MAGNITUDE AND THE WEIGHT OF A BODY BY ITS SPECIFIC GRAVITY.

416 When solids are too irregular to admit of the application of the rules of mensuration, we can find the content and also the weight by reference to their specific gravity.

The *specific gravity* of a body is its weight as compared with an equal bulk of some other body.

417. A cubic foot of rain water weighs just 1000 ounces avoirdupois, this has been taken in this country, as the specific gravity of one cubic foot of water, and comparing other bodies with it, the following table, determined by careful experiments contains the weight in ounces of a cubic foot of each.

418. TABLE OF SPECIFIC GRAVITIES OF BODIES.

SOLIDS.

Platina (pure)	23400	Marble	2700
Gold (fine)	19640	Clay	2160
Lead	11325	Common earth	1984
Silver (fine)	11091	Sand	1520
Copper	8915	Coal	1250
Brass (cast)	8399	Mahogany	1063
Steel	7840	Oak	925
Iron	7615	Fir	600
Iron (cast)	7425	Ivory	1825
Tin	7320	Gunpowder	937
Glass (flint)	3000	Cork	240

FLUIDS.

Mercury	13600	Wine, Port	997
Sea water	1028	Oil of Olives	915
Common water	1000	Alcohol	837
Milk	1032	Sulphuric acid	1848
Wine, Burgundy	992	Human blood	1054

GASES.

Atmospheric air	1.22	Azote or Nitrogen	.972
If the specific gravity of air be 1.000, the corresponding specific gravities of the principal gases are:—		Hydrogen	.069
		Steam	.625
		Ammonia	.590
		Sulphuretted Hydrogen	1.777
Air	1.000	Sulphurous acid	2.220
Carbonic acid	1.520	Chlorine	2.470
Oxygen	1.111	Nitrous acid	3.194

419. Ex. 1. What is the solidity of 1 ton of oak?

By the table ∵ 925 ounces of oak are the weight of one cubic foot.

∴ 1 ton or 35840 ounces of oak are the weight of $\frac{35840}{925} = 31\frac{138}{185}$.

Ex 2. Required the weight of a block of marble whose length is 2.3 yards, the breath 1.25 yards, and the thickness .97 yard.

Here the solidity is $2.3 \times 1.25 \times .97 = 2.78875$ cubic yards.

∴ By the table 1 cubit foot of marble weighs 2700 ounces.

∴ 1 cubic yard weighs $27 \times 2.78875 \times 2700$ ounces $= 5$ tons, 13 cwt., 1 qr., 22 lb, 4 oz.

420. EXERCISES.

1. A waggon is loaded with 24 bars of iron, 2 yards long, 8 inches broad and 8 inches thick; find the weight of the load.

2. The weight or pressure of the atmosphere at the earth's surface, is equivalent to the weight of a column of mercury, the height of which is 29¼ inches. Supposing that the surface of the human body is 3.4 square yards, what is the pressure to which it is subjected?
3. Archimedes discovered the following principle: a body immersed in a fluid, loses as much weight as an equal bulk of the fluid weighs. From this it is required to find the weight of a cast iron ball 6.4 inches in diameter, when immersed in water.
4. What is the weight of the air contained in a room, the dimensions of which are 17 feet 6 inches in length, 14 feet 8 inches in breath, and 10 feet 4 inches in height?
5. What will be the weight of a foot of lead pipe, 3 inches diameter in the bore, and .6 inch in thickness?
6. How many cart loads of clay will there be in a drain 100 feet long, 2⅔ feet broad and 6.4 feet deep, allowing 1¼ tons for each load?
7. Required the weight of an oak board, 5 yards long, 2.7 feet broad, and 4.4 inches thick?
8. What is the diameter of a gold ball of the same weight as a silver ball whose radius is 1.5 inches?

MISCELLANEOUS EXERCISES.

1. Pestalozzi was born on the 12th of January, 1746, and died February 17th, 1827. How old was he when he died, and how long is that ago?
2. From May 1st to October 15th, 1851, the time that the Great Exhibition was open, the number of visitors was 6167944. How many went on an average daily, Sundays not included?
3. If 17 pieces of cloth, 23.75 yards long and .77 yard wide, are to be lined with stuff, $\frac{7}{8}$ yard wide. How many yards will be required?
4. There were 3300 iron pillars in the Crystal Palace, on the average 18 feet in length and 8 inches in diameter. If laid side by side, what surface of ground would they cover; and what would be their length, if placed lengthwise?
5. At the opening of a dock, which measures 870 yards by 166 yards, the water was admitted at an average of 4500 gallons per second. and continued to flow in for 12 hours. What was the depth of water?
6. How much further will a man's head travel than his feet, in going round any great circle, supposing his height to be 6 feet?
7. If 100 oranges are bought at 3 a penny, and 100 more at 2 a penny, at what price must they be retailed, so as to gain 25 per cent.?
8. Two travellers sat down to dinner, one had 5 loaves and the other 3; a stranger passing by asked to eat with them, to which they consented. The stranger dined, laid down 8 pieces of money, and departed. How much must each traveller receive?

9. What is the weight of a block of marble, specific gravity 2700, length 63 feet, and breadth and thickness each 12 feet (being the dimensions of one of the stones in the walls of Balbec)?

10. A gentleman dying left in his will a certain amount, to be disposed of as follows: 1st, for funeral expenses £500; 2nd, doctor's bill 150 guineas; 3rd, to each of his two executors £500; 4th, lawyers' bills 300 guineas; 5th, to each of his 13 servants £50: 6th, to his wife he left money for an investment, which, at 4 per cent., would produce her an income of £1000 a year; 7th, the remainder of his property to be divided among his five sons, in such a manner that B shall have 10 per cent. less than A; C, 10 per cent. less than B; D, 10 per cent. less than C; and E, 10 per cent. less than D. Now, E's income, at 4 per cent. from his fortune, was £656.2 per annum. Required, the fortune of each son, and the total amount of property the gentleman left.

11. A could do a work in 16 days, B in 15, and C in 18. Required, the time they will do it in together.

12. How many moidores, guineas, marks, nobles, and crowns are there in £1598. 14s., when there is an equal number of each?

13. If 60 gallons of water, in one hour's time, runs into a cistern, containing 240 gallons, and by a pipe in the same cistern, there runs out 42 gallons per hour. In how many hours will it be filled, supposing there is no water in the cistern, and both taps are open at the same time; the rate of influx and efflux being uniform?

14. The difference of income for one day, between the common and leap year, is 1.346972 farthings. Required, the income per annum.

15. If a solid inch of mould candle will burn 28 minutes, how long will a whole one last, the length of which is 9 inches and diameter .75 inch, supposing the candle to burn uniformly?

16. A body is immersed in a cylindrical vessel of water, the diameter of which is 3 feet 6 inches; and on taking it out, I found the fall of water to be $.\dot{6}$ of an inch. What was the solidity of the body?

17. Three merchants form a partnership: A and B together put in £1300, and C 700. A gained £5 more than B, and C £5 more than A. The whole gain was £600. What was the gain of each, and what stock had A and B respectively?

18. A druggist bought $2\frac{1}{4}$ cwt. avoirdupois of Epsom salts for £7. 16s. How should he sell 1 oz. Troy to gain 250 per cent.?

19. A man of war displaces 64240 cubic feet of sea water. The weight of the vessel is required.

20. Divide £$229.\dot{6}$ among A, B, and C, so that for every $5\frac{5}{8}$ shares taken by A, B takes up $4\frac{3}{7}$; and for every $4\frac{1}{9}$ shares taken by B, C takes $3.\dot{2}$. What did each receive?

21. If 12 oxen be worth 29 sheep, 15 sheep worth 25 hogs, 17 hogs worth 3 loads of wheat, and 8 loads of wheat worth 15 loads of barley, how many loads of barley must be given for 24 oxen?

22. At what time, between 4 and 5 o'clock, will the hour and minute hands of a watch make an angle of 180 degrees?

23. In a shoal of herrings, 6 miles in length, $2\frac{1}{4}$ in breadth, and 240 yards in depth, how many herrings, allowing 184 to the solid foot; and how many casks would they fill, each cask containing 875?

24. A, B, and C can trench a field in 14 days; C, D, and A in 12 days; and D, A, and B, in 18 days. In what time will it be done by them together, and by each singly?

25. Required, the prime cost of a ream of paper, weighing 16 lbs., made from rags at £42. 10s. per ton; waste in manufacturing 25 per cent., expenses 7s. 6d. per ream, and a duty of 3d. per lb. on the paper.

26. It has been computed that the weight of the atmosphere is 13719772666590000000 lbs. The carbonic acid gas is $\frac{1}{200}$ part of its whole weight; and in every 100 lbs. of air there are 23 parts of oxygen, and 77 nitrogen. How much nitrogen is in combination with oxygen in the air?

27. It is found that 61.024 cubic inches of air weigh 20.0624 grains, it is required to determine the weight of the same

quantity of oxygen, nitrogen, hydrogen, and chlorine gas, in the same circumstances. (See table of specific gravities.)

28. It has been ascertained that bodies falling freely in space, describe distances increasing as the square of the number of seconds: thus in 1 second a body falls $16\frac{1}{12}$ feet, in 2 seconds $2^2 \times 16\frac{1}{12}$ or $64\frac{1}{3}$ feet, in 3 seconds $3^2 \times 16\frac{1}{12}$ or $144\frac{3}{4}$ feet, &c. What is the space traversed by a stone which has been 7 seconds in falling?

29. The time of oscillation of a pendulum increases as the square root of its length, in the latitude of London, the length of a pendulum which vibrates seconds is $39\frac{1}{8}$ inches. Find the length of a pendulum that vibrates 4 times in a second.

30. How many oscillations will a pendulum 75 inches long make in a minute?

31. A merchant received 18 guineas for a piece of cloth, by which he gained 3s. 6d, per yard, at the rate of $16\frac{2}{3}$ per cent. Required the length of the piece and the invoice price per yard.

32. When will the hour and minute hands of a clock be at right angles with each other, after 2 o'clock?

33. If by selling goods at 2s. 3d. per lb. I clear 100 per cent., what shall I gain per cent. by selling them at 9 guineas per cwt.?

34. In common air, sound travels at the rate of 1142 feet per second. What is the depth of a well, in which the sound of a stone arriving at the bottom is heard $4\frac{1}{2}$ seconds after its being dropped?

35. How long after firing a gun will the report be heard, at the distance of $5\frac{3}{4}$ miles?

36. In a storm, thunder was heard $3\frac{1}{2}$ seconds after the flash of lightning. At what distance was the cloud from which it proceeded?

37. Light travels at the rate of 192500 miles per second, and the light from the sun is 8 min. 13 sec. in reaching us. What distance is the earth from the sun?

38. What would be the difference of income made by the transfer of £4500 stock from the $3\frac{1}{4}$ per cent., at 84, to the 4 per cent., at 96?

39. Suppose a wolf could devour a sheep in an hour, a tiger in 20 min., and a lion in 30 min.; and suppose the wolf eats 10 min. by himself, after which the tiger arrives, and eats along with him ten minutes more; then the lion arrives, and they all eat together. Find the time in which the sheep will be devoured.

40. If a factor's commission, at $3\frac{1}{2}$ per cent., be £16. 14s. 6d., what money has he disbursed on his employer's account?

41. On starting for a walk, I found the hour and minute hands of my watch exactly together, between 2 and 3 o'clock. On my return, having walked at the rate of 3 miles an hour, the hands were again together, between the hours of 5 and 6. How many miles did I walk?

42. Bought, 23 bags of hops, at £3. 17s. 6d. per bag, by which I expected to clear 22 per cent.; but when I had sold 12 bags at that rate, the price lowered, and I could only sell the remainder at 4 guineas per bag. How much did the whole gain fall below my expectations?

43. A stationer sold pencils at 2s. 6d. per 100, by which he gained 25 per cent.; but afterwards, the pencils becoming very scarce, he raised the price to 6s. per 100. What did he gain per cent. by the latter price?

44. In the atomic theory, substances are supposed to combine with each other by atoms, and the weights of these atoms express the proportion in which they are combined; thus, taking the weight of an atom of hydrogen, or $H=1$; the weight of an atom of oxygen, or $O=8$. Now, since the atom of water consists of one atom of hydrogen and one of oxygen, it follows that the weight of one atom of water, or $HO=1+8=9$. Find the weight of hydrogen and of oxygen in 50 gallons of water.

45. Express the weight of one atom of the three following gases: 1st, the oxide of carbon, which is formed of 2 atoms of carbon and 1 of oxygen; 2nd, carbonic acid, formed of an atom of carbon and 2 atoms of oxygen; and 3rd, carburetted hydrogen,

formed of an atom of carbon and an atom of hydrogen. (The atom of carbon is represented by $C=6$. The number of atoms of one element is expressed by an index, thus, C^2 signifies two atoms of carbon.)

46. When phosphorus burns in the air, it produces phosphoric acid, formed of an atom of phosphorus, represented by $Ph=32$, and of 5 atoms of oxygen. Find the number and the form representing this acid.

47. For one atom of carbon there is one atom of water in sugar. Find the expression and the composition, in vulgar and decimal fractions, of this substance.

48. The analysis of an organic substance produced 60 parts of oxygen, 7.5 of hydrogen, and 45 of carbon. Find the atoms of each in the substance.

49. Bought, a quantity of cloth, for as many pence per yard as there were yards in the whole quantity; and by selling it again at 16 per cent. profit, I received for the whole £302. 1s. 8d. How many yards were there?

50. A owes B £360, which becomes due in 9 months; but in 3 months he pays him £80, and in three months after that £100. Required, how much longer than 9 months B should, in equity, defer demanding the balance.

51. A contractor, for digging a canal, when he had finished a mile in length, gave in his account to the proprietors. The canal was 20 feet broad at the top, 16 feet at the bottom, and 7 feet deep. How many cubic yards should be charged for, and what is the amount, at 1s. 3d. per yard?

52. If the end of the minute hand of a clock moves over a distance of 8 inches in 4 min, 30 sec., what is the length of the index?

53. How many trees may be planted on an English acre, at the distance of 6 feet from each other?

54. C barters with D 140 yards of cloth, at 24s. 8d. per yard, for 48 dozens of wine, at 28s. per dozen, and £40 in cash; the balance to be paid in stuff, at 2s. 6d. per yard. How many yards must be given?

55. A ship was bought for £1600. What must be charged for $\frac{2}{3}$ of $\frac{2}{5}$ of $\frac{7}{8}$ of her, to gain £350 on the whole?

56. I reckoned 8 pulsations during the fall of a stone from the top of a rock. What height did it fall, the pulse beating 72 times in a minute?

57. A cubical box, the diagonal of which is 16 inches, is filled with marbles, .60 inch in diameter. Required, the number it contains; and also the quantity of water, in feet, which might be poured into the cavities between the balls?

58. A general, detaching $\frac{4}{13}$ of his army to occupy a certain height, and $\frac{17}{21}$ of the remainder to watch the enemy's movements, had only 170 men left. The strength of his force is required.

59. In order to improve the quality of some home-made wine, which cost 4s. per gallon, I mixed in a cask, containing 45 gallons of it, 3 gallons of brandy, at 33s. per gallon, and 10 gallons of foreign wine, at 20s. per gallon. What is the price per gallon of the whole mixture?

60. What is the simplest vulgar fraction that is equivalent to .088134765625 ?

61. A bought some coffee; by selling it at $11\frac{1}{4}$d. per lb., he loses 3s. 3d.; by selling it at $13\frac{1}{4}$d., he gains 6s. 6d. How many lbs. were bought?

62. A cubic inch of glass is blown into the form of a globe, that will hold a pint of wine. What is the thickness of the glass?

63. A travels from P to Q, at the rate of $5\frac{1}{2}$ miles an hour. After 6 hours, B, who travels at the rate of $7\frac{2}{3}$ miles per hour, departs also from P, to overtake him. When will P be up to A?

64. A body weighs 12 lbs. 6 oz. in the air, and 7 lbs. 8 oz. in the water. Find its density, or its weight, compared to that of the same quantity of water. $\left(\frac{\text{Weight lost}}{\text{Whole weight}} = \frac{1000}{x}\right)$.

65. A stone weighs 15 oz. in the air; a vessel, filled with water, weighs 16 lbs. 6 oz.; the stone is plunged into it, and the weight is 16 lbs. 14 oz. Find the density of the stone.

66. The freezing point of water, by the centigrade thermometer, is 0°, and the boiling point 100°. In Reaumur's thermometer, the freezing point is 0°, and the boiling point 80°, Find what degree of the centigrade corresponds to 24° Reaumur.

67. In the thermometer of Fahrenheit, the freezing point is 32°, and the boiling point 212° Reduce 48° Fahrenheit into degrees centigrade.

68. Reduce 15 degrees centigrade to degrees Fahrenheit.

69. Bring 75° Fahrenheit into degrees Reaumur.

70. The expansion of iron is .0000122 of its length for every degree centigrade of increase in temperature. What will be the expansion of an iron ruler, $3\frac{1}{2}$ yards in length, for 32° of heat ?

71. Mercury expands $\frac{1}{5550}$ of its bulk for every degree of heat. What will be the increase of $1\frac{2}{3}$ cubic inches of mercury, by raising the temperature from 16° to 48° ?

72. To clothe 25000 men, it takes 125000 yards of stuff, $\frac{3}{4}$ wide. How many yards, $\frac{7}{8}$ wide, are required to clothe 3840 men ?

73. Multiply $\frac{3}{7}$ of $\frac{5}{9}$ of $16\frac{1}{2}$ by $\frac{2}{3}$ of $\frac{7}{8}$ of 15 ; and divide $\frac{4}{11}$ of $27\frac{1}{2}$ by $\frac{3}{10}$ of $21\frac{1}{4}$.

74. How many revolutions will a wheel, $4\frac{2}{3}$ feet in circumference make, during a journey of 24 miles ?

75. What decimal of a pound is 1.045 of a shilling ?

76. A clock, which gains $7\frac{1}{2}$ min. in 24 hours, is set right at noon on Monday. What will be the time by it at 6 o'clock in the evening of the following Saturday ?

77. What income will arise from the investment of £2400 in the 3 per cent. stock, at 71 ?

78. If cloth, when sold at 14s. 3d. per yard, realises a profit of $14\frac{1}{2}$ per cent., at what price must it be sold to gain $20\frac{3}{4}$ per cent. ?

79. A father divided his property among his three children, in the following manner: to the eldest, he gave $\frac{9}{20}$ of the whole ; to the youngest, $\frac{1}{4}$; and to the second, £3000. What was the property left ?

80. What sum will amount to £217. 0s. 7½d. in 3½ years, at 4½ per cent., S.I.?

81. What weight of water will a cistern contain, the depth of which is 3 feet 8 inches; its length, 5 feet 4 inches; and its breadth, 2 feet 6 in.?

82. In how many years will 100 guineas amount to £140. 8s. 9d., at 3¾ per cent., S.I.?

83. Subtract $\frac{5}{16}$ of 5 guineas from $\frac{7}{9}$ of £5. 17s. 9d.

84. A merchant sells goods for his correspondent at different times, for which he is to receive £64. 10s. 8d. at the end of 3½ months, £72. 10s. at the end of 5½ months, and £81. 10s. 6d. at the end of 6 months. What is the equated time to pay the whole?

85. Dr. L. Playfair states that in smelting iron 81½ per cent. of the fuel is wasted by imperfect combustion, and this amounts, according to the present quantity of iron smelted, to the enormous waste of 5400000 tons, $\frac{1}{7}$th of the whole quantity of coal annually consumed in the United Kingdom. How much coal is consumed in smelting the iron, and how much is consumed in the United Kingdom.

86. A and B at the opposite extremities of a wood, 248 yards in compass, begin to go round it the same way, at the same time, A at the rate of 25 yards in 2 minutes, and B at the rate of 38 yards in 3 minutes. How many rounds will each make before the one will overtake the other?

87. 35lbs. of tea being mixed with 20lbs. of a better quality, the mixture is found to be worth 7s. 4d. per pound. Find the value of each kind, the difference of their values being 1s. 10d. per pound.

88. Venus revolves round the sun in 224 days, 16 hours, 49 minutes, 11 seconds; and the earth in 365 days, 6 hours, 9 minutes, 12 seconds. Required the seven first approximate ratios of these periods.

89. A load of corn was sold for £14 17s., by which a loss of 17½ per cent. was sustained, what should it have been sold for, to realize a profit of 12½ per cent.?

90. If the interest of £250 amount to £50. 12s. 6d. in $4\frac{1}{2}$ years, what is the rate per cent.?

91. The weight of a bar of iron 3 feet long, 2 inches broad, and $1\frac{1}{2}$ thick, is $30\frac{1}{3}\frac{5}{2}$ pounds, what is the weight of another bar $8\frac{1}{2}$ feet long, 4 inches broad, and $2\frac{1}{4}$ inches thick.

92. If a pound of tea be worth $2\frac{1}{2}$ of coffee, and a pound of coffee worth $3\frac{1}{2}$ of sugar, what will be the value of 56 pounds of tea when sugar is worth 7d. per pound?

93. By purchasing railway shares at $22\frac{1}{2}$ per cent. discount, and selling them again at 9 per cent. premium, I gained 300 guineas, what was the original sum I invested?

94. What per centage is gained on an article which is purchased for £10. 5s. and sold for 12 guineas?

95. If 4180 pounds of beef serve 176 men 19 days, how many days will $1991\frac{1}{4}$ pounds of beef serve 59 men.

96. Find the compound interest on £850 for $2\frac{1}{2}$ years, at $4\frac{1}{2}$ per cent.?

97. Suppose the surface of a man's body to contain 34 square feet, and that there are 784 pores in a square inch, what is the diameter of each pore, supposing it circular, and the membrane between the pores one quarter of the diameter, how many pores are there in the body?

98. If 464 men in 5 days, of 12 hours each, dig a trench 340 yards long, 3 wide, and 3 deep, in how many days of 9 hours each could 75 men dig a trench 720 yards long, 4 wide, and 2 deep?

99. If a person gain $8\frac{1}{3}$ per cent. by selling apples at the rate of 8 for $6\frac{1}{2}$d., how much does he gain per cent. by selling them at 3 for $2\frac{1}{2}$d.?

100. Three gentlemen contribute £164. 5s. towards the building of a church, at a distance of 2 miles from the first, $2\frac{7}{8}$ miles from the second, and $3\frac{1}{2}$ miles from the third; and they agree that their shares shall be proportional to the reciprocals of the distances from the church. Required their subscriptions.

101. If beer which is brewed with 3 bushels of malt to the barrel, cost 1s. 3d per gallon when malt is at 62s. 8d. per quarter, how much will beer cost per gallon which is brewed

with 5 bushels of malt to the barrel when a quarter of malt costs 50s.?

102. If 10 cannons, which fire 3 rounds in 5 minutes, kill 270 men in $1\frac{1}{2}$ hours, how many cannons which fire 5 rounds in 6 minutes will kill 500 men in 1 hour at the same rate?

103. At what time between 4 and 5 o'clock are the hour and minute hands of a watch together, at right angles, and in opposite directions?

104. A stationer sold quills at 11s. a thousand, by which he cleared $\frac{3}{8}$ of the money, and he afterwards raised them to 13s. 6d. a thousand. What did he gain per cent. by the latter price?

105. Steam requires about 1000° of heat when generated from water, at 212° or 180° above freezing point; this heat not being sensible to the thermometer is *latent heat*. Then steam may be considered as water in a high state of rarefaction, containing about 1000° of heat above what its state indicates. From this it is required to find the temperature of 20 gallons of water at 60°, in which there is 2 gallons of steam at 212°.

Solution:—

20 gallons at 60° or 28° of heat .... = 560°
2 gallons at 212° or (1000+180) or 1180 of heat = 2360°
∴ The temperature of the 22 gals. .... = 2920°
∴ " " mixture .... $= \frac{2920°}{22} +$
$32° = 164\frac{8}{11}°$.

106. To liquify a given quantity of ice requires as much heat as would raise the same weight of water 140° in temperature, or from 32° to 172°. So that if a stone of ice at 32° was mixed with a stone of water at 172°, the result would be 2 stones of water at 32°. Then ice on the point of melting may be considered as water at 172° below 32°, or 140° below freezing point. This is the *latent heat* of water acquired in liquefaction.

From this let us find the temperature of a mixture of 6 lbs of water at 122° with 3 lbs. of ice at 32°.

Solution: Since ice on the point of melting may be considered as water 140° below freezing point:

∴ 6 lbs. of water at 122° or 90° above freezing point = 540°
3 lbs. of ice at 32° or 140° below freezing point = 420°

∴ The difference is 120° above 32° which is disseminated into 9 lbs. of the mixture, ∴ every lb. will be at $\frac{120^\circ}{9}+32^\circ$ or $45\frac{1}{3}^\circ$.

107. What would be the result of mixing 3 lbs. of water at 100° with 2 lbs. ice at 32°?

108. In 20 lbs. of water at 32° there are floating 2 lbs. of ice, into this 3 lbs. of steam are admitted. Find the temperature of the mixture.

109. A mixture is made of 16 gallons of water at 45°, and of 12 gallons at 62°. Required, the temperature of the liquid.

110. What would be the temperature of a mixture of 24 lbs. of water at 77° with 42 lbs. of ice at 25°?

111. If the pendulum of a clock vibrate the distance of $9\frac{3}{4}$ inches in a second of time, how many inches will it vibrate in the course of 7 years 14 days 2 hours 1 min. 56 sec., each year consisting of 365 days 5 hours 48 min. 48 sec.?

112. A boy bought some apples for 6d.; the $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$ of what he had amounted to 54. How many could he get for a half-penny?

113. Received an order for cloth, at 16s. 6d. per yard. Now, it will take 2 lbs. 2 oz. of wool per yard; the cost of making is 4s. per yard, dressing 2s. per yard, and dying the wool 1s. per lb. How much must I give per lb. for my wool, so as to gain $12\frac{1}{2}$ per cent. by this trade?

114. If a cylindrical vessel be 7 feet deep, and will hold 1010 gallons when filled, it is required to know what quantity will run over by putting in the largest heavy cube possible.

115. A room is 32 feet long, 27 feet broad, and 20 feet high. Find the length of the longest pole that can be placed therein.

116. A merchant insured a ship and cargo at the rate of $1\frac{1}{8}$ per cent. Now, if .001 of the value of the ship were multiplied by .001 of the value of the cargo, the product would be £1275; and if the cube of .001 of the ship's value were subtracted from the cube of .001 of the cargo's value, the

remainder would be £610750. The sum paid for insurance is required.

117. A city is supplied with water by means of five pipes, and their diameters are, respectively, 4, 4½, 5½, 6½ and 7½ inches. The smallest pipe supplies 10 gallons per minute; and the supply of the others is as the squares of their diameters. How many families were supplied on an average, each family consuming 10 gallons of water daily?

118. If 1 lb. Troy of English standard gold, $\frac{11}{12}$ fine, be worth £46. 12s. 6d., what is the value of a coin, weighing 7 dwts. 11 grs., in which 924 parts in 1000 are pure gold?

119. A clock gains 3½ min. per day. How should its hands be placed at noon, to point on the true time at 7½ in the evening?

120. A person performs $\frac{2}{7}$ of a piece of work in 13 days; he then receives the assistance of another person, and both together finish it in 6 days. In what time could each do it separately?

121. If 3 miles 4 furlongs 93 yards be run in 6 min. 4 sec., how much is that short of a mile a minute?

122. Find the edge of a cubical block of coal, weighing 2000 tons. (See table of sp. gr.)

123. How much per cent. is 14s. 6d. gain on £4. 5s.?

124. The difference between the year in the Julian calendar and the true year is .007736 days, which the Gregorian calendar corrects, by omitting 3 days in 400 years. Find how much the error would have accumulated under the Julian calendar, from A.D. to A.D. 1854.

125. The external and internal diameters of an iron shell are 8.36 and 6.45 inches. Required, the weight of the shell, and the quantity of gunpowder it will contain?

126. A cubical box, 1 foot in height, is filled with water; and 8 equal spheres, ½ foot in diameter, are placed in it. What volume of water remains?

127 A scale of German inches is placed by the side of a scale of English inches, so that the first division of the two scales coincide. Find the divisions of the English scale which coin-

cides nearest with the German scale. (The length of each scale is 10 feet, and 100 German inches are the same as 106.577 English inches.)

128. If a person invest in the $3\frac{1}{2}$ per cent., at $92\frac{1}{4}$, including the broker's commission. What interest will he get per cent.?

129. How many ounces of tea, at 3d., 4d., 5d., and 8d., are required to make a mixture at 6d.?

130. If 24 pioneers, in $2\frac{1}{2}$ days of $12\frac{1}{2}$ hours long, can dig a trench 139.75 yards long, $4\frac{1}{2}$ wide, and $2\frac{1}{2}$ deep, what length of trench will 90 pioneers, in $4\frac{1}{5}$ days of $9\frac{2}{3}$ hours long, dig, the trench being $4\frac{7}{8}$ yards wide and $3\frac{1}{5}$ deep?

131. Express $2\frac{1}{2}$ Spanish piastres in French money, exchange being at the rate of $25\frac{1}{3}$ fraucs per £1 sterling, and £3. 7s. 6d. for 20 piastres.

132. A can mow $2\frac{1}{2}$ acres of grass in $4\frac{2}{3}$ days, B $2\frac{1}{6}$ acres in $3\frac{1}{2}$ days; they mow together a field of 10 acres. In what time will they do it, and how many acres will each mow?

133. The water in a river, 16 feet deep and 144 yards wide, flows at the rate of $3\frac{1}{2}$ miles per hour. Find how many cubic feet of water run into the sea per minute; also the number of tons, supposing a cubic foot of water to weigh 1000 ounces?

134. Through a piece of wood, 2 feet 4 inches in length, a hole is made, the section of which is a square of $8\frac{1}{2}$ inches on a side, and the largest possible cylinder is inserted in the hole. Find the space left vacant.

135. If a gallon of water were resolved into the oxygen and hydrogen of which it is composed, it is required to determine the volume into what it would thus be expanded, water being 741 times heavier than an equal volume of oxygen, and 9699 times heavier than an equal bulk of hydrogen.

136. A person mixes 11 lbs. of tea with 5 lbs. of an inferior quality, and he gains 16 per cent. by selling the mixture at 7s. 3d. per lb. Find the prime cost, when a pound of the one costs 1s. more than a pound of the other.

137. Find the converging fractions approaching to $\frac{5\text{ h. } 48\text{ m. } 48\text{ s.}}{24\text{ hrs.}}$

138. At what rate per cent. per annum must £1200 be lent for 2 years, to amount to £1348.32 ?

139. How must goods bought at 8s. 4d. a yard, be sold to realize 15 per cent., allowing 5 per cent. on the cost for expenses ?

140. A train moves the first minute at the rate of 6 miles an hour, each minute its speed increases by $\frac{1}{3}$ of 8 miles. At what rate per hour will it travel in 10 minutes ?

141. £62 10s. is paid, the first payment being £1 11s. 3d., the others increasing three-fold, find the number of payments.

142. Exchange between Russia and England is 6.28 rubles, per £1, and between Russia and America, 6.28 rubles for 4.55 dollars; also between America and England 4.85 dollars, per £1. Find the gain on transmitting 6000 rubles through England to America ?

143. The Britannia tubular bridge consists of two rectangular tubes of iron, each 1513 feet in length, 26 feet in average depth, and 14 feet 8 inches in width, the internal depth and width being respectively reduced by construction to 22 and 14 feet. Find the volume of air in the tube, and also the contents of the construction itself ?

144. The length of Jupiter's orbit is found to be 3104000000 miles, which is described in 4332.62 days. The length of the earth's orbit is 596900000 miles, which the earth describes in 365.26 days. What is the difference of the uniform rate of speed per hour of the two planets.

145. It is supposed that one man consumes 25 cubic feet of oxygen in 24 hours; and supposing he takes 21 respirations in a minute, how much does he consume at each respiration ?

146. An omnibus conductor was convicted of defrauding his master, who stated that if he were thus defrauded, of only 3d. for every journey made by each of the omnibusses of which he was the proprietor, he should lose £2901. 15s. per annum. Supposing each omnibus took 6 journeys daily, how many omnibusses had the master ?

147. A principal of £10000 is lent at compound interest. At what rate per cent. per annum must it be lent to amount to £11576. 4s. in three years?

148. If a man earn 3s. $6\frac{1}{2}$d. a day by mowing grass at 5s. 8d. per acre, when he works 12 hours a day, how long must he work a day, and what quantity must he mow to earn 3s. 9d. a day, when he is allowed 5s. an acre?

149. How many revolutions will the driving wheel of a locomotive 18 feet in circumference make between London and Birmingham, a distance of $112\frac{1}{2}$ miles? And how many times will the cylinder discharge its steam on the journey (the cylinder discharges steam 4 times in each revolution.) Also how long will it be going, supposing it to move at an average rate of 30 miles an hour when at work, and stopping at 12 stations; at the first station 1 minute, and at the twelfth 15 minutes, increasing the period of stopping in arithmetical progression? Also how much coke will the engine consume supposing that while in motion it requires 6 bushels per hour, and when stopping 4 bushels per hour? And to what sum will the price of the coke amount at 5d. per bushel?

150. The rent of a house is £120 per annum. It is assessed at the rate of $\frac{2}{3}$ the rent, the poor rate is 7s. 6d. in the pound, the paving rate is 1s. 9d. and the church rate 4d. How much does the tenant pay altogether for his residence?

151. A person increases his estate annually by £100 more than the quarter of its value, and at the end of 4 years he found his estate amounted to £10342. 3s. 9d. What had he at first?

152. A person after paying an income tax of 7d. in the pound, and a rate equal to $\frac{3}{4}$ of $\frac{5}{7}$ of it, has £458. 10s. left. Find his income

153. Prove that the simple interest of £260 for 4 months at $3\frac{7}{8}$ per cent. per annum, is equal to the discount of £263 7s. 2d. for the same time at the same rate.

154. What is the difference between the true and bankers' discount, on a bill of £1209. 12s., drawn October 13th, at 4 months, and discounted December 5th, at 4 per cent. per annum, C. I.?

155. A cubic foot of wood weighs $11\frac{10}{11}$ lbs. What is the weight of a beam, 24 feet long, $2\frac{3}{4}$ feet wide, and $2\frac{1}{4}$ feet thick; and what is its value at 2s. 2d. per cubic foot?

156. A wishes to buy property that will cost £50000, for which purpose he sells the following stocks, of which he is the holder, viz.: £1750 income in the 5 per cent., at 121.40, and £800 in the 3 per cent., at 83.50. How much had he left after he bought the property ?

157. A has 3 lumps of metal of the same volume, but of different weights: 5 cubic inches of the 1st weigh $69\frac{3}{4}$ oz., $3\frac{1}{3}$ cubic inches of the 2nd weigh 41 oz., and $4\frac{2}{7}$ cubic inches of the 3rd weigh 91 oz. Find the weight of 1 cubic inch of the mixture of these metals.

158. Brass contains 3 parts of copper to 7 of zinc; copper costs 1.60s., and zinc .30d. per lb. Find the price of 1 cwt. of brass.

159. A bought oil for £240; $\frac{1}{4}$ of the payment is to be made in 3 months, $\frac{1}{3}$ in 6 months, and the rest in 10 months. In what time should the whole sum be paid in one payment?

160. Divide 20s. among 6 men, so that each shall have $2\frac{1}{2}$d. more than the other.

161. In a field of 12 acres there are grazing 2 horses, 3 cows, 12 sheep, and 1 ass; each horse eats as much as 1 cow, 1 sheep, as 1 ass; each cow as much as 1 sheep and 1 ass, and 1 sheep one-third part of the ass. What part of the pasture does each kind of animal eat?

162. The united weight of two persons is 148 lbs., their difference 48lbs. Find the weight of each in ounces; and also how many cubic inches each contains, the specific gravity of living men being .891.

163. The relation of two magnitudes is expressed by the continued fraction:

$$\cfrac{2}{3+\cfrac{4}{5+\cfrac{1}{2+\cfrac{1}{4+\cfrac{1}{1+\cfrac{2}{1+\cfrac{1}{3}}}}}}}$$

Find the converging fractions showing the approximate ratio of these magnitudes.

164. I bought goods for £150, upon which I want to gain 10 per cent. and allow 12 per cent. discount to the purchaser. What will be the selling price?

165. A, B and C perform $\frac{4}{5}$ of a work together in 24 days, but if A or B, who do the same work in equal times, had been taken away, then $\frac{3}{8}$ of it would only be accomplished in 28 days. In what time can each do it separately?

166. What is the vulgar fraction equivalent to .08134765625 ?

167. The advance of the hour hand of a watch beyond the minute hand is measured by $24\frac{2}{3}$ of the minute divisions, and the time is between 6 and 7 o'clock. Find the time by the watch.

168. A work is to be performed by 15 workmen in 24 days; after having been 9 days working, 4 men fell ill, who returned after being absent 7 days. How long were they doing the work?

169. A wall can be built in $8\frac{3}{4}$ days by 16 men. The first day all the men take part in the work, and every succeeding day their number is diminished by one. How long will it take building the wall ?

170. 21204 yards of stuff are to be woven in 20 days by 21 men, working 8 hours a day, when they had been 13 days working, 9 of the men ceased attending. How many hours per day must the remaining weavers work, in order to have performed their task in the required time?

171. What principal must be lent at 4 per cent. per annum, so that its interest after 7 months, being lent at 5 per cent. for six months, shall bring £8. 15s.?

172. A bought for £1200, merchandise which he sold to B at 5 per cent. profit; B sold it to C, at $4\frac{3}{4}$ per cent. profit; C to D, at $4\frac{2}{5}$ per cent. profit; D to E, at $3\frac{1}{4}$ per cent. profit. What did the goods cost each man? How much is the profit of each individual?

173. P had goods to the amount of £25640 prime cost, which he disposed of during the year. At how much per cent. did

he retail his goods, having covered the prime cost, his annual expenses of £2460, and increased his capital by £1680?

174. The latitude of Greenwich is 51° 28′ 40″, and at the solstices the sun is 23° 27′ 34″ from the equator. Find the zenith distances of the sun at both solstices.

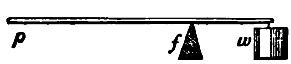
175. Find the meridian altitude of the sun at the summer and winter solstices, in the latitude of Greenwich.

176. The proportion of the solid area of our planet is to that of the liquid parts as 10 to 27. If the mean depth of the sea were 484 yards, what would be the weight of the water, and what would be the weight of the salt and other impurities wherewith it is impregnated?

177. A cloth merchant values his cloth at 18s. per yard in bartering with B, which is $12\frac{1}{2}$ per cent. above his invoice price. Required, how B should rate his goods, which cost 2s. 6d. per yard, to be even with him, and the prime cost of the cloth.

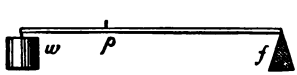
178. I have a case of liquors in bond on which I wish to pay the duty; the case contains 48 bottles of $3\frac{3}{4}$ gills, each bottle weighs 14 ounces. What is the duty, knowing that the duty on foreign liquors is £1. 10s. 4d. per gallon, and on glass 3s. per cwt.

179. A lever is an inflexible rod turning upon a fixed point, called a fulcrum. Levers are of three kinds according to the relative situations of the power and weight:



1st kind, as balances, scissors, &c.

2nd kind, as oars.



3rd kind, as toys, the bones and muscles of animals, &c.

Now in every lever the power × the units in its arm = the weight × the units in its arm; or in other words, the momentum of the power = the momentum of the weight. Suppose

the weight is 20 lbs., its arm 3 feet, and the power's arm 5 feet, what must the power be in case of equilibrium?

$$\text{Power} = \frac{\text{Weight's arm} \times \text{Weight}}{\text{Power's arm}} = \frac{3 \times 20}{5} = 12 \text{ lbs.}$$

180. A weight at the end of its arm 9 inches in length, balances 2 lb. 14 oz. at the end of the other arm 5 inches in length. Find that weight.

181. Five persons of equal strength hold the ends of a pole 10 feet long, to which a weight is hung. Two persons are at one end, and three at the other. Where must this load be fixed, so that every bearer may have the same weight to carry?

182. In a false balance, (one in which the arms are of unequal length,) a body appears to weigh when in one scale, and the weight in the other, 24 lbs., but when weighed in the other $18\frac{1}{2}$ lbs. The real weight is required.

In this question we have $24 \times a = \text{W} \times b$, and $18\frac{1}{2} \times 6 = \text{W} \times a$.
$\therefore\ 24 \times a \times 18\frac{1}{2} \times 6 = \text{W}^2 \times a \times 6$, &c , &c.

183. It is required to make a steel-yard with a rod, the unvariable arm of which is 4 inches, and the moveable weight used 2 lbs. At what distances from the fulcrum must the weight be placed so as to equipoise 1,2,3,4,...12 lbs.

184. There is a cistern whose length is $\frac{7}{3}$ of its breadth which is $\frac{7}{3}$ of its depth and whose solidity is $1157\frac{5}{8}$ cubic feet. Required its length, breadth, and depth, as well as how many imperial gallons it contains. Also what difference is there in the weight of water it contains, allowing a cubic foot to weigh 1000 oz. and the imperial gallon to weigh 10 lbs.?

185. Divide £$229\frac{3}{4}$ between A, B, and C so that for every $5\frac{5}{8}$ shares taken by A, B takes $4\frac{3}{7}$; and for every $4\frac{1}{6}$ shares taken by B, C takes $3\frac{5}{8}$; what did each man receive?

186. A received £2100 which was $\frac{3}{4}$ of B's money, now three times B's money was half C's; what was C's money?

187. Suppose I plant a bean, and its produce be 21 the first year, how much land (allowing 6 inches for each bean) will it take to plant 10 years' produce?

188. A garden roller 38 inches long, outside diameter 22 inches, and the thickness $2\frac{1}{2}$ inches, is made at the rate of 3d. per lb.

Supposing the cubic inch to weigh 6 oz. and the axle and handle to weigh $\frac{1}{12}$ of the roller, what is the value of the roller?

189. What length of a gun $6\frac{3}{4}$ inches in diameter, will be filled with 12 lbs. of powder? (See Art. 418.)

190. A lady having bought some silver articles which weighed 6 lbs. $0\frac{3}{5}$ oz. troy, but suspecting she had been defrauded, ordered them to be weighed in the opposite scale, when they weighed only 5 lbs. Required the true weight and the amount at 4s. $10\frac{1}{2}$d. per ounce.

191. In 1842 the number of letters which went through the post office of this country was $208\frac{1}{2}$ millions, and in 1852 the number was $379\frac{1}{2}$ millions. What is, on the average, the yearly increase, and also the increase per cent.?

192. Since the pressure of a fluid against any upright surface, as a canal gate, is equal to half the weight of a column of the fluid whose base is the surface pressed, and its altitude the same as the altitude of that surface, find the pressure of the water against a canal gate, the depth of which is 15 feet and breadth 12 feet.

Weight of half the column of water is $\frac{15}{2} \times 1000$ ounces, or 7500 ounces; area of gate is 15×12 or 180 square feet.

$\therefore$ Pressure $= \frac{15}{2} \times 1000 \times 15 \times 12$ or $7500 \times 180 = 1350000$ or 84375 lbs. or $37\frac{2}{3}$ tons nearly.

193. What is the pressure against the gate of a sluice, the dimensions of which are: depth 9 feet 6 inches, and breadth 3 feet 8 inches?

194. The pressure on the bottom of a pail of water, the radius of which is 1 feet 6 inches, is 108 lbs.; find the pressure per square inch.

195. A spherical leaden ball is suspended by a string in a cylindrical vessel containing water; determine the additional pressure sustained by the base. (See table of specific gravity and exercise 3.)

196. Required the contents of an irregular block of marble, which weighs 12 cwt. (See table of specific gravity.)

197. How many cubic feet are there in 3 tons 7 cwt. of mahogany? (See table of specific gravity.)

198. What is the weight of a block of iron 12 feet long, 9 inches broad, and 6 inches deep ?

199. What is the weight of 4 gallons of olive oil ?

200. The internal diameter of a leaden pipe is 3.6 inches, the thickness .25 inches. Required the weight of 20 feet in length.

201. Pure alcohol contains 1 atom of oxygen, 2 atoms of carbon, and 3 atoms of hydrogen ; acetic acid, 4 atoms of carbon, 3 of oxygen, and 3 of hydrogen; oxalic acid, 2 of carbon, 6 of oxygen, and 3 of hydrogen. Determine (in vulgar and decimal fractions) the atomic expression and the composition of these substances. (See Ex. 44 and following.)

202. A farmer has a field 348 yards long and 168 yards broad, which he exchanges for one of the same area, the breadth of which is 1240 links. Its length is required.

203. The income of N is derived from money in the $3\frac{1}{4}$ per cent. stock at $89\frac{1}{2}$, which he spends in the following manner: household expenses £46 10s, 6d. weekly, other expenses £464 7s. 8d. annually. Of how much stock is he the holder and what is his fortune ?

204. When flour is at 1s. 10d. per stone. $4\frac{1}{4}$ lbs. of bread cost $6\frac{1}{2}$d. ; what must be the price of bread per lb. when flour is at 2s. 2d. per stone ?

205. Twenty weavers in 7 weeks, working 6 days a week, and 11 hours a day, make 164 pieces of stuff, each piece 36 yards long and $1\frac{1}{4}$ broad ; how many pieces will 12 weavers make in 5 weeks, working 6 days a week and 12 hours a day, when the pieces are 40 yards long and $1\frac{1}{12}$ yards broad ?

206. Three merchants enter into partnership. A puts in £400 for 8 months, B £500 for a certain time, C's share continued 6 months. Now A's gain was £300, B's £540, and C's £660. How long did B's money continue in ; how much was C's share, and how much was the whole stock ?

207. What principal must be lent at $3\frac{3}{4}$ per cent. per annnm to yield 17s. 6d. interest daily ?

208. At what rate per cent. per annum must £1500 be lent for 5 years to produce as much as £1200, at $4\frac{1}{2}$ per cent., for $7\frac{1}{2}$ years.

209. S owes £900, to be paid as follows: £400 at the end of the first year, £300 at the end of the second, and the remainder at the end of the third; but wishing to settle immediately, what discount at $4\frac{1}{2}$ per cent. per annum is he allowed?

210. If I buy 36 pieces of stuff at £30. 12s. per piece, and sell 15 pieces at £5. 10s. per piece, at what rate per piece must I sell the remainder to gain 24 per cent. by the whole?

211. A merchant bought $15\frac{1}{2}$ hogsheads of wine at £38. 12s. per hogshead, which he is compelled to retail at £35. 15s. per hogshead. What is the loss per hogshead, per gallon, on the whole, and per cent.?

212. Sold goods at £4. 15s. per cwt., and lost 18 per cent., but the price rising, they are sold at such a rate that I gain 10 per cent on the buying price. How much were they sold at in the last instance?

213. Two grocers gave an order for some sugar, for which they paid £36. 16s. A's share of it was $\frac{1}{8}$; B sells $\frac{3}{4}$ of his share, at $8\frac{1}{2}$d. per lb. and gains thereby 32 per cent. Find how much sugar was ordered, the quantity each grocer received, and the buying price per lb.

214. If 5 gallons of wine at 12s. 6d. per gallon, be mixed with 6 gallons of another sort at 11s. 4d., 2 gallons of brandy at 18s. 6d. and 4 gallons of water; find the price of the mixture per gallon.

215. How much pure gold is there in a nugget 7 lbs. 6 oz. weight, and 15 carats fine?

216. In a composition 18 carats fine, the gold weighs 2 lbs. 5 oz., find the weight of the whole mass.

217. A silversmith melts silver 14 carats fine, with silver 6 carats fine. How much of each must be take to make 9 oz. of a composition 11 carats fine?

218. With goods worth 18s. 6d. and 13s. 3d. it is required to make a mixture worth 15s. 9d. How must it be done?

219. A nugget weighs 6lbs. 7 oz., and is 17 carats fine, what is its value at £3. 16s. 8d. per ounce of pure gold.

220. I bought 280 gallons of wine for £245. It consisted of a mixture of three kinds, viz.: at 20s., 18s., and 12s. per gallon.

How much would there be of each kind in that compound?

221. Express in metres 1 inch, 1 foot, 1 yard, 1 mile.

222. Reduce to grammes 1 grain, 1 pennyweight, 1 ounce, 1 pound troy; also 1 pound avoirdupois; 1 hundred weight; 1 ton. (1 pound troy = 373.2 grammes.)

223. A lever 5 feet in length weighs 10 lbs., what weight on the shorter arm will balance 20 lbs. on the longer, the fulcrum being one foot from the end?

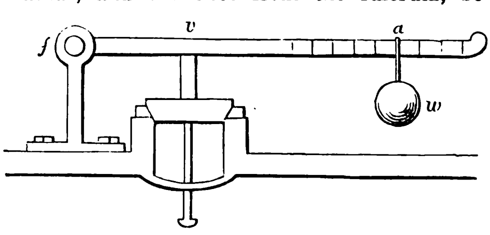
Solution: Here we have a third force taken into account, which was not considered in Ex. 179. It must be viewed as acting in the centre of gravity of the lever, and is added or subtracted from the weight according as it tends to favour or to oppose it. And the general expression for cases of this sort is: $wa \pm wg = pb$.

Where a represents the weight's arm; b the power's arm; w the weight of the lever, and g the distance of the new force from the fulcrum.

Thus in our example we have

$$w \times 1 - 10 \times 2 = 20 \times 4$$
$$\therefore\ w = 100 \text{ lbs.}$$

224. If the safety valve v of a steam engine boiler, $2\frac{1}{2}$ inches radius, and 3 inches from the fulcrum, be pressed up by a force of 60 lbs. per square in., what must the weight w, (placed at a on the lever $f\,a$, 30 inches in length,) be to equipoise the force of the steam; the weight of the valve is 6 lbs., and the weight of the lever 8 lbs.?



Solution: Area of $v = 5^2 \times .7854$, or 19.635 square inches.

$19.635 \times 60 = 1178.1$ lbs. total pressure of steam on v.

$1178.1 - 6 = 1172.1$ lbs. effective pressure of steam on v.

$30 \times w =$ effective power of lever.

$15 \times 8 = 120$ lbs., effect of weight of lever. (See Ex. 223.)

$$\therefore\ w \times 30 + 15 \times 8 = 1172.1 \times 3.$$

$$\therefore\ w=\frac{3516.3-120}{30}=\frac{3396.3}{30}=113.21 \text{ lbs.}$$

225. When the safety valve is 3 inches in diameter, its distance from the fulcrum 3 inches, and its weight 4 lbs.; length of the lever 24 inches, and its weight 10 lbs.; and the weight of w 20 lbs., find the pressure of the steam per square inch on the valve necessary to balance the weight w.

226. If a mixture of $1\frac{1}{2}$ lbs. of snow, at 32° F., and 3 lbs. of water, at 162° be made, find the temperature of the mixture. (See Ex. 105.)

227. What will be the temperature of a mixture of 5 lbs. of water, at 84° F., and 7 lbs. at 112°, the loss resulting from mixing being neglected.

228. A person has a glass filled with wine, out of which he drinks the half, and fills it up again with water; now he drinks one-third part of it, and fills it up a second time with water; out of this he drinks one quarter. Find how much wine there is left in the glass.

229. A square field, the perimeter of which is 964 yards, is exchanged for another of the same perimeter, but whose area is found to be $\frac{7}{16}$ of the former. Its dimensions are required.

230. A owes to B £322, payable in 12 months; £287 in 8 months; £612 in 4 months; £1260 in 1 month; and £2450 ready money. Find the equated time to pay the whole.

231. A has two bills to be drawn on Paris, one of 2450 francs, to be paid in 66 days; and the other of 4500 francs, to be paid in 36 days. What is the equated time for drawing both bills?

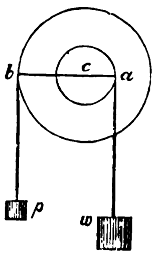
232. In the Linnean classification of plants the number of classes established is equal to the cube root of 13824. Find it.

233. When the year begins on a Tuesday, on what day of the week will the anniversary of the battle of Waterloo occur, which was fought on the 18th of June, 1815?

234 Find the cube root of the cube root of 7691483, correct to two decimal places.

235. Find the square root of the square root of 13672, correct to three decimal places.

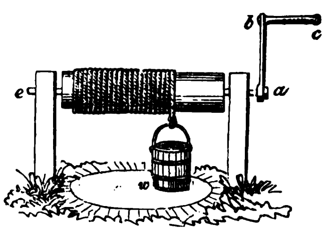
236. In the wheel and axle $b\ a$, let the radius of the wheel b $=18.5$ inches, and the power p applied to it 40 lbs. Required, the weight w acting on the axle a; the radius of which is 6 inches, to produce an equilibrium, neglecting the friction.



Solution: Here we have merely a modification of the lever, so that $a\ b$ is a lever moveable about the centre c; the power p acts by the arm $c\ b$, and the weight w by arm $c\ a$; and thus we have, as in Ex. 179; $p \times c\ b = w \times c\ a$, and for our question we have $40 \times 18.5 = w \times 6$.

$$\therefore\ w = \frac{40 \times 18.5}{6} = 123\tfrac{1}{3} \text{ lbs.}$$

237. The handle $a\ b\ c$ of a windlass is 1 foot 4 inches, and the radius of the axle $a\ e$ $3\frac{1}{2}$ in. A man acts with a power of 36 lbs. in the direction of the tangent of the circle described by the handle. What weight would he balance were there no friction?



Solution: The annexed figure differs from the preceding in having the wheel replaced by a handle; its mechanical property is the same. Then, $p \times$ the length of the handle $= w \times$ the radius of the axle.

Or, in this case, $36 \times 16 = w \times 3\frac{1}{2}$.

$$\therefore\ w = \frac{36 \times 16}{3\frac{1}{2}} = 164\tfrac{4}{7} \text{ lbs.}$$

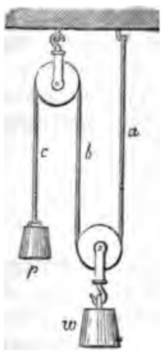
Hence, the power gained by this machine is about fourfold.

238. A power of 8 lbs. keeps in equilibrium a weight of 320 lbs. by means of a wheel and axle; the diameter of the axle is 9 inches. What is the radius of the wheel?

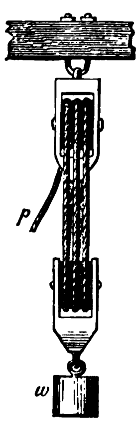
239. A man exert a power of 72 lbs. upon a windlass, the handle of which is 1 foot 8 inches, and the radius of the axle 4 inches. What weight will he balance, and what is the advantage gained by the machine?

240. What must be the length of the handle of a windlass, the radius of the axle of which is $3\frac{1}{2}$ inches, so that a power of 60 lbs. shall keep in equilibrium 720 lbs.?

241. A pulley is a small grooved wheel, which moves round an axis fixed in a block or case. It is made to turn by means of a cord passed round its circumference, which transmits the force applied in any convenient direction. If a power sustain a weight by means of a *fixed* pulley, the power and weight are equal; but when the power sustains a weight by means of *moveable* pulleys, the weight raised is always greater than the power applied. In this single moveable pulley, the cord has the same stretch or tension. Then *w* is supported equally by the two cords *a* and *b*; hence each sustains one half *w*. But *c* has the same tension as *b*, therefore to produce a state of equilibrium the power must be equal to half the weight. The velocity of the power is evidently double the velocity of the weight.



In this system of pulleys, let the weight to be raised be $220\frac{1}{4}$ lbs. Required, the power when equilibrium takes place.

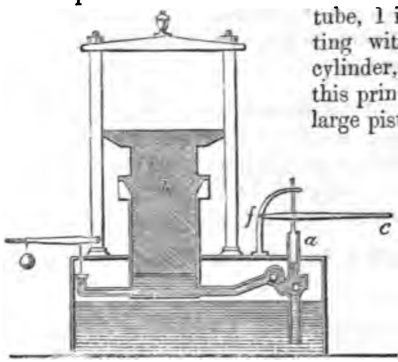


242. The same principle is applicable to all systems of pulleys having one fixed block, any number of moveable wheels and a single cord; for, in the annexed system, the same rope passes over all the pulleys, and the tension of the cord is uniform throughout; and as the weight is supported by 6 cords, therefore each carries one-sixth of *w*, and the power must also be one-sixth of the weight. The velocity of *p* will necessarily be 6 times the velocity of *w*. In this system, there being 6 pulleys, what weight can be supported by a power of 12 lbs.?

243. The length of a cistern is 9 feet, its breadth $4.\dot{6}$, and depth 3.5. What will be its contents, in gallons?

244. A cistern is to contain 1000 gallons; its length is 8 feet, and breadth 6.5. What is its depth?

245. It is ascertained that liquids transmit equally, and in all directions, the pressure exerted on any part of them, so that if a pressure of 12 lbs. be exerted on the water in a small tube, 1 inch in area, communicating with the piston of a large cylinder, 64 inches in area; by this principle, the pressure on the large piston will be 64×12 lbs., or 768 lbs.



The Bramah, or hydrostatic press, is a machine in which this property, combined with the lever of the second class, produces an enormous pressure. This was illustrated in a very striking manner at the raising of the tubes for the Britannia Bridge, across the Menai Straits.

In a hydrostatic press (see the figure) the surface of piston *a* is 3 inches, of *b* 280 inches; the lever *c f* is 18 inches long, and acts on the piston at 2 inches from the fulcrum *f*. What pressure will be produced upon *b* by a power of 80 lbs. applied to the lever?

Solution: By the lever, we obtain a pressure of $\frac{18 \times 80}{2}$, or 720 lbs. upon the small piston, or upon 3 inches of surface; therefore, $\frac{280}{3}$ or $93\frac{1}{3}$, viz., as many times as the surface of the large piston contains the surface of the small one, so many times will the pressure of 720 lbs. be increased. $\therefore 93\frac{1}{3} \times 720 = 67200$ lbs., or 30 tons, is the pressure on the large piston.

It will be noticed that whatever may be the gain in power, it is bought by an equivalent loss of motion; for, if the small piston be made to move the distance of 1 inch, the water is

thrown out into the large cylinder $93\frac{1}{3}$ times as large; therefore the piston will be raised $\frac{1}{93\frac{1}{3}}$, or $\frac{3}{280}$ of 1 inch.

246. The diameter of the small piston of a hydrostatic press is 4 inches, that of a large one 2 feet 6 inches; the lever is 3 feet long, and the piston rod is 3 inches from the fulcrum. If a power of 1 cwt. be applied to the lever, what pressure will be produced upon the large piston ?

247. In a hydrostatic press the diameters of the two pistons are, respectively, 3.75 inches and 30.4 inches; the lever is $2\frac{3}{4}$ feet, and the piston rod is $2\frac{1}{4}$ inches from the fulcrum. When a power of 100 lbs. is applied to the lever, what pressure will be produced upon the large piston ?

248. From what height will a body fall in 6 seconds ? (See Ex. 28.)

249. In what time will a body fall from a height of 402 feet? (See Ex. 28.)

250. The power of a steam engine is easily calculated: supposing the diameter of a cylinder to be 24 inches, the area will be $24^2 \times .7854$, or 452.39 square inches, let the pressure of the steam on every square inch of the piston be 12 lbs.; therefore, 452.39×12, or 5428.68 lbs., is the force with which the piston is pressed. Now, if the length of the stroke be 5 feet, and the piston makes 44 strokes in a minute, it will move through 44×5 feet, or 220 feet in one minute; and the power of the engine will be equivalent to 5428 lbs. raised 220 feet in a minute. But it is found convenient to estimate the power of a steam engine by horse power, and Watt admitted that a horse could raise 33000 lbs. one foot high in one minute. Then, for our example, we have:

33000 lbs.	raised	1 foot	high in	1	minute	$= 1$	horse power;
$\therefore$ 1	"	1	"	1	"	$= \frac{1}{33000}$	"
$\therefore$ 5428	"	1	"	1	"	$= \frac{5428}{33000}$	"
$\therefore$ 5428	"	220 feet	high in	1	"	$= \frac{5428 \times 220}{33000}$, or 36	

horse power.

Then, generally. let d inches be the diameter of the piston;
p lbs. the effective pressure of the steam upon each square inch;
l feet the length of the stroke of the piston;
n the number of strokes per minute.

$$\therefore \text{the H.P. of a steam engine} = \frac{d^2 \times .7854 \times p \times l \times n}{33000}.$$

It is necessary to diminish the given pressure by about $\frac{1}{5}$, for friction, &c.; thus, if the pressure of the steam is 15 lbs. to the square inch, the effective pressure is $\frac{4 \times 15}{5}$, or 12 lbs. to the square inch.

251. The diameter of the piston of a steam engine is 24 inches, the length of the stroke of the piston 4 feet, the number of strokes per minute 48, and the pressure of the steam 30 lbs. per square inch. Required to find the H. P. of the engine?

252. The diameter of the piston of a steam engine is 2 feet 6 inches, the length of the stroke 6 feet, the number of strokes per minute 40, what pressure must the steam exert per square inch, so that the engine performs the work of 80 H. P.?

253. Suppose the engine of the last example is used to raise water from a mine 240 feet deep, how many cubic feet will it raise per hour.

Solution:

An engine 80 H.P., in 1 hour, raises 1 ft. high $80 \times 60 \times 33000$ lbs.

$\therefore$,, 80 ,, 1 ,, 240 ,, $\frac{80 \times 60 \times 33000 \text{ lbs.}}{240}$ or 660000 lbs.

But $\because$ 62.5 lbs. is the weight of 1 cubic foot of water,

$\therefore$ 660000 lbs. ,, $\frac{660000}{62.5}$, or 10560 cubic feet.

254. How many cubic feet of water will an engine raise in 8 hours, from a mine 120 yards deep, the diameter of the piston is 2 feet 4 inches, the length of the stroke 4 feet, the number of strokes per minute 36, and the pressure of the steam 24 lbs. per square inch?

255. What is the H. P. of an engine which raises 20000 gallons of water from a mine 80 feet deep, in 12 hours?

256. In what time will a 24 H. P. engine empty a reservoir 600 yards long, 360 broad, and 10 feet deep; the water to be raised 40 feet?

257. What must be the pressure of the steam, so that an engine may raise 3000 gallons of water per hour from a mine 560 feet deep, the length of the stroke of the piston is 5.5 feet, the number of strokes 32 per minute, and the diameter of the piston 18 inches?

258. To ascertain the capacities of three casks, it is known that if the first be filled with the contents of the second, there remains in the latter $\frac{2}{9}$ of its capacity: if the second be filled with the contents of the third, there is left in the latter $\frac{1}{3}$ of its capacity; lastly, if the contents of the first were poured into the third, there would yet be room for 50 gallons more. Find the gallons which each cask contains.

259. I have laid out £8000 in bills to India, at the rate of 1s. $10\frac{1}{4}$d. per rupee, and £5000, in like bills, at 1s. $9\frac{3}{4}$d. per rupee. At what rate per rupee should the bills purchased be sold, in order to realize the sum invested and 10 per cent. profit; also what would the difference be, had the bills been sold at 1s. $10\frac{1}{3}$d.?

260. A lends two sums of money to B: on the 15th of March, 1853, £1200, at 4 per cent.; and on the 1st of September, 1853, £1600, at 5 per cent. A wished to receive the interest of both on the same day, and at the same rate per cent. On what day can this be effected, and at what rate per cent.?

261. A father left his 6 sons £14670 in cash, and 4 bills of £750 each, due at the end of three, four, five, and six months respectively. The eldest son had, by the will, $\frac{1}{8}$ of the property, and having the management of the whole, he paid his brothers their shares (which were equal) in cash. How much were they, discount being reckoned at $4\frac{1}{2}$ per cent. per annum?

262. What would be the amount of 5 yearly payments, each of £1500, put into a bank for five years, at $3\frac{1}{2}$ per cent. per annum, C.I ?

263. In the following accounts the pupil is required to find the quantities left blank:—

London, October 30th, 1853.

Mr. W. Langton,

To A. Jourdan.

		£.	s.	d.
1853.				
Jan. 23.	212 tons 16 cwt. 3 qrs. of coals, at £1 4s. per ton			
Mar. 15.	12 pieces, each 25 ells (English), of Irish linen, at 6s. 4d. per yard..................................			
24.	Sold £1650 in the 3 per cent., at 84¼, brokerage ⅛ per cent.			
June 6.	——— francs, exchange at 24.12 francs per £1 sterling	154	8	8
July 1.	2410 francs, exchange at——francs per £1 sterling	99	15	1¼
10.	Proceeds of two bills, each £114. 16s. for 36 and 48 days, at 3½ per cent.			
	Received	£2750	14	9
	Balance.....................			

Liverpool, November 6th, 1853.

264. Mr. O. Cambridge,

To C. Oxford.

		£.	s.	d.
April 4.	564 doz. of silk handkerchiefs, at 32s. 7d. per doz.			
May 8.	Factor's commission on £1760. 19s. 4d., at 2½ per cent..			
24.	Cost of £ ——— 3 per cent. consols, at 92¾, and brokerage ⅛ per cent.	560	14	4
July 16.	27.4 bales of coffee, each 3 cwt. 2 qrs. 20 lbs. gross; tare 15½ lbs., tret and cloff as usual, at £5. 14s. 6d. per cwt. net			
Aug. 20.	Insurance on £6480. 15s., premium £3. 16s., and policy ⅓ per cent.			
Oct. 6.	Amount of £840 for 124 days, at 6½ per cent. per annum			
31.	Sale of £—— Bank Stock, at 196¼, and brokerage ⅛ per cent..	1260	12	0
Nov. 3.	—— milrees —— rees, exchange at 62½d. per milree..	864	10	0
	Received	£3224	16	9
	Balance....................			

265. I met a man and a woman begging, to the man I gave $\frac{3}{5}$ of $\frac{3}{4}$ of my money, to the woman $\frac{3}{8}$ of $\frac{3}{8}$ of the remainder, and then I had £2. 16s. 4½d. left. What sum had I at first?

266. A can do as much work in 5 hours as B can do in 7 hours, or as C in 8 hours. How long will A be completing a work, of which B, working 12 hours, and C 16 hours, can do together two-fifths?

267. I have a horse to sell which cost me £112. I wish to get 10 per cent. profit, and to allow the purchaser 12 per cent. discount. What will be the selling price?

268 Two clocks point out 12 at the same time, one of these gains 8 seconds, and the other loses 9 seconds in 12 hours. After what interval will one have gained an hour over the other, and what time will each then show?

269. How many guineas, sovereigns, crowns, half crowns, shillings, sixpences, and pence, of each the same number, are there in £735?

270. If the height of a cube be 1 foot 6 inches, what is the altitude of another cube, which is three times the magnitude of the first?

271. Required, the diagonal of a square field, containing $12\frac{1}{4}$ acres.

272. A corn factor sold 80 quarters of corn for £150. 4s., at 48s., 44s., and 41s. How many quarters were there of each kind?

273. If an engine of 85 horse power, with a driving wheel $7\frac{1}{2}$ feet diameter, will draw a train of 45 tons at the rate of 35 miles per hour, what weight can an engine of 120 horse power, with a driving wheel of $6\frac{1}{2}$ feet diameter draw, at 30 miles an hour?

274. If I transfer stock to the value of £720 from the 4 per cent. at $92\frac{3}{4}$ to the $3\frac{1}{2}$ per cent. at $86\frac{1}{4}$, what shall I gain or lose annually?

275. A company consisting of 1 captain, 2 lieutenants, 8 sergeants, 16 corporals, and 650 men, took the enemy's chest of £5400, which is divided among them according to their pay and the time of their service. The captain has been in the army for 6 years, and is paid 12s. 6d. a day; the lieutenants have served $4\frac{1}{4}$ years, and are paid 7s. 6d. a day; the sergeants have served 8 years, and are paid 2s. 6d. a day; the corporals have served 5 years, and are paid 1s. 8d. a day; and the men have served $9\frac{1}{2}$ years, and are paid 1s. 4d. a day. What *portion* of the prize should each receive?

276. Find the interior edge of a cubical vessel which will hold exactly 2 cwt. 3 qrs. 16 lbs. 12 oz. of water.

277. How many yards of silk may be had for 4720 lbs. of cotton, if for $1\frac{1}{2}$ yards of silk, 13 lbs. of coffee may be had; for 7 lbs. of coffee, 2 lbs. of tea; and for 16 lbs. of tea, 49 lbs. of cotton?

278. A grocer, by means of false scales, defrauds to the extent of 15 per cent. in the buying, and 15 per cent. in the selling of his goods. What is his whole gain per cent.?

279. Six porters, M, N, O, P, Q, R, have 5 bales of goods to carry 560 yards, and each is to carry a bale a distance according to his strength. Their physical powers are represented thus: 16, 15, 14, 13, 12, and 10 respectively. How many yards will each carry a bale, and how far will each set of porters travel?

280. A and B are at two opposite corners of a square field, whose diagonal is 360 yards, and set off exactly at the same time, the same way, to go round it. A goes $28\frac{3}{4}$ yards in $2\frac{2}{3}$ minutes; B, 34 yards in 3 minutes. How many times will each go round the field before the faster overtakes the slower?

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